Attenuation and its Impact on Elastic Waves

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Introduction

Converted waves have the potential for providing higher resolution than P-waves, due to the shorter wavelengths associated with the same temporal frequencies. In practice it is often observed that converted wave resolution does not reach this ideal, particularly at depth. One possible reason suggested for this is the stronger effect of Q attenuation upon converted waves. When considering the effect of absorption on converted waves, we must consider two different values Q_P and Q_S, in much the same way as a medium has two different velocities V_P and V_S. One possibility is that Q_S is lower than Q_P, especially for the unconsolidated near surface layers. Deffenbaugh et al. (2000) considered the expected resolutions from PP and PS under assumptions that $Q_S<Q_P$, assuming a zero phase Q response. They conclude that in such a case there should be a “crossover” point of equal PP and PS resolution, with the PS resolution surpassing that of PP above the crossover, but becoming poorer than PP below it. In contrast, when $Q_S>Q_P$, the PS resolution is expected always to exceed that of PP. In this paper we undertake an analysis of the expected Q effect on converted waves compared to P waves, considering amplitude attenuation, resolution, and minimum phase dispersion. We illustrate these effects by modelling Q absorption for different homogeneous and layered models, with a view to improving understanding of observed converted wave behaviour. The modeling confirms that Deffenbaugh et al.’s result holds true when dispersion is included.

Constant Q Attenuation Theory

A widely used model of seismic attenuation in the earth assumes a Q value that depends upon the medium, but not upon frequency – within the bandwidth of interest. This is known as the “constant Q model” of absorption, and a significant body of theory has been developed based upon it (e.g. Futterman, 1962; Kjartannsen, 1979). Aki and Richards (1980) provide a good overview of this theory. Based on this assumption, a differential equation for the amplitude attenuation law can be obtained, which has the following solution:

$$A(x) = A_0 \exp\left(-|\omega| x / 2Qc\right) \tag{1}$$

where A_0 is the initial amplitude of a harmonic wave of frequency ω, and $A(x)$ is the amplitude after propagation by a distance x at velocity c, through a medium with a constant quality factor Q.

The application of equation (1) to a propagating pulse gives rise to a pulse that broadens symmetrically about a central peak. This situation entails a violation of causality, since some energy arrives before it has had the time to physically propagate. In a classic paper, Futterman (1962) showed based only on the assumptions of linearity and causality, that there must inevitably be dispersion accompanying attenuation, and derived the relationship between them for the case where Q is constant over a wide range of frequencies. The resulting revised law of attenuation is given by the following equation:

$$A(x) = A_0 \exp\left(-\frac{|\omega|}{2Qc} x + i\mathbf{H}\left(\frac{|\omega|}{2Qc}\right) x\right) \tag{2}$$

where $\mathbf{H}$ is the Hilbert transform, with respect to ω.

In the case of a layered medium and considering the simplest case of vertical wave propagation (hence using z instead of x for distance), the Q filters may be combined recursively for each layer. The result is that equation (2) holds true where Q is replaced by an “effective Q” value, Q_{eff}, given by:

$$T/Q_{eff} = \sum_{n=1}^{N} \Delta z_n / Q_n c_n \tag{3}$$

where: $$T \equiv \sum_{n=1}^{N} \Delta z_n / c_n$$

Here $\Delta z_n \equiv z_n - z_{n-1}$ is the thickness of layer n, c_n is the velocity and Q_n is the Q value in layer n. T is the total one-way travel time.

An integral expression for effective Q, equivalent to (3), is given in Bickel and Natarajan (1985).

PS Effective Q

Using a similar approach as above, based upon the P and S wave propagation operators for each layer gives the following equation for $Q_{PS,eff}$, the effective Q in the PS case:

$$\frac{T_{PS}}{Q_{PS,eff}} = \sum_{n=1}^{N} \left(\frac{\Delta z_n}{Q_{P,n} v_{P,n}} + \frac{\Delta z_n}{Q_{S,n} v_{S,n}} \right)$$

$$T_{PS} \equiv \sum_{n=1}^{N} \left(\frac{\Delta z_n}{v_{P,n}} + \frac{\Delta z_n}{v_{S,n}} \right) \tag{4}$$

