Reconstruction of incomplete seismic data using a minimum weighted norm least-squares algorithm

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Summary

In seismic data processing we often need to interpolate missing source and receiver positions. The interpolation/resampling problem can be posed as an inverse problem where from inadequate and incomplete data one attempts to recover the Fourier transform of the seismic wavefield. The estimated Fourier spectrum is used to reconstruct the wavefield. The Fourier transform can be inverted using a minimum weighted norm least squares algorithm.

Introduction

The interpolation problem can be posed as an inverse problem. One can try to estimate the Fourier transform of the regularly sample data from irregularly acquired data. In this case a least-squares minimization can provide a solution to the problem when the data are band-limited (Cary, 1997, Duijndam et al. 1998, Duijndam et al., 2000). Least squares solutions with zero-order quadratic regulararization (damping) exhibit poor performance when facing the problem of interpolating large gaps of data. This is why we have modified the least-squares approach to include a weighted norm regularization term that helps to model the spectrum of the unknown signal. Cabrera and Parks (1991) have proposed a similar approach to interpolate time series. In their approaches, the missing samples are inverted rather than the coefficients of the discrete Fourier transform.

Theory

We define the discrete 2-D (e.g. source and receiver) inverse Fourier transformation for any source and receiver pair location (x_S, x_R) as

$$u(x_S, x_R, \omega) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} U(k_S(m), k_R(n), \omega) e^{jk_S(m)x_S} e^{jk_R(n)x_R} , \qquad (1)$$

where x_S and x_R are the spatial variables along source and receiver coordinates, k_S and k_R are the corresponding wave-numbers, and ω is the temporal frequency. Equation (1) gives rise to a linear system equations

$$u = FU \qquad (2)$$

where the u and U denote the known data and unknown coefficients of the DFT, respectively. A unique solution of U can be obtained by minimizing the following expression:

$$\| u \|_Q + \| FU - u \|_2 , \qquad (3)$$

where $\| u \|_Q = \sum_k \left\{ \frac{U_k^* U_k}{Q_k} \right\}$ is a weighted DFT-domain norm (Cabrera and Parks, 1991) and the weighting function Q has the same spectral support as U_k. The solution of equation (3) can be shown to take the form:

$$\hat{U} = (F^H F + Q^{-1}) F^H u . \qquad (4)$$

The elements of Q are computed from the amplitude spectrum of U. Ideally, one should know the amplitude spectrum of the data. Unfortunately, U is the unknown of our problem. The latter can be overcome by defining Q in terms of the DFT of the irregularly sampled data $F^H u$ and smoothing the result to attenuate the artifacts introduced by the irregularity of u (Ning and Nikias, 1990). The scheme can be summarized as follows:

1. Start with an initial $\hat{U}$

2. Compute $Q = S(\hat{U}^{*}\hat{U})$, where S is a smoothing filter

3. Solve $\hat{U} = (F^{H}F + Q^{-1})F^{H}u$ using Conjugate Gradients

4. Iterate until convergence.

Example

The 2-D reconstruction is illustrated on synthetic shot gathers. Figure (1) shows 20 shot gathers with the shot #2, #3, and #11 removed. The reconstruction was carried along shot and receiver direction using the algorithm described above. The reconstruction is shown in Figure (2). The missing shots were completely reconstructed.

Conclusions

We have presented a method to interpolate pre-stack data. In this method, a frequency domain weighting function is used in the 2-D minimum norm least squares extrapolation. The algorithm generally gives a better reconstruction than the method of minimum norm least square without weighting. It is efficient since the algorithm is carried out in Fourier domain and, in general, is not restricted to the narrow-band case.

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References

Cabrera, S.D. and Thomas, W.P., 1991, Extrapolation and spectrum estimation with iterative weighted norm modification: IEEE Trans. Signal Processing, vol. 39, no. 4, 842--850.

Cary, P., 1997, High-resolution "beyond Nyquist" stacking of irregularly sampled and spare 3D seismic data: Annual Meeting Abstracts, CSEG, 66-70.

Duijndam, A.J.W., Schonewille, M. and Hindriks, K., 1999, Reconstruction of seismic signals, irregularly sampled along on spatial coordinate: Geophysics, 64, 524-538.

Hindriks, K. and Duijndam, A.J.W., 2000, Reconstruction of 3-D seismic signals irregularly sampled along two spatial coordinates: Geophysics, 65, 253-263.

Luenberger, D.G., 1969, Optimization by vector space methods. New York: Wiley.

Ning, T. and Nikias, C.L., 1990, Power Spectrum Estimation with Randomly Spaced Correlation Samples: IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 38, no. 6, 991-997.

