What happens to AVO when we assume dipping layers are flat?

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Introduction

Surface seismic is recorded at offsets, but quantitative AVO analysis requires angles of incidence, necessitating a translation from offset to angle. To go from the offsets to the incident angles, we have to ray trace through some velocity model. The question arises: do we need to take into account the slope of a dipping reflector or can we simply ray trace for the equivalent horizontal layer? In performing AVO analysis on dipping reflectors, it is common to ignore the dip and assume the reflector is flat when calculating angles of incidence. Is this a valid assumption? What difference to the estimation of angles of incidence does it make to assume flat layers? What difference does the flat dip assumption make to the extracted AVO attributes?

In this short paper, we investigate, both mathematically and by a modeling study, the influence on the incident angle due to the dip of the reflector.

Summary of Findings

1) The error in estimated angle of incidence is less than 3^o for a layer with 30^o dip when encountered by a wavefront at actual 34^o incident angle (*Figure 4*). Only for dips about 45^0 and offset/depth ratio close to 2 (which make about 45^0 incident angles) does the difference becomes significant: up to 8^0 to 10^0.

2) The results of AVO extraction and inversion indicate that the error in these attributes introduced by assuming dipping layers to be flat is relatively small. Quantitative AVO analysis is stable, even when we assume dipping layers are flat, and even at high angles of incidence.

3) The AVO attributes which are most sensitive to the flat layer assumption are S-reflectivity, Fluid Stack, S-impedance and Mu*Rho. This is not unexpected: the S-response is an inferred attribute, estimated from the amplitude behavior of the pre-stack P-wave data.

4) This modeling investigation does not account for anisotropy, for non straight dipping situations such as anticlines or synclines, or for the situation of very large offsets. The conclusions may be considered valid in only the most straightforward of cases.

Theory

It is well known that for a homogeneous medium the dip affects zero-offset time by multiplication by cosine of the angle, and the stacking velocity by division by the cosine of the boundary angle. Consider a layer with velocity V and dipping boundary with a dip angle φ. Assume that for the ray tracing, we take the layer with a horizontal boundary. We say that a new velocity model (with the horizontal reflector) is equivalent to the original model (with the dipping reflector) if the incident angles for all offsets within fixed midpoint are the same. It means that for the given CDP gather, we can use the equivalent horizontal layer to find the incident angles instead of using a layer with the dipping boundary. We will show that for any dipping boundary, there exists an equivalent horizontal boundary, which preserves all incident angles. We will also describe how to find this horizontal reflector using zero-offset time and a stacking velocity. In many cases, for the inhomogeneous medium, we can find an approximate equivalent medium using zero-offset times and stacking velocities.

It turns out that to determine the depth of the equivalent horizontal boundary, we take the depth at the middle point (CDP point) of the dipping boundary. For this horizontal boundary, incident angles will be the same as for the dipping reflector. To prove this let's consider *Figure 1*. Here S and R are the source and receiver respectively and φ is the dip angle for the boundary I. Let d be a distance from the source S to the receiver R, d = |SR| and H is a distance from the midpoint X to the dipping boundary I. Let the line SR be the horizontal axis x, and vertical axis y coincides with the line XM. Then it can be shown that a reflection point N has the coordinates x_R, y_R:

$$x_R = \sin\varphi\,(4H^2 + d^2\cos^2\varphi)/(4H), \qquad y_R = \sin\varphi\,(4H^2 - d^2\sin^2\varphi)/(4H). \qquad (1)$$

Using these formulas, we can prove that the angles 1 and 3 are the same. The idea is to find these angles (angle 1 = angle 2 and angle 3 = angle 4) from the triangles SMR and SNR respectively. For these triangles, we could find all the legs (using formulas (1)) and apply the cosine theorem, but this approach requires quite a lot of mathematical calculations.

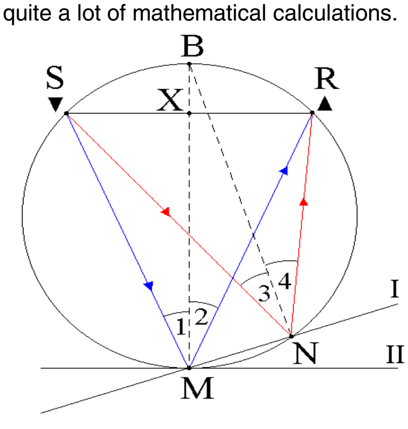

Figure 1: Incident angles for the horizontal and dipping reflectors with source S and receiver R.