where $v_{P,n}$ and $v_{S,n}$ are the P and S wave interval velocities, and $Q_{P,n}$ and $Q_{S,n}$ are the P and S wave Q values, in layer n^{th} layer. This can also be expressed in terms of the effective Q values for P and S, $Q_{P,eff}$ and $Q_{S,eff}$:

$$\frac{T_{PS}}{Q_{PS,eff}} = \frac{T_P}{Q_{P,eff}} + \frac{T_S}{Q_{S,eff}}$$

$$T_{PS} \equiv T_P + T_S \tag{5}$$

where T_P and T_S here are one-way P and S times. Alternatively, the effective Q for PS data can be given in terms of average vertical P and S-wave velocities, V_P and V_S, as follows:

$$\frac{1}{Q_{PS,eff}} = \frac{V_{PS}}{2} \left(\frac{1}{Q_{P,eff} V_P} + \frac{1}{Q_{S,eff} V_S} \right) \tag{6}$$

where: $\frac{1}{V_{PS}} = \frac{1}{2}\left(\frac{1}{V_P} + \frac{1}{V_P}\right)$

V_{PS} is the usual definition of average PS velocity in the sense that it relates vertical two-way time to depth for a PS wave, and also appears in the context of converted wave zero-offset migration (Harrison and Stewart, 1993).

Q Attenuation Compared in Time and Depth

Equation (2) may be recast in the time domain as follows:

$$A(t) = A_0 \exp\left(-\frac{|\omega|}{2Q}t + i\mathbf{H}\left(\frac{|\omega|}{2Q}\right)t\right) \qquad (7)$$

This describes attenuation for a wave travelling for a time period t, equal to t_P for P-waves and t_S for S-waves, in a medium with attenuation given by Q. A comparison of equation (7) for P and S cases readily shows, in the case $Q_S=Q_P=Q$ (which also implies $Q_{PS}=Q_P$), that for any given temporal frequency, ω, the S-wave attenuation is stronger than the P-wave attenuation, because $t_S>t_P$. This is intuitively sensible, since the slower shear velocity implies that the wave uses more cycles propagating from any given depth as a shear wave than as a P-wave.

However, this conclusion is unduly pessimistic for the following reason: ultimately the important criterion to the geologist is the ability to resolve strata in depth, not in time. Whether or not this involves transforming explicitly from time to depth, or not, we should compare PP and PS resolution based on a common vertical coordinate. Thus the resolution difference in the temporal frequency domain will be compensated by the compression of the PS time scale required to compare with PP time.

Another way of thinking about this, is to consider attenuation as a function of depth, z, and wavenumber, $k = \omega / c$:

$$A(z) = A_0 \exp\left(-\frac{|k|}{2Q}z + i\mathbf{H}\left(\frac{|k|}{2Q}\right)z\right) \qquad (8)$$

Comparing P and S filter responses for the *same* wavenumber (necessarily corresponding to *different* temporal frequencies), we see that in the case that $Q_S=Q_P=Q$, the predicted attenuations are the same. To see why this is so, recall that the number of cycles executed for a particular wavelength is given by distance traveled divided by wavelength, and does not depend upon the velocity.

Modeled Q Impulse Responses

Vertical incidence Q responses are computed using a 1-D plane-wave modeling program written in Matlab, based on equation (2). One-way constant Q attenuation for layered media is modeled for both P waves and S waves, and are then combined to compute the PP, PS and SS responses. The model used is homogeneous, with events generated corresponding to a reflector at 2100m depth, corresponding to 2 sec. PP time. The P-wave velocity is 2100m/s, and the S-wave velocity 700m/s, thus the V_P/V_S ratio is 3.0, giving a PS time equal to twice PP time, and SS time 3 times PP time. For this study Q_P and Q_S are both set equal to 50.