Sacchi, M.D. and Ulrych, T.J., 1996, Estimation of the discrete Fourier transform, a linear inversion approach: Geophysics, 61, 1128-1136.

Satya, D. and Arun, K.S., 1997, Bandlimited extrapolation using time-bandwidth dimension: IEEE Trans. Signal processing, vol. 45, no. 12, 2951-2966.

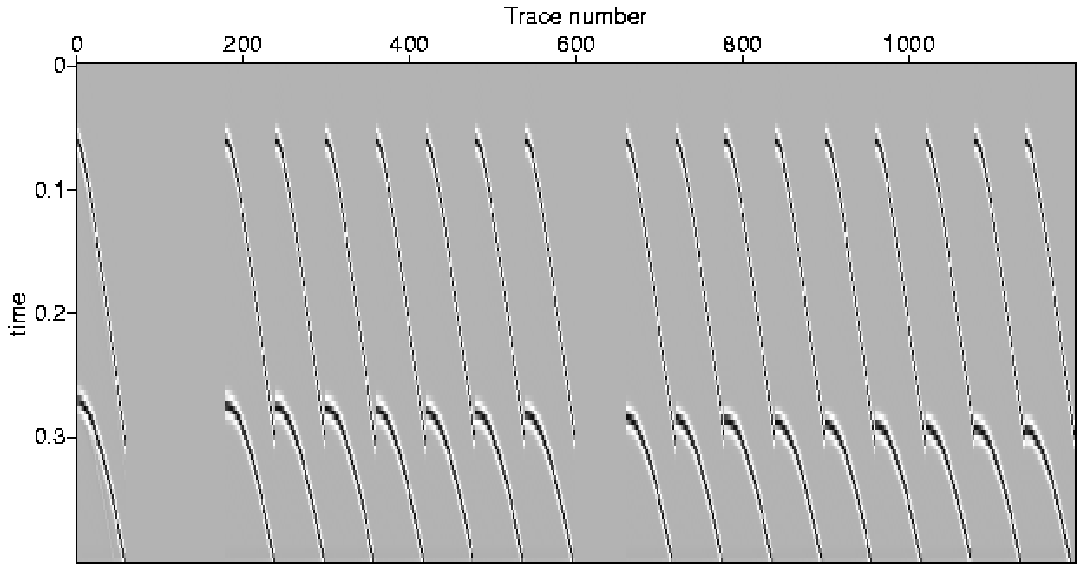

Figure 1: Synthetic shot gathers. Shot #2, #3, and #11 were removed.

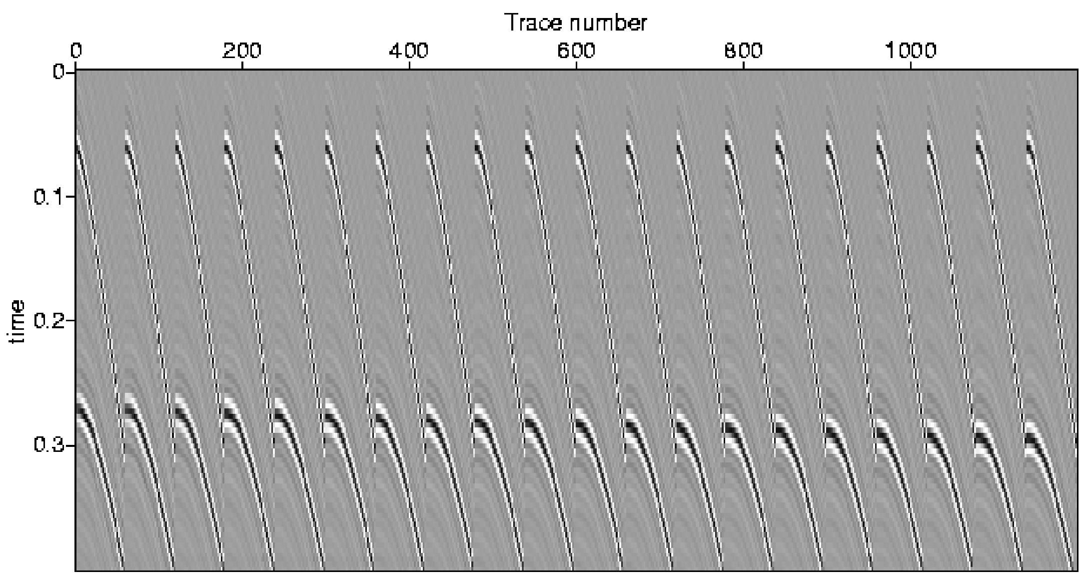

Figure 2: Reconstructed shot gathers from Fig. (1). The missing shots are completely reconstructed using minimum weighted norm least squares algorithm.