A hybrid Linear-Hyperbolic Radon transform

Daniel Trad[1], Mauricio D. Sacchi [2], and Tadeusz J. Ulrych[1]
[1]The university of British Columbia, [2]University of Alberta

Summary
The main application of the Radon transform (RT) is to map events with different curvature in the data space to different areas of a model space, where these events can be separated or filtered before going back to the data space. This ability of the RT is in practice diminished by the fact that data contain events with different shapes. Linear events due to ground roll (GR), for example, are superimposed on hyperbolic events from reflections. Both events cannot be simultaneously focused with the standard RT. A linear RT localizes the GR and not the reflections, and vice-versa, a parabolic RT for NMO corrected data does not localize the GR. In this paper we present a hybrid Radon transform to perform a sparse representation in the model space of data containing events with different shape. The forward modeling is performed by two operators, each one with a different set of basis functions. The same idea can be extended to more than two operators for data with more complicated combination of events.

Introduction
The Radon transform (RT) is usually implemented to focus events with three different shapes: linear, parabolic and hyperbolic. The linear RT (LRT) has been used mainly for processing directly in the slant stack domain (multiple attenuation, stack, migration, etc.). The parabolic and hyperbolic RT (PRT and HRT) are used for multiple attenuation and sometimes for interpolation and aperture extension. In all these cases there is an operator that maps the model space to the data space, and its adjoint (or conjugate transpose) that performs the reverse mapping, from data to model.

However, there is no reason why, when inverting for the Radon model, the data should be modeled by a single class of operator. A combined operator has more flexibility to fit different shapes in the data with more parsimony and hence, to better separate different characteristic of data. In this paper we apply a hybrid operator for ground roll removal to understand whether it is possible to map different events to different spaces. We will show that this is indeed possible, as long as the two spaces do not overlap too much each other, in other words, the basis functions look different and a sparse inversion is applied. In this case a very simple filtering procedure can be applied by inverting the data for the combined model with all the operators, and reconstruct the data with only the desired operator.

A multiple representation of the same data event in different model areas is undesired. A further improvement is to focus the solution using model weights. These weights allow the mapping of a particular event with preference for one of the operators.

Theory
Whatever the shape of the basis functions, the Radon Transform (RT) can be defined in terms of summation along paths. By defining an operator **L** that performs such a summation, the Radon transform is in general computed by inversion of the following matrix equation

$$\mathbf{d}=\mathbf{L}\mathbf{m}$$

where, **m** is the Radon transform of the data **d**. Thorson and Claerbout (1985) proposed the computation of the Radon transform by a stochastic inversion technique that is able to retrieve a sparse Radon panel. A similar technique has been developed by Sacchi and Ulrych (1995) to invert time-invariant Radon operators.

It is often the case that strong linear events are present in the data superposed to hyperbolic reflections. An appropriate forward modeling of these data is given by

$$\mathbf{d}=\mathbf{L}_1\mathbf{m}_1+\mathbf{L}_2\mathbf{m}_2$$

where the forward operator is $[\mathbf{L}_1\ \mathbf{L}_2]$ and the adjoint operator is

$\begin{bmatrix}\mathbf{L}_1^H\\ \mathbf{L}_2^H\end{bmatrix}$, where $\mathbf{L}^H_{1,2}$ are the complex conjugate transpose matrices of $\mathbf{L}_{1,2}$. This combined operator and its adjoint is all we need to solve for the model $\mathbf{m}=[\mathbf{m}_1\ \mathbf{m}_2]^T$ using, for example, a least square conjugate gradient algorithm (Claerbout, 1992).

In the examples in this paper we use $\mathbf{L}_1$ as the Linear RT operator that maps lines to points, and $\mathbf{L}_2$ as the pseudo hyperbolic Radon transform (PHRT) operator of Foster and Mosher(1992). In general we could apply any combination of Radon operators. This kind of hybrid RT could be applied whenever the data have events with different shapes. Similar approaches are used to decompose images in a dictionary of basis or atoms, called (basis pursuit), where the decomposition is done using an optimization method with constrains given by a l_1 norm for the model, leading to sparse representations (Chen et. al., 1998)

example

An example is shown with real data set, the shot gather number 25 from Yilmaz (1987) (Fig.1a). Because a shot gather is in general asymmetric, any RT other than linear requires a different model for negative and positive offsets (this is another problem where a RT with two different operators can be used). Fig 1b shows 320 traces in the Radon domain. The first 160 traces correspond to the negative offsets, the 161-320 traces correspond to the positive offset. Each half shows the two spaces side by side, left hand side is the linear RT, right hand side is the pseudo hyperbolic RT. By applying a muting to the pseudo-hyperbolic space, or equivalently, by using only the linear operator of the hybrid Radon transform, the linear ground roll can be predicted (Fig. 1c) and separated for subtraction (Fig. 1d).

A very small part of the ground roll remains in the small offset traces, and can perhaps, be removed with an additional filtering. This part of the data has not been predicted for two reasons: the slopes become very large (approaching infinity) and the linear events are heavily aliased. In fact *f-k* filtering has similar difficulty in removing the ground roll in the near zero offset traces.

