Parallel Implementation of an Integral Wavefield Extrapolator

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Summary

Highly accurate seismic depth imaging (migration) is required by the oil and gas exploration industry in regions of complex geology. Fourier-domain depth-imaging techniques provide the needed accuracy although the computation cost can be large when the lateral velocity variation is rapid. Parallel computer network provide an efficient solution to overcome the computational barrier of Fourier-domain imaging techniques. In this paper, a new Fourier-domain imaging technique, based on nonstationary filtering and wavefield extrapolation theories, is described. The parallel version of the algorithm is implemented using C++ and Fortran 90 on an Alpha cluster workstations. The Message Passing Interface (MPI) library was used for data distribution and collection. The algorithm, found to be accurate but slow on serial machines, can achieve a speed acceptable for most industrial applications.

Introduction

Seismic depth imaging repositions the scattered energy from the subsurface to its correct spatial location according to a velocity model. Produced by a true-amplitude depth imaging technique, a subsurface image also carries amplitude information that is directly proportional to reflection coefficients.

Kirchhoff depth-imaging techniques are very common in the exploration industry due to robustness and high efficiency. Theoretically the method is based on Green's theorem and it is implemented by summation along iso-time surfaces computed by raytracing. Difficulties arise when it is used in regions of complex geology. Fourier-domain imaging algorithms, especially for complex media, are often slower than Kirchhoff-type techniques. However, they provide a higher accuracy solution by allowing the energy to propagate in all possible directions instead of only the Snell's law paths. Fourier-domain techniques can be made efficient by making reasonable approximations and operating in efficient data domains.

Fourier domain wavefield extrapolation and integral wavefield extrapolators

The wavefield at depth z can be obtained by extrapolation the wavefield at depth 0, given knowledge of the velocity field between 0 and z. Consider a mono-frequency wavefield at depth 0, $\varphi(\mathrm{k}_x, z=0, \omega)$, which has been 2D forward Fourier transformed to frequency-wavenumber (ω,k_x) domain, the wavefield at depth z can be written as (Gazdag, 1978),

$$\varphi(\mathrm{k}_x, z, \omega) = \varphi(\mathrm{k}_x, 0, \omega)\alpha(\mathrm{k}_x, z, \omega),, \tag{1}$$

where $\alpha(\mathrm{k}_x, z, \omega) = e^{\pm iz\sqrt{\frac{\omega^2}{v^2}-k_x^2}}$ is the (ω,k_x) domain wavefield extrapolator operating from 0 to *z.* The plus and minus signs denote upward and downward extrapolation when the z-axis points downward. For heterogeneous media, the extrapolation operator is known only approximately and the step size is limited. The extrapolation is done recursively with small steps, so that local wave propagation can be approximated by homogeneous and isotropic propagation theory.

There are several ways to compute an approximate wavefield extrapolator for heterogeneous media. Typical algorithms are phase-shift-plus-interpolation (PSPI) originally developed by Gazdag and Sguazzero (1984) and the split-step Fourier algorithm originally developed by Stoffa et al. (1990), which is also called the phase-screen algorithm by Wu and Huang (1992). There are many further developments of the above algorithms in order to deal with extreme lateral velocity variation and achieve high efficiency (Jin and Wu, 1998; Popovici, 1996; etc.) These algorithms have also been implemented on parallel computers, for example, Tanis and Stoffa (1997).

Nonstationary Fourier wavefield extrapolators

Using a complete set of reference velocities, the PSPI algorithm becomes an integral over horizontal wave number k_x, which performs wavefield extrapolation simultaneously with an inverse Fourier transform (Margrave and Ferguson 1999a)

$$\psi(\mathrm{x}, z, \omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\alpha(\mathrm{x},\mathrm{k}_x,z,\omega)\varphi(\mathrm{k}_x,0,\omega)\exp(-i\mathrm{k}_x x)d\mathrm{k}_x\,, \tag{2}$$

where $\alpha(\mathrm{x},\mathrm{k}_x,z,\omega) = e^{\pm iz\sqrt{\frac{\omega^2}{v^2(x)}-k_x^2}}$ is the nonstationary wavefield extrapolator. $\psi(\mathrm{x}, z, \omega)$ is the (ω,x) domain expression of the wavefield at depth z and $\varphi(\mathrm{k}_x,0,\omega)$ is the (ω,k_x) domain expression of the wavefield at depth 0. Though there is no interpolation in equation (2), the expression PSPI is used for equation (2) because it is a limiting form of Gazdag's PSPI.

Equation (2) has a complementary form, called nonstationary phase-shift (NSPS), which performs wavefield extrapolation simultaneously with a forward Fourier transform (Margrave and Ferguson, 1999a)

$$\varphi(\mathrm{k}_x, z, \omega) = \int_{-\infty}^{+\infty} \alpha(\mathrm{x}, \mathrm{k}_x, z, \omega)\psi(\mathrm{x}, 0, \omega)\exp(i\mathrm{k}_x x)d\mathrm{x}, \tag{3}$$

where $\alpha(\mathrm{x}, \mathrm{k}_x, z, \omega)$ is expressed as before.

