From stacking to interval velocities in a media with non-horizontal interfaces and inhomogeneous layers (explicit formulae approach)

Emil Blias - Scott Pickford, A Core Laboratories Company

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Introduction

We can divide reflected travel-time investigations into two parts. Into the first category, we may place a large number of studies, which deal with numerical methods for travel-time inversion (Goldin, Hubral and Krey, Al-Chalabi, Shah, Chernyak, Gritsenko and others). This approach requires numerical ray tracing. For the laterally varying layered media, within this approach, we cannot derive direct analytical relationships between interval and stacking velocities as we can for the horizontally homogeneous media. That means that we cannot obtain explicit general quantitative conclusions about how laterally varying boundaries (that is, boundaries with structures) influence stacking velocities.

Into the second category, we may place general investigations of direct analytical relationship between subsurface parameters (reflected boundaries and interval velocities) and stacking velocities. Most of these investigations were of laterally homogeneous layered medium [Bolshih, Taner and Koehler, Puzirjov].

Let us ask several questions: *What happens if the boundaries or (and) interval velocities vary laterally? What lateral changes of the normal incident time t_0 and stacking velocities V_{stack} can we expect? Do we need to take into account lateral velocity changes? Can we go from stacking velocities to the interval ones through the Dix formula and then to time-to-depth transformation? What are the restrictions for using this formula? What are the necessary corrections and when do we have to make them?*

To answer these questions, the perturbation method was used to derive explicit formulas of stacking velocities and normal incident time for a laterally varying layered velocity model. The main question that I try to answer is: "*What happens with the normal incident time t_o and stacking (NMO) velocity when boundaries and interval velocities start to deviate from constant values?*"

Theory

In this paper, for simplicity sake, I consider 2-D media but the same scheme can be applied to 3-D media as well. Let us consider layered media with the boundaries $z = F_k(x)$ and layered velocities $v_k(x)$, k=1, 2, …, m. Let $\Upsilon(S, R)$ be the reflected wave raypath from the source S(X-l/2,0) to the receiver R(X+l/2,0) where X is the surface midpoint and l is the distance between S and R, Fig.1; $P_k(\xi_k,F(\xi_k))$ –intersection points of the downgoing path, $Q_k(\eta_k,F_k(\eta_k))$ –intersection points of the upgoing path, k = 1,2,…,n, n – number of intersecting layers . Then for time t(S, R), from the source S to the receiver R, we can write

$$t(S,R) = \sum_{k=1}^{n}[t_k(\xi_{k-1},\xi_k)+\tau_k(\eta_{k-1},\eta_k)] = T(S,R,\xi_1,\xi_2,\ldots,\xi_n,\eta_1,\eta_2,\ldots,\eta_n). \quad (1)$$

