Multiple removal using hyperbolic Radon operators

Qiang Li and M D. Sacchi, University of Alberta

Abstract
We developed an algorithm to compute high resolution Hyperbolic Radon Transforms (HRT). High resolution is achieved by introducing a regularization norm that mimics a sparse distribution of reflections in the velocity space. Specifically, we have adopted the Huber norm to regularize the HRT inversion. The algorithm is tested with a Gulf of Mexico subsalt data set.

Introduction
Multiple suppression techniques based on moveout discrimination constitute a cost efficient way of removing undesired reflections. Hampson (1986) proposed a method to eliminate multiples using Parabolic Radon transforms. In this approach, hyperbolic events are approximated by parabolas (after application of NMO correction). The parabolic Radon transform is a time-invariant operator. Therefore, it can be efficiently implemented in the frequency domain. On the other hand, HRT are time-variant operators that need to be applied in the time-offset domain (Thorson and Claerbout, 1985). This implementation can be computationally expensive. In 1995, Sacchi and Ulrych proposed an implemention of the parabolic Radon transform using a regularization strategy that would serve to enhance the focusing power of the transform. They adopted the Cauchy norm as a way of inverting high resolution parabolic Radon transforms. However their implementation in the *f-x* domain did not allow for sparseness constraints in the time. Based on this observation, Cary (1998) proposed a Radon transform where the sparseness is imposed in the time-*q* (*q*: residual move-out at far-offset) domain.

Like Cary, our constraints are in the $\tau - q$ domain, but our operator is a Hyperbolic Radon Transform instead of a Parabolic Radon transform. The regularization is carried out using the Huber criterion (Huber, 1981). The latter allows us to improve focusing power of reflections in the transformed space.

Theory
We consider the data as a result of a transformation from a space **M** to the data space **D**:

$$\mathbf{d} = \mathbf{L}\,\mathbf{m} \quad , \tag{1}$$

where **L** indicates the hyperbolic stack operator or Hyperbolic Radon transform. This is an operator that converts a spike in model space into a hyperbola in data space. For completeness, we will consider that the operator **L** can be decomposed into two operators:

$$\mathbf{L} = \mathbf{W}\,\mathbf{H} \ , \tag{2}$$

where **H** is an operator that spreads a point in M space into a hyperbola in data space and **W** is a band-limiting operator or, in other words, a simple convolution with a seismic wavelet. We found that the wavelet is needed to stabilize the inversion. In general, the wavelet is unknown; therefore it is either estimated from the data or replaced by a simple band-limiting operator.

In our approach **m** is retrieved by minimizing the following function

$$\mathbf{J} = \mathbf{R(m)} + \| \mathbf{Lm} - \mathbf{d} \| \ , \tag{3}$$

the functional **R(m)** indicates the Huber regularization term:

$$\mathbf{R(m)} = \sum_i \rho(m_i) \tag{4}$$

where

$$\rho(m_i) = \begin{cases} m_i^2/2 & \text{if } |m_i| \le a \\ a|m_i| - m_i^2/2 & \text{if } |m_i| > a \end{cases} \tag{5}$$

Equation (3) is minimized using the method of Conjugate Gradients. We have found that this regularization scheme leads to an algorithm that provides a good resolution enhancement in a few iterations. The cut off parameter *a* in the Huber norm is iteratively explored until a value that gives a good trade-off between resolution and numerical stability is found. In general, when *a* is too large the Huber norm becomes the Euclidean norm and the final estimator of the velocity panel is the one we would have estimated using standard least squares.

Examples

We applied our multiple suppression scheme to a data set from the Gulf of Mexico (see for instance, Verschuur and Prein, 1999). The data is provided by Western Geophysical. The data were processed using standard least-squares Radon inversion and the Huber norm regularization. Figure 1 and 2 show the velocity stack and the multiples estimates using the Huber norm. By comparison, Figure 3 and 4 show the corresponding results from least squares inversion. The velocity stack obtained from Huber regularization is sparser than that obtained with least squares. Consequently the filtering operation in velocity space becomes an easy task. After completing the multiple removal, the CMPs were stacked. The final sections are portrayed in Figures 5 (Huber norm inversion) and 6 (LS inversion). Now the salt body can be clearly seen.

Conclusions

We have applied the Hyperbolic Radon transform to the problem of multiple removal from a marine data set. Because the algorithm does not need NMO correction and gives high resolution velocity stack, it appears to be a feasible alternative to the parabolic Radon transform.

References

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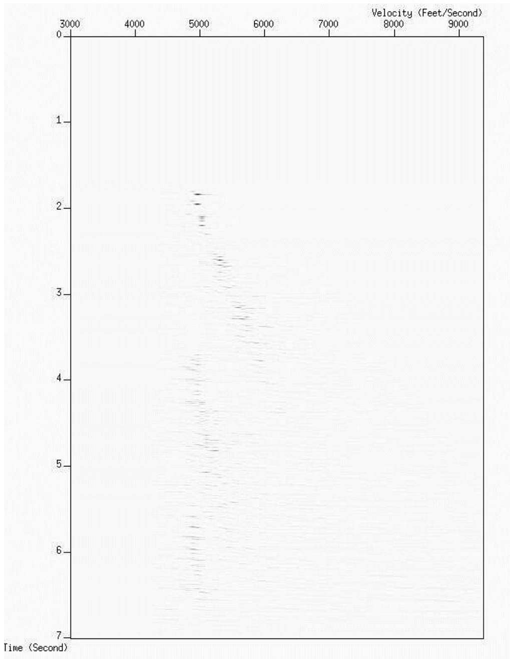

Figure 1 Velocity stack obtained using Huber norm regularization.

