Ray Tracing and Travel Times in Anisotropic Media

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Summary

Traditionally seismologists have considered Earth models in which P-wave velocity varies with depth (Gray, 1999; Zhu et al., 1998). In recent years, however, seismic anisotrpoy has become of growing concern. Seismic anisotropy deals with the lateral variation of P-wave velocity and ignoring the effects of this phenomenon may result in the lateral and/or depth misplacement of the subsurface image. In the case of transverse isotropy (TI) all directions perpendicular to the axis of symmetry of the media are equivalent (i.e. the P-wave velocity is symmetric with respect to the axis that is perpendicular to the surface of the anisotropic layer). Therefore, along the symmetry axis the phase and the group velocity of the P-wave coincide. In this work we present a ray tracing algorithm for a two layer model. The top layer is isotropic and the bottom layer anisotropic. A computer code based on this algorithm is then written to compute the aplanatic surfaces for total travel times from S to E and from E to R, where E is any point in the subsurface.

Procedure

Our model is based on the foothill anisotropic thrust model of Leslie and Lawton (1998). Consider the two layer model shown in figure 1. The top layer is isotropic with a P-wave velocity v_1=2740 *m/s* and the bottom layer is anisotropic with parameters: δ=0.081, ε=0.15 and α_0=3365 *m/s* where α_0 is the P-wave velocity along the symmetry axis of the anisotropic layer. We assume that a P-wave ray sent by a source penetrates into the anisotropic layer, is reflected back to the surface and is detected by a receiver. The source *(S)* and receiver *(R)* are positioned on the surface at x_s, and x_r respectively. We further use the secondary source principle, i.e. we assume that at time t_0 point *E* in the subsurface is a scattering point. The rays from E are then detected at S and R at times t_1 and t_2 respectively.

The rays from the source and receiver strike the surface of the anisotropic layer at the angles of incidence θ_i and θ'_t respectively, and are refracted in the layer at the angles of transmission θ_i and θ'_t respectively.

From the geometry of figure 1 we can calculate the travel time for a ray from the source S to the point E as

$$t_1 = \frac{AC}{v_1} + \frac{CE}{v_2} \qquad (1)$$

where v_2 is the P-wave velocity in the anisotropic layer. Similarly

$$t_2 = \frac{IG}{v_1} + \frac{GE}{v_2} \qquad (2)$$

is the travel time for a ray from R to point E. If we define the horizontal positions of points E, C and G as x, x_1 and x_2 respectively, we can write:

$$\begin{aligned} (x_1 - x_s) &= h \ \tan(\theta_i) \\ (x - x_1) &= (z-h)\tan(\theta_t) \\ (x_2 - x) &= (z - z_1)\tan(\theta_t - \beta) \\ (x_r - x_2) &= z_1 \tan(\theta_i - \beta) \end{aligned} \qquad (3)$$

where h is the height of the isotropic layer before the dipping begins, β is the dipping angle of the anisotropic layer and z_1 is the vertical position of point H. Therefore, in terms of θ_i

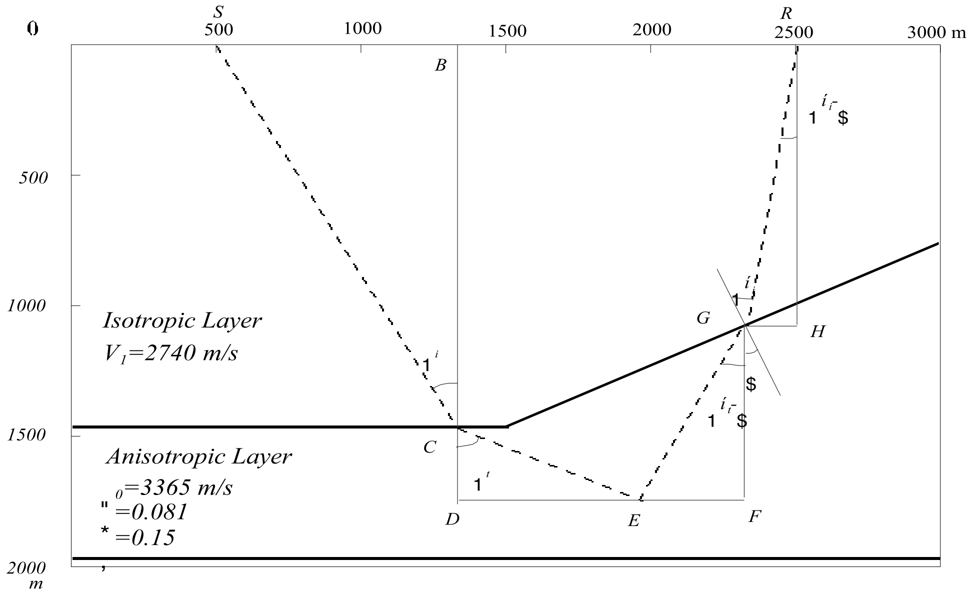

Figure 1. The two layer model used in this work.

$$x_1 = x_s + h\ \tan(\theta_i) \tag{4}$$

To solve for x_2 in terms of the angle of incidence we proceed as follows.