Impulse responses corresponding to unit amplitude reflections at 2100m depth, are shown in Figure 2, displayed in depth for PP, SS and PS modes. The modeling is done both without and with the dispersion term, giving a zero-phase response (Figure 2(a)), and a minimum phase response (Figure 2(b)), respectively. Note that the pure attenuation response is identical in depth for all three modes, after normalizing amplitudes. (For example it can be shown that the amplitude ratio of impulse responses for PP and PS modes is given by: $(Q_P V_P)/(Q_{PS} V_{PS})$). The differences in dispersion arise because the modeling requires a reference frequency corresponding to the maximum phase velocity, and this frequency corresponds to different wavenumbers for each mode. If reference frequencies are chosen separately for each mode, so

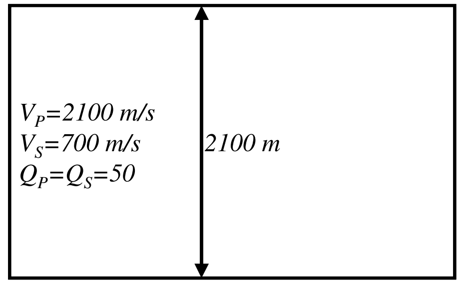
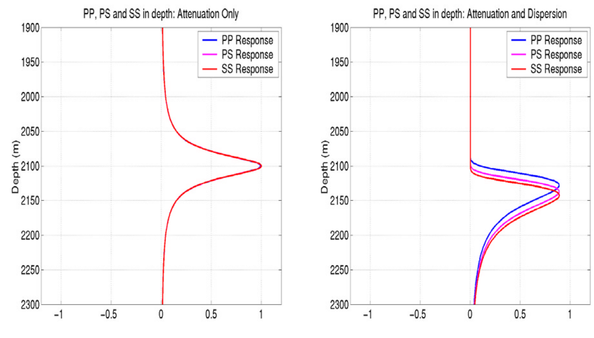


FIG 1. Homogeneous model: PP, PS and SS events are modeled corresponding to unit amplitude reflection at 2100m depth.



(a) (b)

FIG 2. PP (blue), PS (magenta) and SS (red) impulse responses for the reflector at depth 2100m in FIG 1. The responses are displayed in depth and the Q absorption is computed: (a) using attenuation term only, giving a zero-phase response, and; (b) using both attenuation and dispersion terms, giving a minimum phase response. All of the responses have been normalized according to the maximum amplitude of the attenuation response (a) for each mode. Note that pulse shapes are identical for PP, PS and SS cases, but there are differences in onset time, due to the cut off frequency.

as to correspond to the same wavenumber, the curves become identical (plotted in depth).

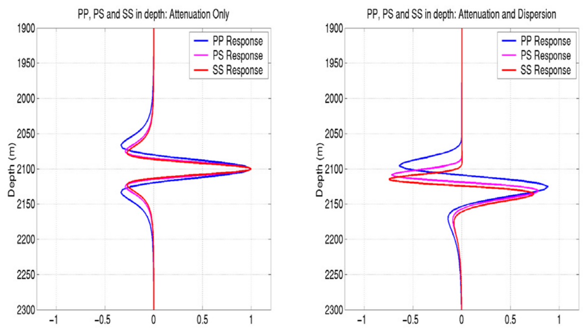
Q Responses to Input Wavelet

To consider the interaction with an initial wavelet, the modeling is repeated starting with a 20 Hz Ricker wavelet as input.

The most obvious effect (aside from the different arrival times) is that the PS mode attenuates more strongly than PP, as seen by comparing the relative sizes of the 2nd and 3rd pulses relative to the 1st, and the SS mode attenuation is stronger still. This is contrary to the observation for pure impulse responses, and suggests that the role played by the wavelet with regard to amplitude attenuation is important.

In order to better compare resolution and dispersion, the events at 2100 m depth are normalized to a peak amplitude of 1, and plotted together as a function of depth, in Figure 3. This should be compared with the impulse responses in Figure 2. The Ricker wavelet is identical in PP, PS and SS time, so that after conversion to depth, PS has a resolution advantage before attenuation. In this case, where $Q_S=Q_P$, the advantage is retained after propagation, as seen in Figure 3(a).