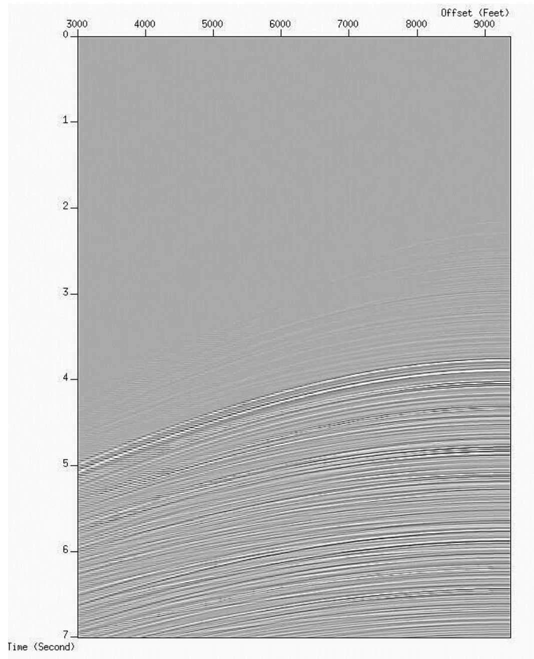

Figure 2 Estimated multiples from figure 1.

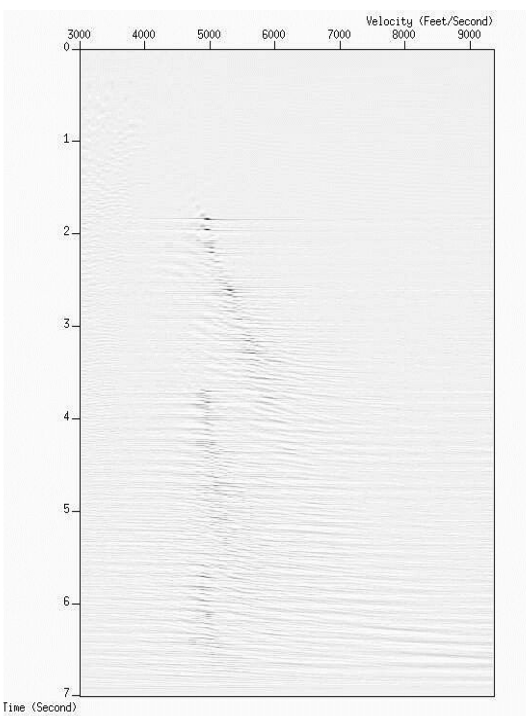

Figure 3 Velocity stack obtained using Least squares.

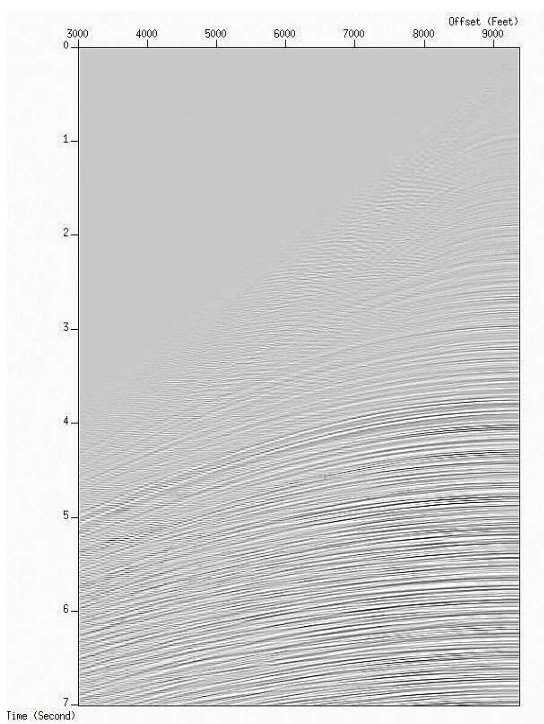

Figure 4 Estimated multiples from figure 3.

