Least-squares DSR migration using a common angle imaging condition

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Summary

Pre-stack wavefield propagators based on generalized phase-shift operators produce accurate depth images of complex geological structures. The standard imaging condition for pre-stack "Double-Square-Root" (DSR) migration extracts the zero time wavefield at zero offset, resulting in a single depth image.

In least-squares (LS) Kirchhoff migration a smoothing constraint along the offset direction in the common reflection point (CRP) domain has been proposed to mitigate artifacts resulting from incompletely and/or coarsely sampled data. In order to make this regularization approach available to least-squares phase-shift migration techniques the common angle imaging condition (CAI) is employed. This imaging condition extracts the zero time wavefield at a set of constant offset ray parameters. This allows for an efficient computation of CAI gathers and the introduction of a ray parameter dependent smoothing in LS migration.

The wavefield propagators for modeling and migration used in the LS inversion are based on the split-step approximation of the "Square-Root operator". They can be improved by utilizing a multiple reference velocity approach.

Introduction

Least-squares (LS) migration based on Kirchhoff modeling/migration operators has been proposed in the literature to account for uneven subsurface illumination and to reduce imaging artifacts due to irregularly and/or coarsely sampled seismic wavefields (Nemeth et al., 1999; Duquet et al., 2000). Duquet et al. (2000) demonstrate how to further improve the artifact reduction of LS migration by applying a smoothing constraint along the offset domain in common reflection point (CRP) gathers. Kuehl and Sacchi (2001a) show that the concept of least-squares migration can also be applied to phase-shift pre-stack "Double-Square-Root" (DSR) migration. This is done by introducing a data weighting operator to account for the missing data.

It is known that (recursive) DSR migration can be modified for lateral velocity variations by using the split-step correction technique (Stoffa et al., 1990) or by introducing the logic of multiple reference velocities into DSR migration resulting in a Phase-Shift-Plus–Interpolation (PSPI) DSR migration (Gazdag and Squazzero; 1984).

In conjunction with the common angle imaging condition, described by Mosher and Foster (2000), these migration operators and their adjoint modeling operators are used to iteratively minimize a least-squares problem with a smoothing constraint along the ray parameter axis.

Theory

Upward and downward wavefield propagators for laterally varying media

Inverse scattering theory provides an instructive framework for deriving phase-shift WKBJ DSR modeling and migration operators in midpoint-offset coordinates (Stolt and Benson, 1986; Clayton and Stolt, 1981) that are valid for vertically varying media. Various techniques exist to modify phase-shift wave equation operators for velocity variations perpendicular to the direction of wave propagation.

The split-step correction, for instance, is based on the following approximation of the square-root terms in the DSR equation:

$$\sqrt{\frac{\omega^2}{c^2(z)}-\rho^2}+\left(\frac{\omega}{c(x)}-\frac{\omega}{c(z)}\right),$$

where $c(z)$ is the average velocity along the lateral direction at depth z, $c(x)$ is the actual background velocity of the medium and ω the frequency. The ρ^2 term is the squared horizontal wavenumber of either the sources or the receivers (Popovici, 1996). The phase-shift DSR operator, symbolically denoted as **DSR**, and the split-step correction operator S yield together a recursive downward propagator for the wavefield $\psi(z)$:

$$\psi(z+dz)=\mathbf{S}\,\mathbf{DSR}\,\psi(z).$$

In order to perform least-squares migration the adjoint upward propagator for modeling is required:

with the prime denoting the adjoint form of an operator.

$$\psi(z-dz) = \mathbf{DSR}'\,\mathbf{S}'\,\psi(z),$$

Alternatively, the DSR operator can be generalized by using multiple reference velocities for the propagation by factorizing the operator into a receiver and a source propagator: **DSR** = $\mathbf{P_r}\,\mathbf{P_s}$. We define a symbolic wavefield copying and wavefield interpolation operator **C** and **I**, respectively. The first operator creates N identical copies of the wavefield, where N is the number of reference velocities to be used. The second operator interpolates the wavefields after each has been downward extrapolated by either the source or receiver propagator using the respective reference velocities. This PSPI DSR migration operator is symbolically expressed as follows:

Again, the adjoint upward propagator is formed by interchanging the order of the operators and taking their individual adjoint forms:

$$\psi(z+dz) = \mathbf{I_s}\;\mathbf{P_s}\;\mathbf{C}\,\mathbf{I_r}\;\mathbf{P_r}\;\mathbf{C}\,\psi(z).$$

$$\psi(z-dz) = \mathbf{C}'\,\mathbf{P_r'}\,\mathbf{I_r'}\;\mathbf{C}'\,\mathbf{P_s'}\,\mathbf{I_s'}\;\psi(z).$$

This operator corresponds to the windowed Non-Stationary-Phase-Shift (NSPS) operator described by Margrave and Ferguson (1999) extended by the adjoint of the linear interpolation operator of PSPI and applied to DSR upward wavefield propagation.

The common angle imaging condition

Mosher and Foster (2000) suggest a common angle imaging (CAI) condition for pre-stack depth migration that replaces the summation over frequency and offset wavenumber for the standard imaging condition of DSR migration with multiple summations over the offset ray parameter $p = k_h/\omega$. In two dimensions this amounts to summing along radial lines in the (k_h, ω) domain with slope p_h . Rather than a single image at each depth step, multiple images are produced, one for each offset ray parameter. Again, the corresponding modeling operator is defined as the adjoint of the CAI imaging condition.