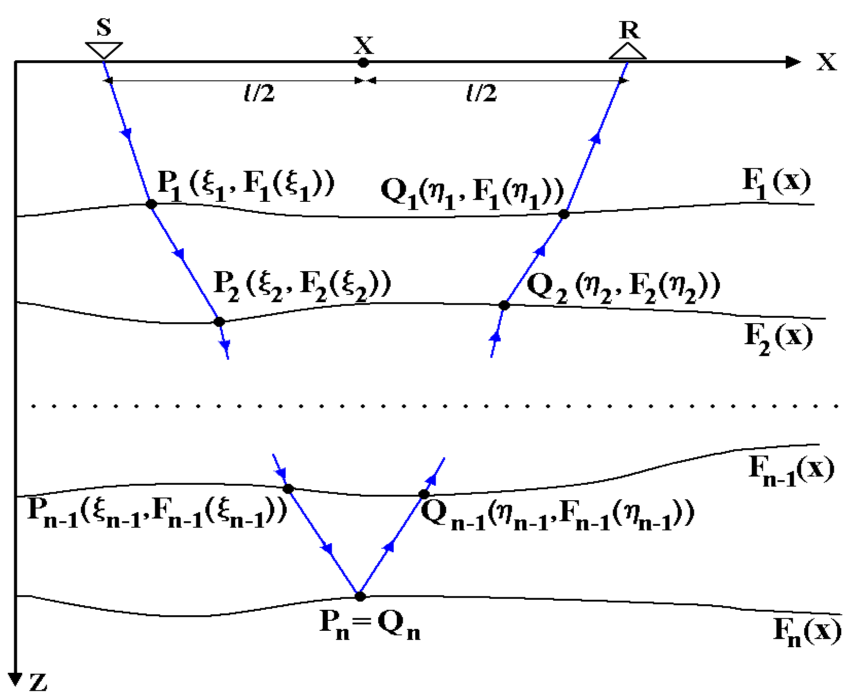

Fig. 1. Ray from the source S to the receiver S and intersection points

Here $t_k(\xi_{k-1},\xi_k)$ denotes traveltime from P_{k-1} to P_k, $\tau_k(\eta_{k-1},\eta_k)$ - traveltime from Q_{k-1} to Q_k. Let x_k be the x-projection of the segment $P_{k-1}P_k$, $x_k = \xi_{k-1} - \xi_k$; y_k – x-projection of the segment $Q_{k-1}Q_k$, $y_k = \eta_{k-1} - \eta_k$. Then

$$\xi_k = X - l/2 + \sum_{i=1}^{k} x_i, \qquad \eta_k = X + l/2 - \sum_{i=1}^{k} y_i \tag{2}$$

and

$$l = \sum_{k=1}^{n} (x_k + y_k) \tag{3}$$

Let us substitute (2) into equation (1). Then function T depends on variables $x_1, x_2,.., x_n, y_1, y_2,\dots, y_n$. According to Fermat's principle $(x_1, x_2,\dots,x_n, y_1, y_2,\dots, y_n)$ is a stationary point of function T under condition (3). If L - Lagrangian function then $x_1, x_2,.., x_n, y_1, y_2,\dots, y_n$ and Langrangian multiplier λ satisfy equation (3) and the system

$$\partial T/\partial x_k = \sum_{j=k}^{n} \partial t_j/\partial x_k = \lambda, \qquad \partial T/\partial y_k = \sum_{j=k}^{n} \partial \tau_j/\partial y_k = \lambda. \tag{4}$$

Thus, the problem of ray path and time calculation is reduced to solving the nonlinear system of equations (3), (4). Let us consider equations (4) as equations with unknown $x_2,.., x_n, y_1, y_2,\dots, y_n$ with the parameter λ. If we find x_k y_k as functions of a single variable λ and substitute into right sides of (1), (3), we will get parametric representationof the traveltime $t(X,l)$:

$$t = t(X,l,\lambda), \qquad l = l(X,l,\lambda), \quad \lambda \text{ - parameter.} \tag{5}$$

Accurate solution of these equations can be obtained only for laterally homogeneous media [Bolshih, Taner and Koehler]. To find approximate solution for laterally inhomogeneous media, we can use perturbation approach. For this, with the original velocity model with the interval velocities

$$v_k = v_k(x), \qquad k=1, 2, \dots, m \tag{6}$$

we consider an auxiliary model with the layered velocities depending on small undimensional parameter ε:

$$v_k(x,\varepsilon) = v_k(X) + \varepsilon u_k(x), \qquad k=1, 2, \dots, m \tag{7}$$

Here $v_k(X)$ are constants (for the fixed midpoint X). We assume that interval velocity variation is small as compared to the average value of this velocity between the points ξ_{k-1} and ξ_k. Then in (3) we can consider $u_k(x)$ to be approximately the same value as $v_k(X)$ and ε to be a small dimensionless coefficient.

We can also use a small dimensionless coefficient μ to describe boundaries:

$$F_k(x,\varepsilon) = F_k(X) + \mu G_k(x), \quad k=1, 2, \dots, m \tag{8}$$

Perturbation theory allows us to acquire the explicit formula of the travel time t_k in the inhomogeneous k-th layer. This explicit presentation has the form of power series of ε:

$$t_k(\xi_{k-1}, \xi_k) = t_k^{(0)}(\xi_{k-1}, \xi_k) + t_k^{(1)}(\xi_{k-1}, \xi_k)\varepsilon + t_k^{(2)}(\xi_{k-1}, \xi_k)\varepsilon^2 + \dots$$

After that, we can write the system (4) - (5) as

$$\partial T/\partial x_k(\varepsilon,\mu) = \sum_{j=k}^{n} \partial t_j/\partial x_k(\varepsilon,\mu) = \lambda, \qquad \partial T/\partial y_k(\varepsilon,\mu) = \sum_{j=k}^{n} \partial \tau_j/\partial y_k(\varepsilon,\mu) = \lambda, \quad k=1,2,\dots,m.$$

This system has a solution that can be written as

$$\begin{aligned} x_k &= x_k^{(0)}(\lambda,l) + x_k^{(1)}(\lambda,l)\varepsilon + x_k^{(2)}(\lambda,l)\varepsilon^2 + \dots, \\ y_k &= y_k^{(0)}(\lambda,l) + y_k^{(1)}(\lambda,l)\varepsilon + y_k^{(2)}(\lambda,l)\varepsilon^2 + \dots, \end{aligned} \tag{9}$$