$$h - z_1 = (x_2 - x_j)\tan(\beta) \tag{5}$$

where x_j is the horizontal position at which the horizontal and the dipping anisotropic layers join. Therefore,

$$z_1 = h - (x_2 - x_j)\tan(\beta). \tag{6}$$

Substituting for z_1 in equation (5) and rearranging terms we get

$$x_2 = \frac{x_r - (h + x_j\ \tan(\beta))\ \tan(\theta_i^{\prime} - \beta)}{1 - \tan(\beta)\ \tan(\theta_i^{\prime} - \beta)} \tag{7}$$

Now, the total travel is $t_t = t_1 + t_2$. In terms of the vertical and horizontal positions, the incident and transmitted angles, and the dipping angle β, the travel times can be written as

$$t_1 = \frac{\sqrt{(x_1 - x_s)^2 + h^2}}{v_1} + \frac{\sqrt{(x - x_1)^2 + (z - h)^2}}{v_2} \tag{8}$$

and

$$t_2 = \frac{\sqrt{(x_2 - x)^2 + (z - z_1)^2}}{v_2} + \frac{\sqrt{(x_r - x_2)^2 + z_1^{\,2}}}{v_1} \tag{9}$$

To find the P-wave velocity v_2 in the anisotropic region we proceed as follows. The slowness p in the isotropic region is given as $p = \sin(\theta_i) / v_1$ (see Leslie and Lawton, 1998). In the isotropic region the phase angle and the group angle for the propagating rays coincide. In the anisotropic region we can solve for the phase angle χ using:

$$\alpha_0\ p(\epsilon-\delta)\sin^4(\chi)+\alpha_0 p\delta\ \sin^2(\chi)-\sin(\chi)+\alpha_0\ p=0 \tag{10}$$

(Slawinski, 1996) where α_0 is the P-wave velocity along the symmetry axis in the anisotropic region. The radius of phase-slowness surface is given as

$$r(\chi)=\frac{1}{\alpha_0(1+\delta\ \sin^2(\chi)\cos^2(\chi)+\delta\ \sin^4(\chi))}. \tag{11}$$

The transmitted angle θ_t (the group angle for the ray propagating in the anisotropic region) is then given as

$$\cos(\theta_t)=\frac{\alpha_0\ \sin(\chi)\sin(2\ \chi)(\delta\cos(2\ \chi)+2\ \epsilon\ \sin^2(\chi))+\dfrac{\cos(\chi)}{r(\chi)}}{\sqrt{\dfrac{1}{r(\chi)^2}+[\alpha_0\sin(2\ \chi)(-\delta\cos(2\ \chi)-2\ \epsilon\ \sin^2(\chi))]^2}}. \tag{13}$$

The group velocity $v(\theta_t)$ is then given as

$$v(\theta_t)=\alpha_0(1+\delta\ \sin^2(\theta_t)\cos^2(\theta_t)+\epsilon\ \sin^4(\theta_t)) \tag{14}$$

Numerical Results

In this section we show results for two different models. The first model consists of a top horizontal isotropic layer and a bottom horizontal anisotropic layer. The P-wave speed and the anisotropic parameters are the same as those described in the previous section. Note that in the plots presented the depth increases upwards. Also whenever travel times are plotted as a function of the horizontal position x (figures 3, 5 and 7) these travel times are plotted for all depths down to the bottom of the anisotropic layer. Figure 2 shows the contour plot of the travel times for a source positioned at x=*500 m* and a receiver at x=*1500 m*. As expected (Liner & lines, 1994) the travel time contours are ellipses with their foci at 500 and 1500 *m*. At a depth just above 1500 *m* we observe a change in the travel-time contours. This change is a manifestation of the transition between the isotropic and the anisotropic regions. Below this transition line the travel time contours are ellipses again. In figure 3 we show the travel-times versus the horizontal distance for the source and receiver described above. The transition line is also manifested in this plot in the form of a hyperbola, at about t=1.5 *ms.* In figure 4 the contour plot of the travel times for the same model with the source and receiver at the horizontal distance x=*1000 m*. In this case the contour lines are circular, as expected. In figure 5 these travel times are plots as a function of the horizontal distance x.

Next we show the results for a model where there is a top isotropic layer and a bottom horizontal anisotropic layer which extends from x=*0* to x=*1500 m*. The top surface of the anisotropic layer then dips upwards at an angle β=30 degrees (see figure 1) . The source and receiver are positioned at x=*1000 m* and x=*2000 m* respectively.

Figure 6 shows the contour plots of the travel times in an *x-z* plane. The contour lines are elliptical, for any give region, with foci at x=1000 *m* and x=2000 *m*. Figure 7 shows the travel-times in the same model as a function of the horizontal distance x. The top surface of the anisotropic layer can be traced in figure 7.