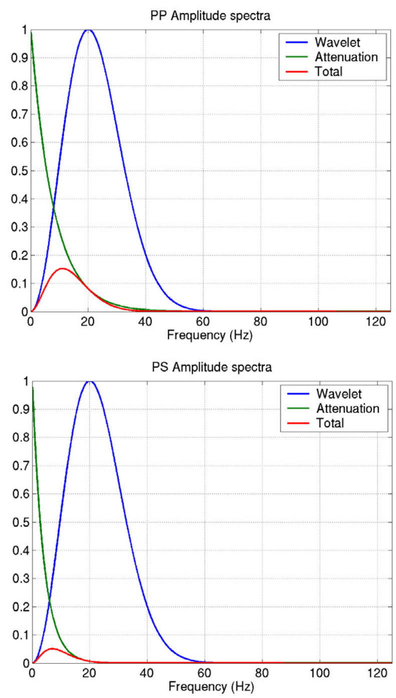
It is instructive to compare these results in both temporal frequency and wavenumber domains. Figure 4 demonstrates how the PP (a) and PS (b) spectra arise from the combination of the initial wavelet spectrum and the Q attenuation spectrum for the reflector at 2100 m depth (of course Q attenuation spectra are depth-dependent). Whilst the wavelet spectrum is identical in both cases, the PS mode attenuation spectrum decays much more quickly as a function of frequency than PP. Compare these with Figure 5, which shows the same spectra, but now plotted as



(a) (b)

FIG. 3. Depth domain seismograms for PP, PS and SS responses at 2100 meter reflector of model in Figure 1. A 20 Hz Ricker wavelet is used and the Q absorption effect is computed: (a) using only the attenuation terms, and; (b) using both attenuation and dispersion terms. Note the resolution advantage of PS and SS, when Q_S=Q_P.

(a)



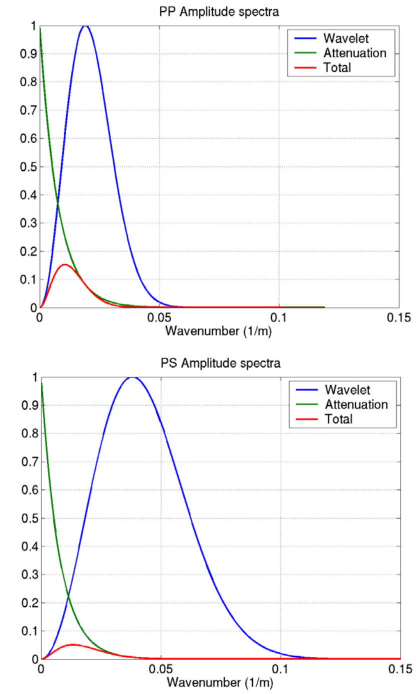
(b)

FIG. 4. Temporal frequency amplitude spectra for (a) PP and (b) PS event at 2100 meters depth. The blue curve is the initial Ricker wavelet spectrum, which is the same for both cases. The green curve is the spectrum of the Q filter for that depth. The red curve shows the resulting spectrum of the pulse at depth. It is the product of the blue and green spectra. The PS event shows both an amplitude drop, and a lower peak frequency relative to PP.

functions of PP and PS vertical wavenumbers. Now the attenuation spectrum (green curves) are seen to be identical functions of wavenumber for PP and PS, as predicted from equation (8). However, the initial wavelet spectrum is stretched in PS wavenumber relative to PP wavenumber.

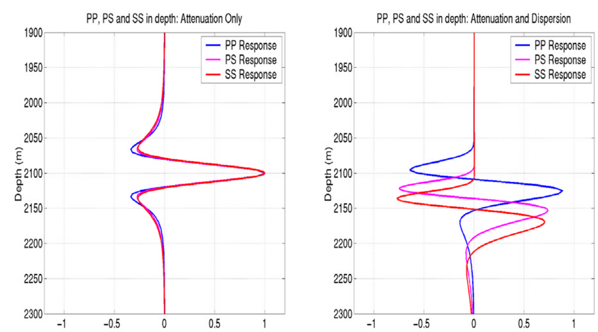
The same modeling is repeated for the case where $Q_P=50$, and $Q_S=30$. The results for the reflector at 2100 m depth are shown in Figure 6, again plotted in depth. These should be compared with the result in Figure 3 for the Q_S=50 case. The crossover behavior predicted by Deffenbaugh et al. (2000) occurs, such that the PS resolution appears to be similar to PP for the case Q_S =30. In fact, the theoretical crossover at this depth occurs when Q_S = 28.6. It should be noted however that: amplitude attenuation is much stronger for PS and SS than for PP, and; that the dispersion