The equivalence of the horizontal and dipping boundaries I and II can be proved using a geometrical approach, which is much easier for this problem. Let II be a horizontal interface and I a dipping boundary with the same depth at mid point X. For the boundary II, SM is an incident ray and MR a reflected ray with the incident angle 1 which is equal to the reflected angle 2 (rays are marked in blue in *Figure 1*). For the dipping interface I, N is a reflection point with the incident and reflected angles 3 and 4 correspondingly. Let us draw a vertical line through the point M to intersect this line with the circumference through the source S, receiver R and reflection point M. Then BM is a diameter of this circumference and angle MNB is 90^0. Angle 3 is equal to the angle 1 because they have the same arc SB. For the same reason the angles 2 and 4 are equal. Angle MNB is a right angle because MB is a diameter. Then it follows that SNB is an incident angle and all angles 1, 2, 3 and 4 are equal. It implies that the boundaries II and I are equivalent for the incident angles. Notice that these boundaries are not equivalent for the stacking velocity V_{stack} and zero-offset time T_0.

From the above consideration, it follows that for the homogeneous medium, incident angle does not depend on the slope of the boundary. It also follows that if we rotate the reflection boundary around the midpoint depth (point M on *Figure 1*), the incident angle does not change at all, so it depends only on the offset but not on the slope of the reflector. In this case, we can write that $\alpha(x,\varphi) = \alpha(x)$ where x is an offset. To find the reflection point for a dipping boundary, one may draw the circle though the source, receiver and midpoint depth. Then the reflection point N can be simply found as an intersection of this circle and the dipping boundary.

To find an incident angle through ray tracing, we need a depth model. To create this model, we take zero-offset time T_0 and multiply it by the velocity. First we consider *non-migrated* CDP gathers. In this case, after multiplying T_0 by the velocity V, we obtain the horizontal boundary II′ with the depth XP at the midpoint, *Figure 2*. From the above it follows that to obtain the equivalent horizontal boundary II, we have to take the depth XM for this boundary. To do this, we correct the boundary II depth XP by dividing it by the cosine of the dipping angle φ. It follows from the equalities XN = XP and XM = XN/cosφ.

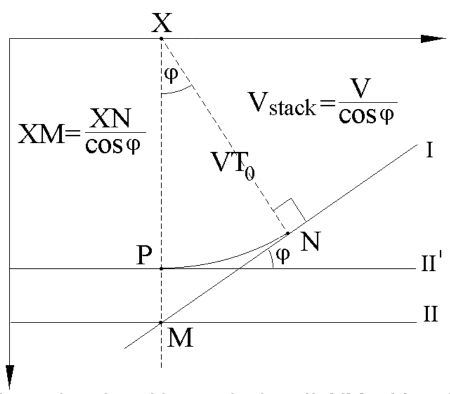

Figure 2: Dipping boundary I and its equivalent II: $XM = V_{stack}\,T_0$. $XP = XN = V\,T_0$

If we use the stacking velocity $V_{stack} = v/\cos\varphi$ to find the depth of the horizontal boundary ($T_0 V_{stack}$), then we do not need to correct the obtained depth because

$$T_0\, V_{stack} = (XM \cos\varphi)\,(V/\cos\varphi) = XM.$$

For *migrated* CDP gathers, the zero-offset time corresponds to the vertical depth. It implies that by using velocity V for the equivalent model, ray tracing can be done without depth corrections.

A layered medium causes the difference between the incident angles of the dipping and corresponding horizontal boundaries (obtained by the assumption of homogeneous medium with average velocity) but in many cases this difference is small and can be neglected. This is because the difference between the incident angles for horizontal and dipping reflectors is caused only by the inhomogeneity and not by the dip itself.

Methods have been developed (Blias,1981; 1987) that allow us to obtain the approximate explicit formulae for the travel time in layered medium with non-horizontal layers. The same approach (using perturbation theory) gives us a tool to obtain explicit expressions for the incident angles in 2D and 3D layered medium with dipping non-parallel curvilinear layers but this is far beyond this paper.

Modeling

Based on sonic, dipole, and density logs from a well, models of a single layer with various dips were constructed. The dip of the oil bearing sandstone reservoir layer was varied from 0 to 45 degrees, at 1 degree increments (45 models, each with a different dip). For each model, angles of incidence were calculated via ray tracing through the velocity field (as defined by the sonic log).

Comparing angles of incidence for any dipping case with the 0^0 dipping case shows the error there would be due to assuming the layer is flat: the error in estimated angle of incidence was found to be less than 3^o for a layer with 30^o dip when encountered by a wavefront at 34^o incident angle (*Figure 4)*.