Equation (2) can be approximately computed by matrix-vector multiplication, which can be written as

$$\underline{\psi}_z = \underline{\underline{A}}\underline{\varphi}_0, \tag{4}$$

where $\underline{\varphi}_0$ and $\underline{\psi}_z$ are column vectors representing a mono-frequency wavefield in (ω, k_x) domain at depth 0 and the extrapolated wavefield in (ω, x) domain at depth z, respectively. Matrix ***A*** is the combination of the wavefield extrapolator and the simultaneous inverse Fourier-transform kernel

$$\underline{\underline{A}} = \begin{pmatrix} e^{i\left(\pm z\sqrt{\omega^2/v_1^2 - k_{x1}^2} - x_1 k_{x1}\right)} & . & . & . & e^{i\left(\pm z\sqrt{\omega^2/v_1^2 - k_{xn}^2} - x_1 k_{xn}\right)} \\ . & . & . & . & . \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ . & . & . & . & . \\ e^{i\left(\pm z\sqrt{\omega^2/v_n^2 - k_{x1}^2} - x_n k_{x1}\right)} & . & . & . & e^{i\left(\pm z\sqrt{\omega^2/v_n^2 - k_{xn}^2} - x_n k_{xn}\right)} \end{pmatrix}. \tag{5}$$

Similarly, integral (3) can also be approximated by matrix-vector multiplication, however with k_x varying in the column direction while x varying in the row direction.

Margrave and Ferguson (1999b) showed that the NSPS and PSPI can be naturally combined into a symmetric wavefield extrapolator (SNPS) by first performing NSPS for the upper half z and PSPI for the lower half z within a single step. A Taylor series derivation of PSPI and NSPS and related error analysis showed that the first-order errors of PSPI and NSPS oppose one another, so that SNPS has a smaller error and is more stable than either PSPI or NSPS alone (Margrave and Ferguson, 2000).

Parallel implementation on MACI Alpha Cluster

The integral SNPS extrapolation algorithm was implemented on the Multimedia Advanced Computational Infrastructure (MACI) Alpha Cluster at the University of Calgary. The cluster consists of 128 Compaq Alpha workstations and each single user can use up to 16 CPUs for a single computing task. A general network configuration of the Alpha Cluster is shown in figure 1.

The Message Passing Interface (MPICH1.2) was used for parallel implementation. Each computing node was assigned a single shot gather migration task. It took about 8 hours to migrate the 240 shot gathers on 16 XP1000 workstations. Figure 2 shows the band-limited (0-20 Hz) reflectivity and the subsurface image computed by the integral SNPS.

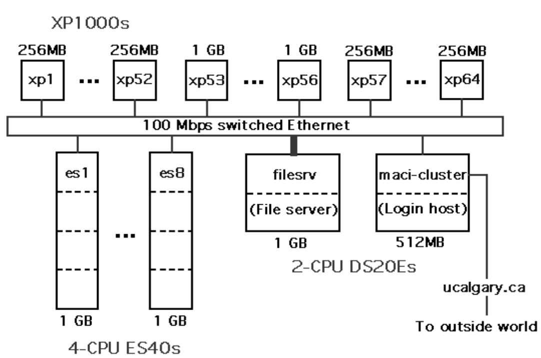

Figure 1. The general network configuration of the Alpha Cluster at the University of Calgary.

Conclusions

Integral SNPS extrapolator produces very accurate subsurface image. Direct computation of equation (2) and (3) can be made faster by utilizing the symmetric properties of the extrapolator and the forward/inverse Fourier transform kernels. Faster while less accurate numerical algorithms to compute the square root and exponential function can also be used to make the algorithm more efficient.

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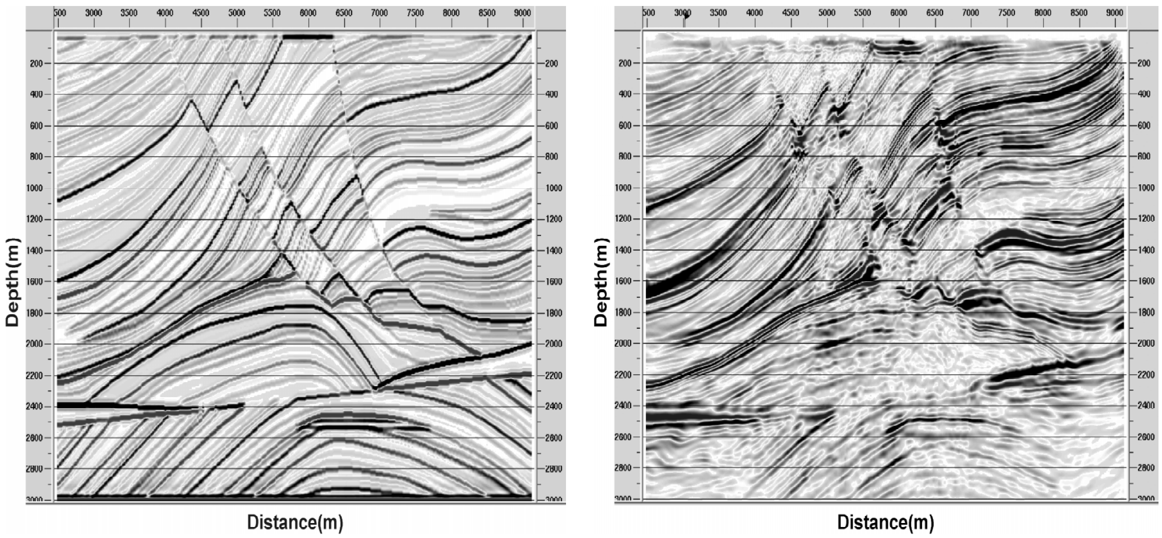

(a) (b)

Fig. 2. (a) 0-20 Hz band-limited reflectivity of the Marmousi model and (b) the subsurface image computed by the integral SNPS

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