(a)



(b)

FIG. 5. Vertical wavenumber amplitude spectra for (a) PP and (b) PS event at 2100 meters depth. The blue curve is the initial Ricker wavelet spectrum, which has a higher wavenumber band for the PS case. The green curve is the spectrum of the Q filter, which is now seen to be invariant in wavenumber. The red curve, which is the product of the blue and green spectra, shows the resulting spectrum of the pulse at depth. The PS event is seen to have a higher peak wavenumber than PP, but lower amplitude.



(a) (b)

FIG. 6. Depth domain seismograms for PP, PS and SS responses at 2100 meter reflector of model in Figure 1, for Q_S=30, Q_P=50. The Q absorption is computed: (a) using only the attenuation terms, and; (b) using both attenuation and dispersion terms. This shows the resolution "crossover" of PP, PS and SS, for a 20Hz Ricker input. We also see significant differences in dispersion.

effects for PS and SS are more pronounced than for PP, as expected for the lower Q value of the S-wave.

Layered Media

The modeling above was also repeated for a layered medium in which a shallow layer 200m thick was assumed to have a low Q_S value, with the medium below it having Q_S=Q_P. In this case we confirmed that the Q response is exactly as would be obtained by using the effective Q_{PS} value as defined in equation (6) above. Assuming a Q_S=10 in the shallow layer, and computing the effective Q value at 2100 m depth, we find that $Q_{S,eff} = 36.2$. For $Q_S = 5$ in the shallow layer, on the other hand, the effective value is $Q_{S,eff} = 26.9$.

Pikes Peak Example

Consider Figure 7, which shows a comparison of PP and PS migrated stacks from a 2-D high-resolution 3-C seismic survey at the Pikes Peak heavy oil field located east of Lloydminster, Alberta/Saskatchewan. A review of the acquisition and processing may be found in Hoffe et al. (2000).

The time axis for the PS section has been compressed by a factor of 2 relative to the PP section, which was found to approximately align corresponding events. This also enables a crude initial comparison of PP and PS resolution on the migrated data. Overall, the data quality is considered to be good. However, it is apparent that in general the PS resolution is considerably poorer than the PP resolution. For example consider the top of the Waseca formation at about 550 ms PP time (1100 PS time).

Analysis of the raw field data (not shown) indicates the following: the radial spectrum is deficient in low wavenumbers compared to the vertical spectrum, and; the radial spectrum decays rapidly after 20 Hz, whereas the PP spectrum has a slower decay after the corresponding frequency of 40 Hz. The first effect is attributable to the initial source spectrum, which is equal in temporal frequency for both PP and PS modes, such that the low-end roll-off covers a wider range of PS wavenumbers. The second effect may well indicate a lower value of Q for shear waves than for compressional waves.

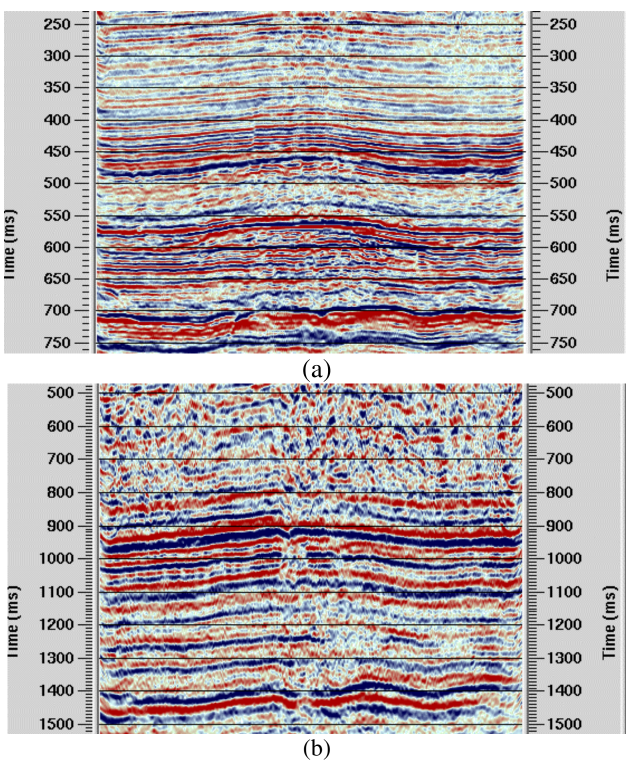


FIG. 7. The Pikes Peak 3-C 2-D line, showing a comparison of migrated PP (a) and PS (b) stacks. The PS stack has been displayed with a compression of 2 on the time axis, which approximately aligns events with the PP section.

