Anelliptic pre-stack time migration

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Summary
The imaging of deep-offshore seismic data acquired with long streamers involves appropriate algorithms, which can take into account the non-hyperbolic moveout of the reflection events. Moreover, the exploration on areas where the anisotropy behavior of the media is observed, the anellipticity of all time-processing operators is required.

Since the anelliptic shifted hyperbola moveout equation (Siliqi and Bousquié, 2000) seems to describe adequately the effects corresponding to bedding and VTI anisotropy, we propose to generalize it to Kirchhoff pre-stack time migration. The effective VTI anellipticity η is the key parameter describing the difference from elliptical shape of traditional operators.

Our migration operators, based on anelliptic shifted hyperbola are pointing out the reduction of migration-errors of dipping events, in comparison with the use of effective VTI Alkhalifah moveout equation. The lateral misfit is drastically improved.

It appears that the anelliptic shifted hyperbola moveout equation is an excellent tool for introducing the effective anellipticity of the media into the time processing.

Introduction
Due to its ability to focus highly structured events well, full pre-stack migration is becoming more and more the favorite technique for time imaging. The Kirchhoff migration, commonly based on hyperbolic moveout, has become a very robust and versatile tool.

Nevertheless, in order to properly image the reflection events, some situations require a more accurate description of the kinematics of the rays. Indeed the occurrence of long offset traces in deep offshore seismic data presents such a challenge, even when no lateral velocity variation exists. In such data the reflection curves often exhibit non-hyperbolic move-out. This kind of behavior may simply illustrate the departure from the Dix approximation due to the layering effect, or it may reflect the existence of in-situ anisotropy in the sub-surface.

Our goal is to improve upon the classical hyperbolic assumption by better approximating the true kinematics of the rays. By limiting ourselves to horizontally layered VTI media we can devise an effective traveltime formulation that will not compromise the robustness of the standard Kirchhoff approach.

It has been shown that time processing in VTI media depends only on two parameters: V_{nmo} and η (Alkhalifah, 1995). V_{nmo} represents the short spread curvature of the reflection curve for the horizontal reflector; the anellipticity parameter η controls the non-hyperbolic behavior of move-out and the focusing of high dips in DMO and migration. The fourth order Taylor's expansion of traveltime-offset relation has been customized for homogeneous VTI media adding an asymptotic behavior of NMO equation (Alkhalifah and Tsvankin, 1995):

$$t^2 = t_0^2 + \frac{x^2}{v_{nmo}^2} - \frac{2\eta x^4}{v_{nmo}^2\left(t_0^2 v_{nmo}^2 + (1+2\eta)x^2\right)} \quad (1)$$

In the case of inhomogeneous VTI media, V_{nmo} and η are effective values.

Siliqi and Bousquié (2000) showed that the extension to anisotropy of the shifted hyperbola (Castle, 1994) yields a more accurate approximation of reflection than the expression of Alkhalifah (1):

$$t = \frac{S-1}{S} t_0 + \sqrt{\left(\frac{t_0}{S}\right)^2 + \frac{x^2}{S V_{nmo}^2}} \quad (2)$$

The Castle layering term S is extended to the effective anellipticity η:

$$S = 1 + \eta \quad (3)$$

Adjusting the layering effect of non-hyperbolic moveout to take into account the in situ anisotropy appears more efficient than the extending of VTI behavior of the homogeneous media to the layered model.

This new VTI moveout equation allowed Siliqi and Bousquié (2000) to propose a coherent DMO operator too. In the present paper the authors would like to show that the anelliptic shifted hyperbola approach continues to be advantageous even in the case of pre-stack time migration.

Migration operators based on VTI Alkhalifah and anelliptic shifted hyperbola moveout equations.

The kernel of Kirchoff pre-stack time migration algorithm consists in computing traveltime maps based on diffraction equation. The standard approach is using NMO equation (locally layered media) with the effective V_{nmo} function. In the case of VTI media, the analytic Green functions have to be anelliptic. Two different equations could be used: Alkhalifah (1) or anelliptic shifted hyperbola (2 and 3).

In order to assess the effectiveness of both approaches, we modeled a set of synthetic data computed by an anisotropic ray-tracing program (ANRAY-University of Prague). The VTI model is composed of six layers (table 1), with four of them exhibiting anisotropic properties. The resulting effective anellipticity η at the bottom is 0.208.