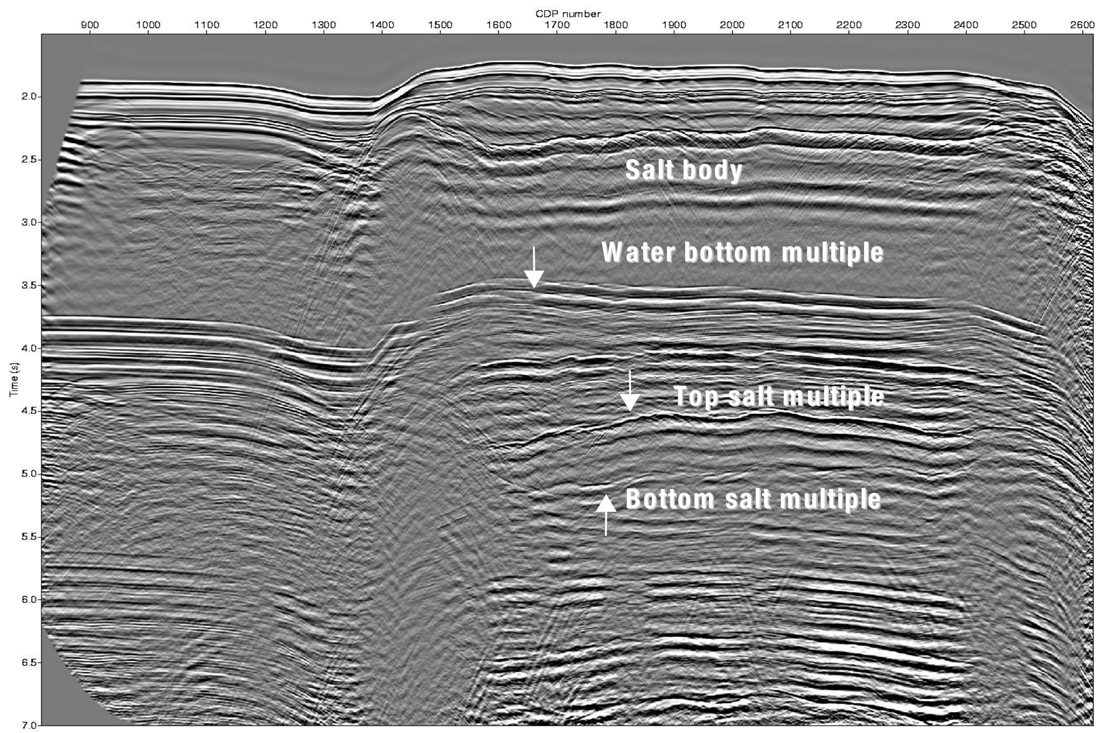

Figure 5 Final stack section of the original data

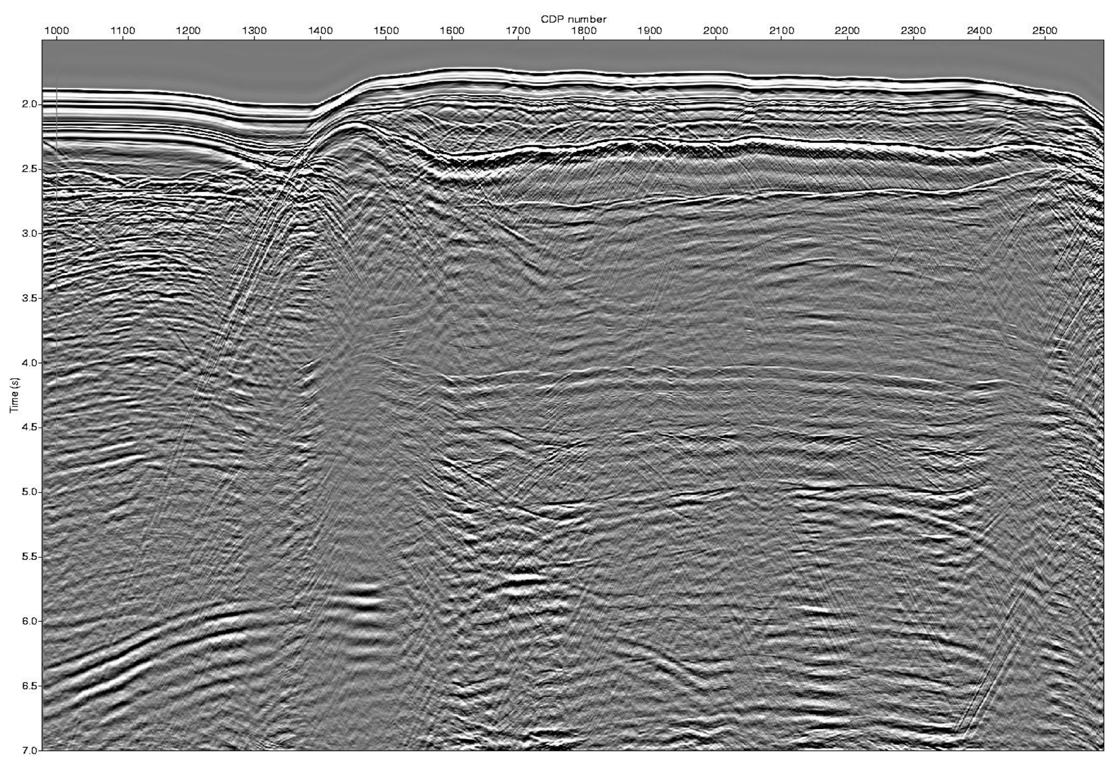

Figure 6 Final stack section after multiple removal by Huber norm inversion