Discussion and Conclusions

The effects of Q absorption may be broadly summarized as:

1. Amplitude decay with propagation distance
2. Broadening of the seismic pulse, due to differential attenuation of higher frequencies compared to low
3. Minimum phase dispersion, consistent with the demands of causality

The intuitive argument that shear waves traveling at a slower velocity than compressional waves have more oscillations and therefore more attenuation is basically correct, but misleading. The reason is that ultimately we wish to compare resolution in depth rather than time, or at least after converting from PS to PP time. Considering the Q filter on its own, we find that the attenuation effect is equal for PP and PS if they have the same Q value. However, it is important to consider the interaction with the initial wavelet. Because the wavelet is a function of time rather than depth, it corresponds to higher wavenumbers for PS than for PP. This might appear to favor the shear waves, and in terms of resolution, it does. However, the missing low frequencies have a confounding effect upon PS amplitudes.

Figures 4 and 5 help to understand these two effects. The resulting PS spectrum of the event has a higher peak wavenumber than PP, consistent with the observation of higher resolution in the depth domain. However, there is less overlap of the initial wavelet spectrum and attenuation spectrum, leading to an overall weaker amplitude. In a sense this could be attributed to lack of low-end frequency content in the initial wavelet, rather than shear wave attenuation. In other words the PS wave can potentially provide equal resolution to the PP in the depth domain but it needs the frequencies at the lower end that correspond to higher frequencies for PP. Robbed of low-end frequencies, what remains is then subject to high absorption, diminishing the amplitude. The problem may then be one of signal to noise ratio, rather than resolution. Hence the emphasis during acquisition should be on increased power at the low end of the spectrum without compromising the high end necessary for PP resolution.

For cases where S-wave Q is less than P-wave Q, modeling confirms the predicted crossover in resolution (Deffenbaugh et al., 2000). Furthermore, in this case the dispersion effect is stronger for PS than for PP data, which may be important to consider when matching events for V_P/V_S computation. For the case where Q_P=50, and $Q_S = 20$, our modelling showed an apparent difference in depth of 50 meters. The implications are that to match PP and PS events a velocity different to the limiting phase velocity would have to be used – for example corresponding to the velocity of the center frequency of the attenuated pulses. Conversely, velocities determined from comparing PS and PP event times are likely to be affected by differences between P-wave and S-wave Q.

Acknowledgements

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References

Aki, K. and Richards, P.G., 1980, Quantitative Seismology: Theory and Methods, Vol.1: New York, W.H.Freeman and Co.

Bickel, S.H., and Natarajan, R.R., 1985, Plane-wave Q deconvolution: Geophysics, **50**, 1426-1439.

Deffenbaugh, M., Shatilo, A., Schneider B., and Zhang, M., 2000, Resolution of converted waves in attenuating media: 70th Annual Internat. Mtg. Soc. Expl. Geoph., Expanded Abstracts, 1189-1192.

Futterman, W. I., 1962, Dispersive body waves: J. Geophys. Res., **67**, 5279–5291.

Harrison, M. P. and Stewart, R. R., 1993, Poststack migration of P-SV seismic data: Geophysics, Soc. of Expl. Geophys., **58**, 1127-1135.

Hoffe, B.H., Bertram, M.B., Bland, H.C., Gallant, E.V., Lines, L.R., and Mewhort, L.E., 2000, Acquisition and processing of the Pikes Peak 3C-2D seismic survey: CREWES Research Report, Volume **12**.

Kjartansson, E., 1979, Constant Q-wave propagation and attenuation: J. Geophys. Res., 84, 4737–4748.