Inversion of incomplete data

The split-step or the NSPS/PSPI downward/upward propagator pair in conjunction with the CAI imaging/modeling operator is used to invert the following linear system in a least-squares sense:

$$\mathbf{d} = \mathbf{Am} + \mathbf{n},$$

where $\mathbf{d}$ is the (noisy) incomplete data, $\mathbf{A}$ the modeling operator and $\mathbf{m}$ the model that contains a set of constant-angle image gathers $\mathbf{m}_p$. The error term $\mathbf{n}$ represents modeling errors, missing data and noise.

$$\min F(\mathbf{m}) = \|\,\mathbf{W(d-Am)}\,\|_2 + \lambda \sum_p \|\,\mathbf{m}_p - \mathbf{m}_{p-1}\,\|_2,$$

We minimize the following objective function using a conjugate gradient (CG) algorithm:

where $\mathbf{W}$ is a diagonal weighting operator with zero weights for dead data traces and non-zero weights for live traces according to their noise level (Kuehl and Sacchi, 2001a) and λ is a trade-off parameter. The second term in the objective function imposes a relative smoothing constraint that suppresses undesired discontinuities in the ray parameter direction due to missing data and noise (compare also to Kuehl and Sacchi, 2001b).

Example

We have generated a midpoint-offset data set using pre-stack Kirchhoff modeling based on a simple syncline structure. We note that to test the robustness of the inversion algorithm the synthetic data has been generated by a different type of operator than the one used in the inversion algorithm. The Figures 1 and 2 show the migrated CAI gathers and a close-up of 6 migrated midpoints of incomplete noisy pre-stack data, respectively (60% of the pre-stack data has been randomly removed prior to migration). The CAI gathers are partially discontinuous along the ray parameter axis. Imaging artifacts and noise are apparent. The least-squares DSR migration using the CAI condition has restored continuity along the ray parameter axis and reduced the noise level and much of the artifacts due to missing data after 4 iterations of the CG algorithm (Figures 3 and 4). The example has been designed such that it can be directly compared to examples given in our related paper (Kuehl and Sacchi, 2001b).

Conclusions

Least-squares DSR migration using a common angle imaging condition and a relative smoothing constraint along the ray parameter domain reduces artifacts due to missing data and enhances the signal to noise ratio of CAI gathers. The recursive modeling/migration operators used for our LS migration can be modified using known correction techniques like the split-step approximation or the PSPI/NSPS approach. The proposed technique is therefore suitable for LS depth migration.

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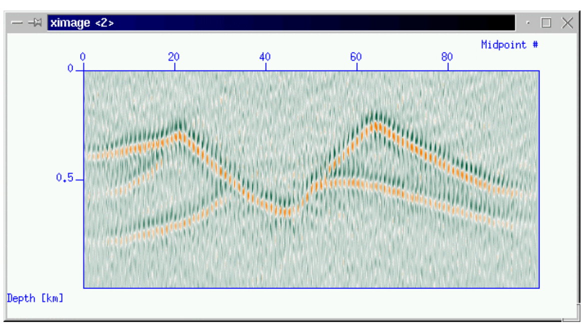

Figure 1: CAI gathers of the incomplete noisy data set (60% of the data has been randomly removed).

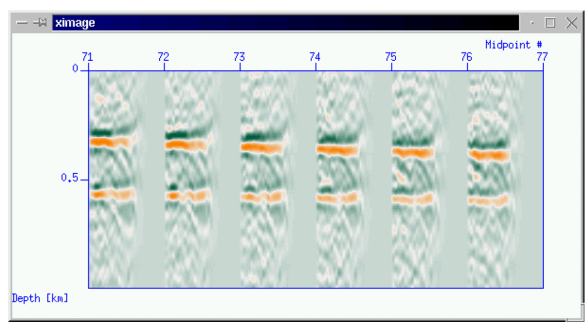

Figure 2: CAI gathers for the midpoints 71 to 77 of the incomplete and noisy data.

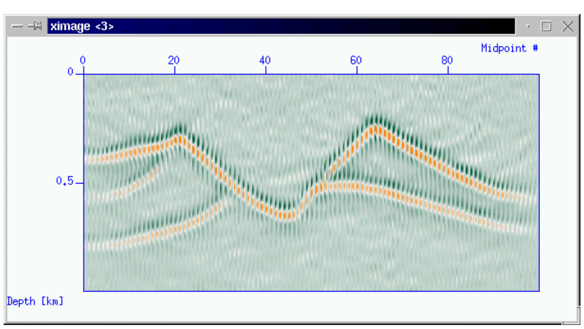

Figure 3: CAI gathers after 4 iterations of the least-squares migration algorithm.

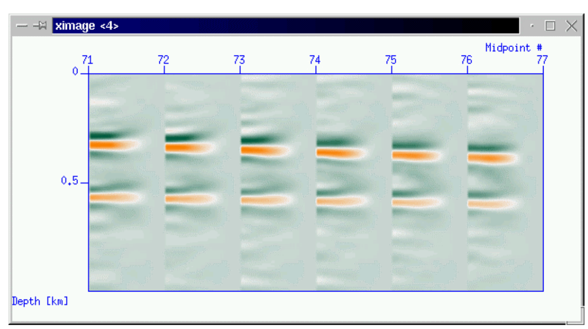

Figure 4: CAI gathers for the midpoints 71 to 77 after 4 iterations of the least-squares algorithm.