The Seismogram and Basic Time Series Analysis

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Summary

The model of the seismogram has remained simple and, essentially, unchanged, since the pioneering work of Enders Robinson in the late 1960's. It is formulated as the convolution of the seismic source signature with the earth's reflectivity together with additive random noise. The simplicity of the model belies the depth of the concepts which underlie it. The purpose of this tutorial lecture is to present these concepts in the frame work of time series analysis. This formulation defines the forward problem, and leads also to the solution of the inverse problem that, under certain constraints, allows the computation of the desired reflectivity.

I begin with the most general statement of the modeling of a stationary time series, the Wold decomposition theorem. In order to take full advantage of this profound theorem, we require the concept of the expectation operator that is introduced in an intuitive manner with just a touch of probability theory. The expectation operator is indispensable in the computation of the many averages that are encountered when dealing with random processes and it is well worth a few moments of exploration.

As a result of appeal to the Wold decomposition theorem, the noiseless seismogram appears as a moving average, MA, model composed of two unknown functions, the seismic wavelet and the earth's reflectivity. It is important to stress that this model stems from the basic underlying assumptions of linearity and time invariance. The separation of these two unknowns from just one equation (a problem referred to as blind deconvolution in much of the current literature) is a classic underdetermined problem that was solved by Enders Robinson with the help of a priori information based on sound physical reasoning. These constraints are well known, minimum phase and whiteness, and I explore them in a little detail. The power of these assumptions is that the MA model is readily transformed into another famous time series model, the autoregressive, or AR, representation. The transformation is readily accomplished by means of the z transform which is rapidly introduced at this stage and related to the well known Fourier transform.

The AR model of the seismogram is a thing of beauty. It is immediately related to prediction, prediction error and deconvolution. Wiener spiking deconvolution is a natural consequence, as is the embedded Toeplitz matrix representation and the Levinson recursion. Another aspect of the AR model which is well worth mentioning is its relationship to maximum entropy spectral analysis.

The ``forgotten'' additive noise is introduced at this stage and leads to yet another time series model, the autoregressive-moving average, or ARMA, representation. This model will be explored from the point of view of a special construction called the Pisarenko representation, that finds important application in the processing of seismic sections in the f-x domain, as well as in high resolution spectral analysis.

For final remarks, I return to the two constraints introduced at the beginning, consider very briefly the situation, the practical situation I hasten to add, when minimum phase and whiteness fail and discuss some possible approaches.