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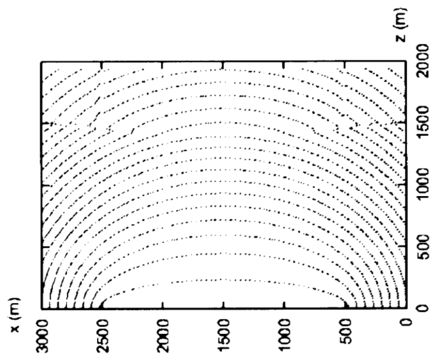

Figure 2. The contour plot of the travel times. The model consists of two horizontal layers, the top layer isotropic and the bottom layer anisotropic.

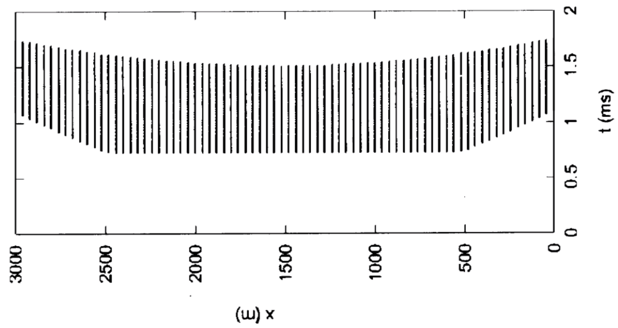

Figure 3. Travel times as a function of the horizontal distance x.

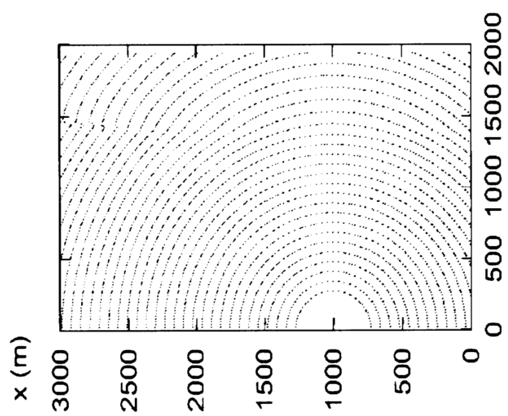

Figure 2. The contour plot of the travel times. The source And receiver are at x=1000 m.

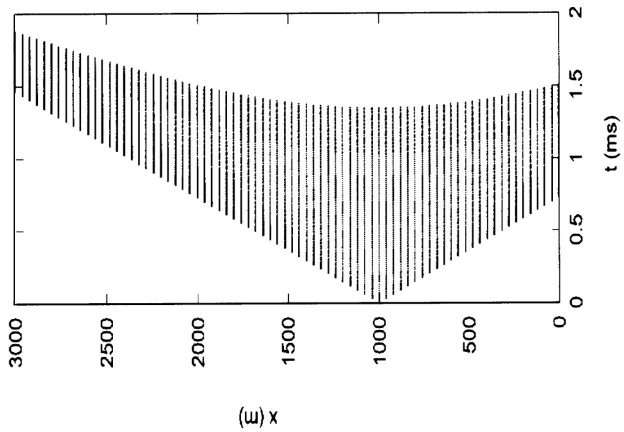

Figure 5. Travel times as a function of the horizontal distance x.

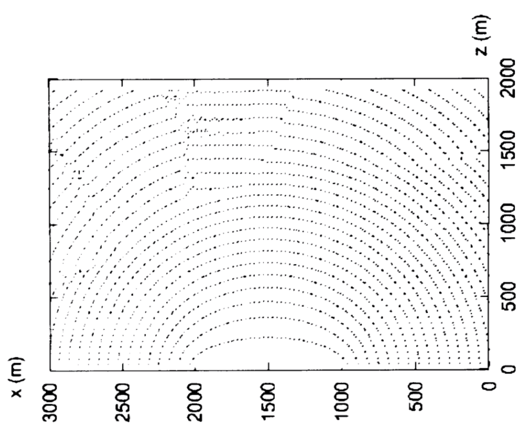

Figure 6. Contour plot of the travel times. The anisotropic layer dips at 30 degrees. The source and receiver are at x=1000 m and x=2000 m respectively.

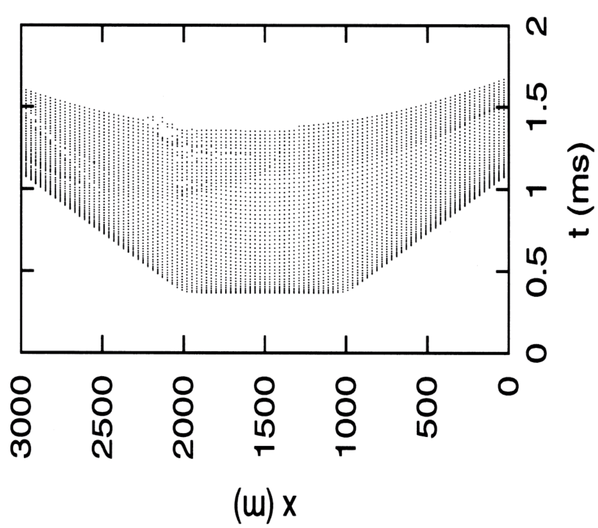

Figure 7. Travel times as a function of the horizontal distance x.