On the use of naturally occurring electromagnetic transients as an energy source for earth resource exploration

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Summary

Thunderstorm activity produces large amounts of electromagnetic energy which is trapped within the earth-ionosphere waveguide. Energy from activity on a near global scale produces a low level, quasi-continuous source field. Very large, or equivalently, relatively nearby lightning discharges produce individual transient events which stand out above the low level background field by more than an order of magnitude. Therefore, substantial increases in signal-to-noise ratio can be realized by recording exclusively signals of a transient nature. However, the transient events are strongly linearly polarized, the polarization diversity of which can affect the estimation of earth response curves.

We introduce a method by which an adaptive time domain averaging of the transient waveforms is conducted. In its static form, this was termed polarization stacking (Kosteniuk and Paulson, 1988).

Our theoretical analysis has shown that the bias in the impedance tensor estimate converges to zero super exponentially in stacked signal-to-noise ratio with the angle between the sources acting as a damping factor in the convergence.

Verification of our theoretical analysis was accomplished with Monte-Carlo simulations using real data. It is found that, given typical polarization characteristics of transient sources, adaptive polarization stacking outperforms conventional least squares and remote-reference analyses in the rate of bias convergence.

Introduction

Naturally occurring fluctuations in the earth's magnetic field induce sub-surface currents to flow, the size of which are determined by the earth's local resistivity structure. By making frequency dependent measurements, resistivity information as a function of depth is obtained with higher frequencies resolving shallower features and lower frequencies deeper ones. The geophysical technique which makes use of these naturally occurring fluctuations is termed magneto-tellurics or simply MT (Cagniard, 1953. Tikhonov, 1950). When measurements are made in the bandwidth above 10 Hz, the modifier audio frequency is added denoted simply AMT (Strangway et al., 1973).

The dominant source of naturally occurring energy in the ELF/VLF [3] bandwidth is due to lightning discharges (Pierce, 1977. Volland, 1982). This naturally occurring energy gives rise to a source field which consists of two main parts; a low level, quasi-continuous component which is due to the random incoherent sum of near-global lightning activity, superimposed on which are *individual* very strong transients which arise in electromagnetic radiation from either relatively nearby and/or very large current moment lightning discharges (Tzanis and Beamish, 1987. Jones and Kemp, 1971). Note that nearby is defined relative to global waveguide attenuation, for example, nearby at 100 Hz may be 6000 km whereas nearby at 5000 Hz may be 1500 km.

Both signal sources can be used to estimate the impedance tensor but substantial increases in signal-to-noise ratio (SNR) are afforded by recording exclusively transients. This is due to the large amplitude of the transient events and their high degree of time localization. Therefore, a time localized recording technique is used (Hoover et al., 1978. Kosteniuk and Paulson, 1988. Vozoff, 1991) to record transient energy with the best possible SNR. This is in contrast to conventional MT/AMT practice, where data is acquired on a continuous basis for some averaging or integration time, typically lasting several seconds to several minutes (Strangway et al., 1973).

To properly process the transient data, we carry out an adaptive time domain averaging to enhance SNR and thereby reduce bias. However, to realize the ideal root-N enhancement in SNR through stacking we require that the noise be independent across recordings and that the same underlying signal be present in every noisy recording. With lightning transients, each received waveform is different, reflecting the amount of attenuation and dispersion suffered in propagating to the measurement site. Also, the magnitude of the current moment in each discharge, and its variation as a function of time affects the shape of each transient waveform. Therefore, each transient is indeed different, but they all have a well defined extremum, each of which can be aligned and summed in phase resulting in an SNR enhancement. The improved signal-to-noise ratio (ISNR) is bounded below by SNR and above by the product of root-N and SNR.

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3 ELF, Extremely Low Frequency (3 Hz to 3 kHz). VLF, Very Low Frequency (3 kHz to 30 kHz) nearby

Theory

The impedance tensor $Z(\omega)$, where ω denotes angular frequency, is a rank 2 tensor which maps magnetic field oscillations $H(\omega)$ onto electric field oscillations $E(\omega)$ with a right-handed co-ordinate system defined according to the geomagnetic convention of positive x to the north, positive y to the east and positive z down;

$$E_x(\omega) = Z_{xx}(\omega) \cdot H_x(\omega) + Z_{xy}(\omega) \cdot H_y(\omega)$$
$$E_y(\omega) = Z_{yx}(\omega) \cdot H_x(\omega) + Z_{yy}(\omega) \cdot H_y(\omega)$$

The regression problem is then one of estimating the complex values of the impedance tensor as a function of frequency. This has traditionally been done by minimizing the sum square error. The least squares (LS) solution involves estimation of auto and cross-spectral densities, unfortunately, auto-spectral densities are biased estimators with finite SNR data, cross-spectral densities may also be biased if the noise is correlated between the two arguments. The fact that biased estimator(s) are used in the LS analysis not surprisingly causes the LS regression to be biased.

This prompted the development of the remote-reference (RR) technique (Goubau et al., 1978, Gamble et al., 1979). Two additional channels are measured so that the regression can be done entirely with cross-spectral densities, typically the horizontal components of the magnetic field are measured at a reference site, thus supplying the two extra reference channels. If the noise between local and reference sites is uncorrelated, unbiased estimation of the impedance tensor may be possible with the RR method.

However, to truly obtain unbiased estimates of $Z(\omega)$ with the RR technique we also need circular polarization in the local and reference source fields and large sample size so that the sample mean approaches the true ensemble mean (Gamble et al., 1979). Neither of these conditions are generally realized when one records transients. The bearing distribution of the entire group of linearly polarized events is typically ‘clumpy’. We may have two or three dominant storm centres with angles between sources, in some cases as small as 20 degrees. The number of events recorded may be 50 or less in some cases. Therefore, when one works exclusively with transients we generally do not have a uniform distribution of bearings nor do we necessarily have a large number of events. Both of these factors violate the conditions needed for the RR method to give unbiased estimates of $Z(\omega)$.

The two-point formula

We have two complex equations and four complex unknowns, therefore, we need at least two independent measurements of $E(\omega)=Z(\omega)\cdot H(\omega)$ to solve for $Z(\omega)$. If we consider the two event case we have the following, where the dependence on ω is implicit.

$$E_{xi} = Z_{xx} \cdot H_{xi} + Z_{xy} \cdot H_{yi}$$
$$E_{yi} = Z_{yx} \cdot H_{xi} + Z_{yy} \cdot H_{yi}$$
$$i = 1,2$$

Solving this pair of 2x2 linear systems with Cramer’s rule yields the two point formulas for the four elements of $Z(\omega)$. Note that the conventional LS estimate, both 5 channel and 6 channel RR estimates (Goubau et al., 1978) collapse to the two point formula in the two event case.

However, prior to using the two-point formula, we perform the signal averaging or stacking, resulting in two sets of SNR enhanced events. To accomplish this, the transients are separated into two groups according to their bearing, this is done in such a manner so as to approximately equalize the signal energy in each group. In this fashion, the location of the stack boundaries is adaptive so that we make the best possible use of the data given the bearing distribution of the current set of recorded transients.

Once the stack membership has been assigned, the only decision remaining is how to add each transient into its respective stack. We have two choices, we can sum in the current transient as it is, or we can multiply by –1 first and then perform the summation. The decision is made by noting which polarity yields the largest resultant stack amplitude.

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