Introducing anellipticity into the diffraction equation we expect improving the high dips of the migration operator.

Z (m)	VP_0 (m/s)	VS_0 (m/s)	•	•	•
450	1500	450	0.00	0.00	0.00
900	1800	594	0.30	0.12	1.70
1200	2500	950	0.19	0.08	0.73
1800	3800	2090	0.21	0.11	0.32
2200	2200	660	0.00	0.00	0.00
2800	3000	1350	0.23	0.07	0.79

Table 1: Layered model used by ANRAY to generate synthetic data. The effective parameters derived at the last interface of the model are V_{nmo}=2614 m/s and η=0.208

A marked decrease in lateral and vertical location errors of the migrated dipping events allows pointing out the accuracy of the diffraction equations.

The moveout advantage of the anelliptic shifted hyperbola over the Alkhalifah VTI equation is preserved even in the diffraction equation case. Figure 1 shows that the vertical time-migration-error for dips higher than 0.4 ms/m, is divided by two, in the case where anelliptic shifted hyperbola equation is used to generate the traveltime map. The effective Alkhalifah VTI equation is an acceptable approximation of migration time operator for dipping events less than 0.35 ms/m.

The same improvement can be observed on lateral misfits (figure 2). It is important to point out that the error is less than 70 meters for the highest dips, whereas it exceeds the 300 meters in the case of Alkhalifah VTI diffraction.

We developed here the migration error aspect only. However, the focusing of large offset data is always improved when anelliptic shifted hyperbola is used. The conclusions on non-hyperbolic moveout drawn by Siliqi and Bousquié (2000) apply to pre-stack migration as well.

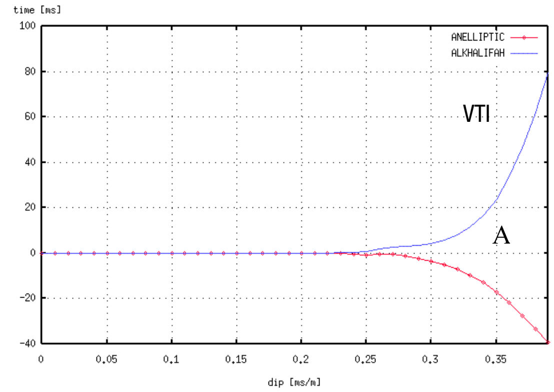

Figure 1: VTI layered synthetic data.
Vertical time-migration-error as a function of zero-offset dip: A) Anelliptic shifted hyperbola; VTI) Alkhalifah VTI.

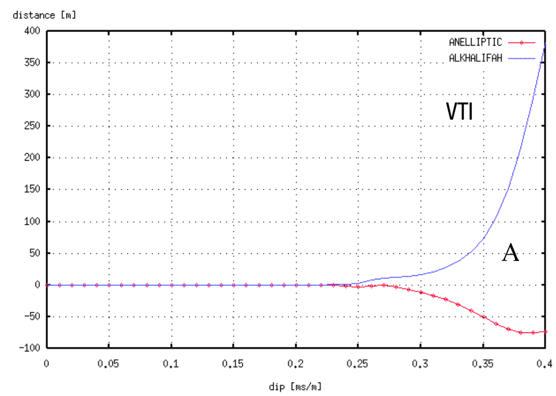

Figure 2: VTI layered synthetic data.
Lateral distance-migration-error as a function of zero-offset dip: A) Anelliptic shifted hyperbola; VTI) Alkhalifah VTI..

Conclusion

The standard time processing operators are evolving to include anellipticity. The reason is not just to extend the Dix theory to layered media. The exploration on critical areas where the presence of the anisotropy of the media is proven requires taking it into account.

Siliqi and Bousquié (2000) observed that rectifying the layering effect of non-hyperbolic moveout in order to take into account the in situ anisotropy offers greater potential than extending the VTI behavior of the homogeneous media to the layered model. The resulted anelliptic shifted hyperbola allowed a better moveout correction than the well-known VTI Alkhalifah NMO equation.

The migration operator based on the anelliptic shifted hyperbola equation produces lateral and vertical migration errors smaller than these ones caused by Alkhalifah VTI moveout, especially in the case of high dips. A synthetic data case study pointed out above all the enhancement of the lateral misfit.

We believe that a homogeneous anelliptic theory based on effective time-processing parameters is a powerful tool of the time-focusing practice.

References

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