After finding the coefficients $x_k^{(j)}$, $x_k^{(j)}$, we substitute (9) into (5), solve the second equation for λ and replace λ in the first equation. This allows us to obtain an explicit formula for the time as a function of the midpoint X and the offset l in the form of power series of l:

$$t^2(X,l) = c_0(X) + c_1(X)\, l^2 + c_2(X)\, l^4 + \dots \tag{10}$$

Here $c_0(X) = t_0^2(X)$, $c_1(X) = 1/V^2_{stack}$, where $t_0(X)$ is the normal incident time at the CDP point X and V_{stack} is the stacking velocity at the same point. For a medium with dipping boundaries, the approximate formula for normal incident tiime t_0, can be written as

$$t_0(X) = 2 \sum_{k=1}^{n} (h_k/v_k) - \sum_{k=1}^{n} h_k v_k \left[\sum_{i=k}^{n} (1/v_i - 1/v_{i+1})\, F_i'(x)\right]^2 \tag{11}$$

Here v_k - is the velocity in the k-th layer, h_k - the thickness of the layer at the point x = X. The first term provides the travel time along the vertical ray in the model with horizontal layers. The second term gives the correction, defining the influence of the non-horizontal reflecting and transmitting interfaces. Practically, the second term defines the influence on the time t_o which is caused by the bias of the central ray from the vertical one.

For the stacking velocity we obtain:

$$\frac{1}{v_{stack}^2} = \frac{1}{v_{rms}^2}\left[1 + \sum_{k=1}^{n-1} (1/v_k - 1/v_{k+1})\, F_k''(x)\, a_k\right] \qquad (12)$$

For the coefficient a_k we have

$$a_k = \left(\sum_{i=k+1}^{n} h_i v_i\right)^2 / \left(\sum_{i=1}^{n} h_i v_i\right)$$

Formulae (11), (12), obtained for the normal incident time t_0 and stacking velocity V^2_{stack}, allow us to answer the questions written in the introduction.

Medium with the curvilinear waterbottom line

Let us consider a relatively simple but important marine case with a curvilinear waterbottom. It has been empirically noted that the lateral behavior of stacking velocities in marine seismic often shows a relationship with the structure of the waterbottom. Work that I have published in the Russian literature (Blias 1981, 1984, 1987) shows how a structured water bottom will change seismically measured stacking velocities with respect to the RMS velocities (Fig. 2). The difference between RMS and stacking velocity from deep boundaries can reach 30% and more. Therefore, when the Dix formula is used to obtain interval velocities, we get large errors since this formula assumes that our picked stacking velocities are close to the values of the RMS velocities.

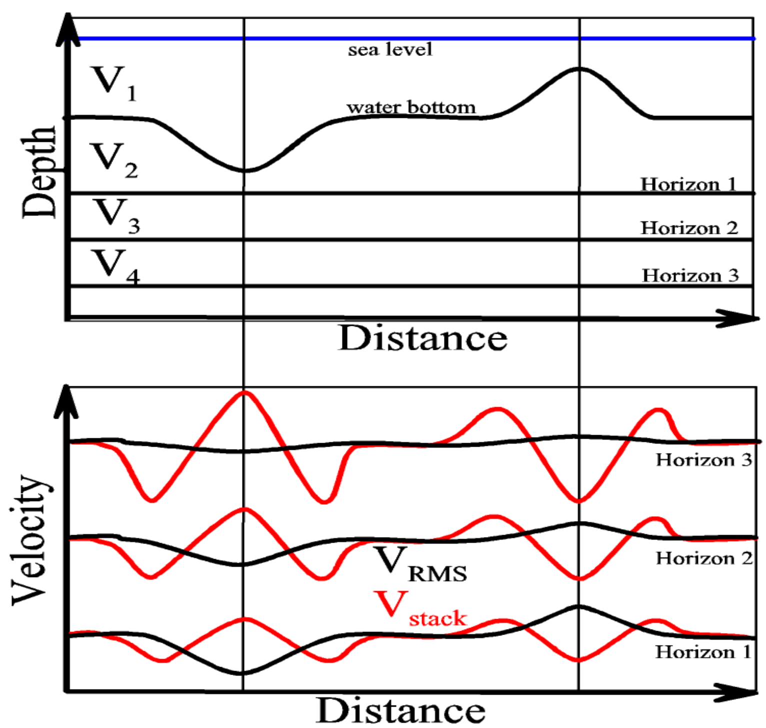

Figure 2

To determine realistic interval velocities (and eventually realistic depths) we need to make corrections to the Dix formula. The main influence on lateral velocity changes is due to the structural curvature of the waterbottom. This suggests that the corrected Dix formula must include the second derivative of the water bottom horizon. Formula (12), connecting interval and stacking velocities, contains the second derivative of the transmission boundaries with some coefficients.