Discussion and Conclusions

In this paper a combined linear - pseudo hyperbolic Radon transform has been introduced, the purpose of which is to decrease the nullspace of the transform, and to increase sparseness in the model. Applications of this hybrid approach could be coherent and/or incoherent noise removal. Real and synthetic examples have shown a successful separation of events with different shape into the two Radon domains.

Although a two operator approach is quite simple, the sparseness constraint is essential to prevent the linear event to be mapped as an incomplete hyperbolic event and vice versa. If a simple l_2 constraint is used, the method resembles the method of frames, where the solution is an average of all possible solutions. If a l_1 constraint is imposed, a theoretically perfect separation is possible. In this case our method resembles a basis pursuit approach (Chen et. al., 1998).

As the data are scanned for all the basis functions, the separation is more difficult when the same event can map to different areas of the model space. For example we tried, unsuccessfully, to apply the idea to two pseudo hyperbolic operators with different focusing depths. Model weights may be able to overcome this problem, but we have not been completely successful in this regard. A linear programming algorithm might produce better results, at the expense of more computation time.

A successful separation can be obtained in reasonable time using a iterative Re-weighted Least Squares method (Scales and Smith, 1997), and, if possible, by choosing a range of parameters for the operators that produce different basis functions.

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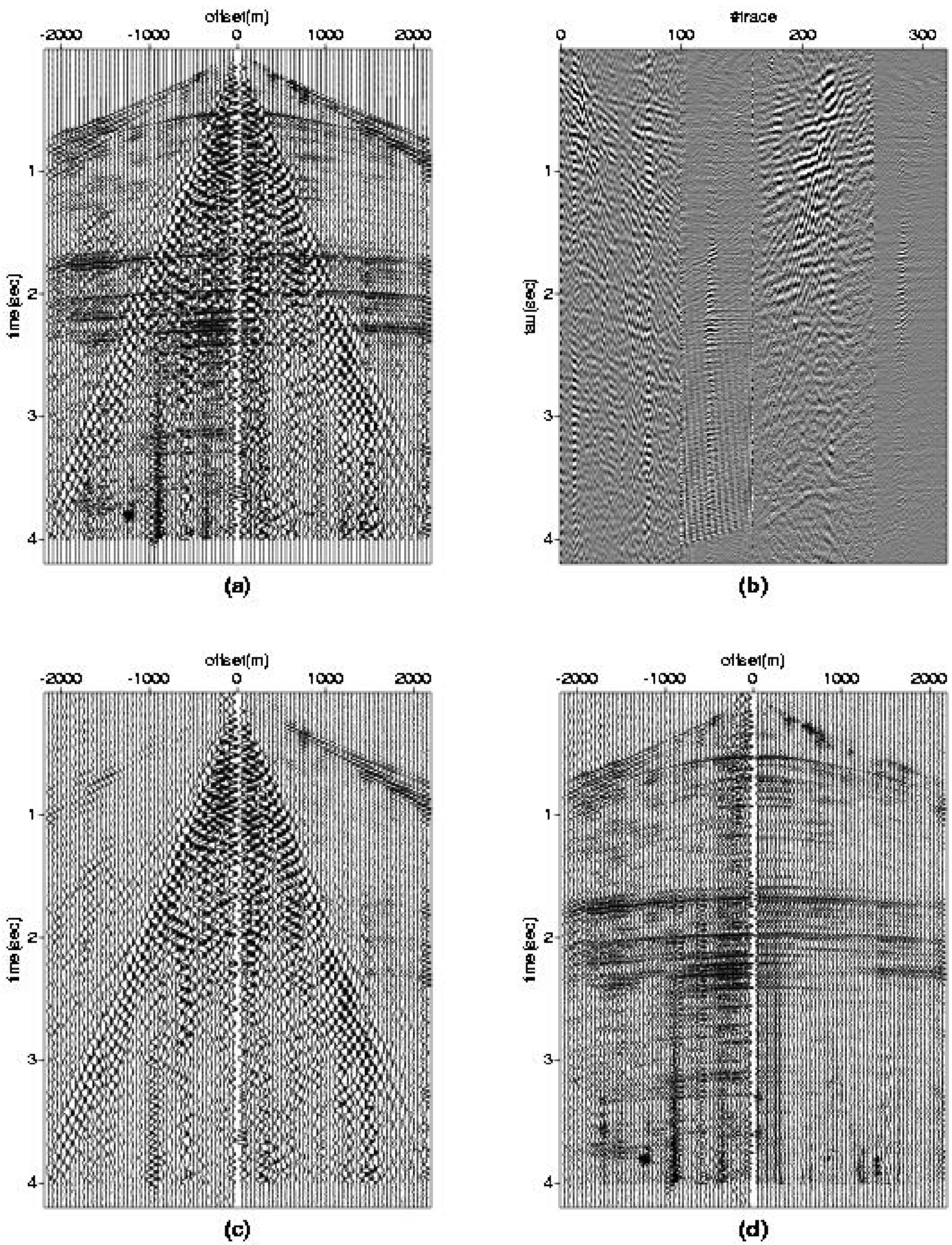

(a) Shot gather. (b) Combined Linear + Pseudo Hyperbolic Radon domain, for negative offsets (traces 1-160), and for positive offsets (traces 161-320). (c) Recovered gather from linear space only. (d) Signal obtained by difference between (a) and (b).}