For a simple example, we can consider a 2-D case with a curvilinear boundary defined by the function F(x), which we will take to be the water bottom. If h_k is the thickness of the k-th layer and v_k is the velocity in this layer, then approximate formulae that connect stacking velocity V_{Stack} with the interval velocity are

$$\frac{1}{\quad} = \frac{1}{\quad}\left[1 + (1/v_1 - 1/v_2)\, F''(x)\, A_n\right]$$

$V_{stack}^2 \quad V_{RMS}^2$

$$A_n = \left(\sum_{i=2}^{n} h_i v_i\right)^2 / \left(\sum_{i=1}^{n} h_i v_i\right).$$

Here $F''(x)$ is the second derivative of the water bottom. If $F''(x) = 0$ (a flat water bottom) then we simply have the stacking velocity equivalent to RMS velocity. Any difference between RMS and stacking velocities will then depend on the expression $(1/v_1 - 1/v_2)\, F''(x)\, A_n$. Here v_1 is the velocity of the water and v_2 is the velocity of the first layer below the waterbottom. This value depends not only on the differential slowness between the water and the first sub-water layer $(1/v_1 - 1/v_2)$ but also on the value of A_n. One can see then that as depth increases the number A_n also must increase since the numerator is the square of the summation and the denominator is just the first power of this sum. Lateral behavior of RMS velocity repeats behaviour of the second derivative of the function $z = f(x)$, fig. 3.

A similar formula for the general case of a 3D medium with curvilinear boundaries and laterally varying interval velocities has been obtained. The next step to solve the problem is to invert these formulae in order to obtain the true interval velocities.

There can be several different approaches to apply these formulae for the case of a structured water bottom that we've been discussing. For instance, we could try to correct the stacking velocities by using information about the water bottom (its second derivative and velocities above and below) and some rough information about the interval velocities and boundaries – essentially we would have to make some estimation of the value of A_n. This is not particularly attractive since the interval velocities making up A_n are the values we are trying to estimate in the first place.

An alternative approach would be to calculate corrected interval velocities using the picked stacking velocities and normal incident times. In other words, we can use stacking velocities and time t_0 (with their first and second derivatives) to make corrections to the interval velocity values. In this case, we can extract 'corrected' interval velocities originally affected not only by the water bottom but could also correct the effects of geologic structure deeper than water bottom. In this second approach, we do not need any estimation of A_n in the area since these values will be obtained from the stacking velocities and times T_0. At the same time we will need to use not only the times T_0 but also its first and second partial derivatives. The corresponding formulae then become much more complicated.

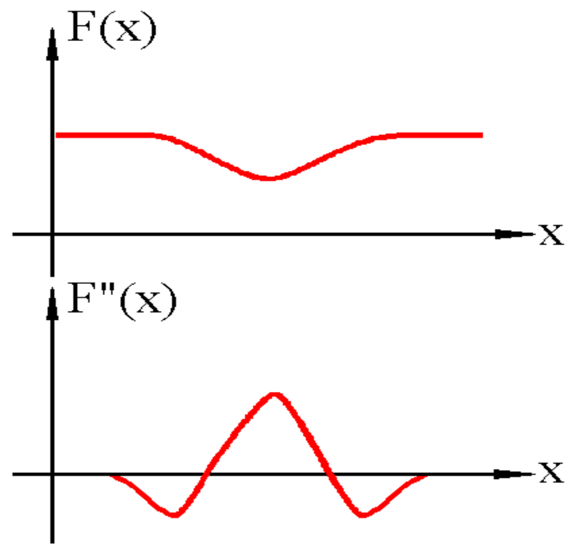

Fig. 3. Boundary and the second derivative

The simplest way, then, to solve the problem of extracting correct interval velocities below curvilinear water bottom is to use approximate explicit formulas (modifications of Dix formula discussed above). For the input data, one will need stacking velocities V_{Stack} and normal incidence times $T_0(x,y)$ with their (time) first and second derivatives. It should be mentioned that to decrease the error and make the solution more stable (and possibly to estimate its quality), it is much better to pick stacking velocities in perpendicular directions and use them for interval velocities calculations.

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