To investigate the effect of the flat layer assumption on AVO attributes, synthetic seismic gathers were calculated for each dipping model using Zoeppritz's equations. Amplitude variations with angle were analysed for each modeled gather (0 to 45 degrees dip). AVO attributes P-reflectivity and S-reflectivity were extracted from the synthetic gathers using Fatti et al.'s (1994) approximation to the Zoeppritz equations. Data was analyzed out to a maximum incidence angle of 36 degrees. Fluid Stack, P-impedance, S-impedance, and LambdaRho™, MuRho, and Lambda/Mu were calculated from these seismically-derived AVO attributes.

Results & Implications

The results of AVO extraction and inversion indicate that the error *in these attributes* introduced by assuming dipping layers to be flat is relatively small. Quantitative AVO analysis is stable, even when we assume dipping layers are flat, and even at high angles of incidence.

Future work

This simple modeling investigation does not account for the mis-positioning that occurs when reflections are received from dipping layers. That is, although the error in estimated angle of incidence due to flat dip assumption is small and has manageable or negligible effects on AVO attributes, the attributes will still be mis-positioned due to dip.

Further, it is acknowledged that the orientation of the model – with receivers updip from the source, so that the source is shooting 'into the dip' – is simpler than the case where the receivers are downdip from the source. In such a case, there would be a more severe imaging problem because the image reflection point is not between the source and receiver. Future work should investigate this shooting orientation as well.

Another assumption inherent in the Zoeppritz modeling is the plane wave assumption. This may be an insufficient approximation to actual spherical wavefronts, particularly at shallow reflectors, where waves have not traveled far enough to become 'planar'. Modeling with the full elastic wave equation, although considerably more expensive, would be more accurate than modeling with the Zoeppritz equations; another future work item.

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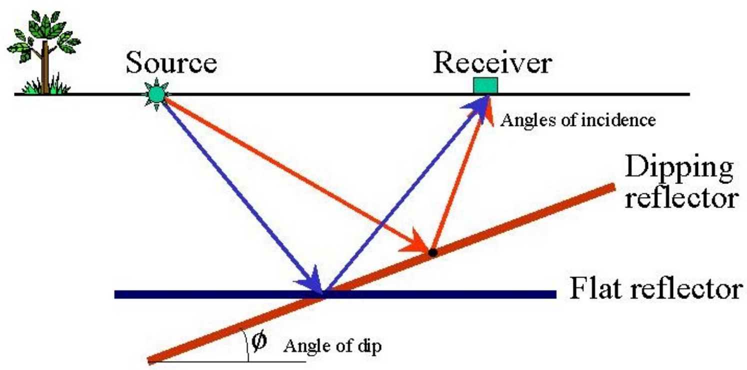

Figure 3: Orientation of source , receiver, and reflector for modeling study.

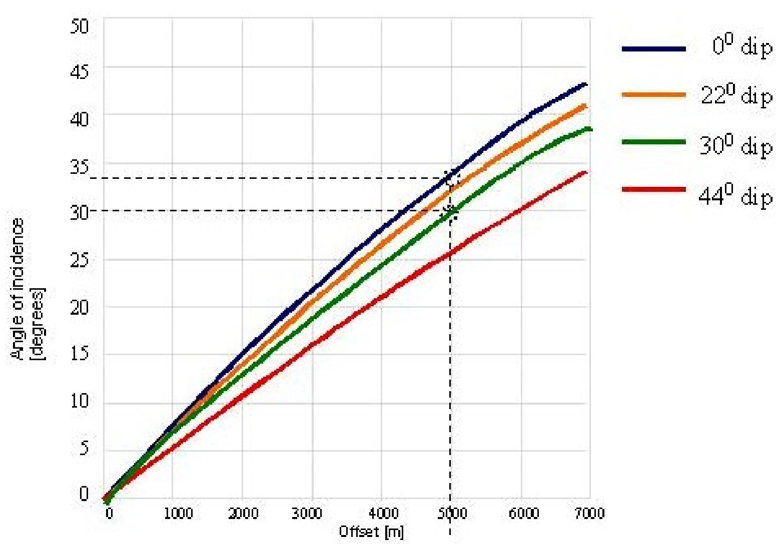

Figure 4: Actual angles of incidence vs. offset for reflection from top of dipping layer, for dips of 0^0, 22^0, 30^0 and 44^0.

Comparison of angles of incidence from dipping reflectors with angles of incidence from flat reflector (0^0 dip) shows the error in estimated angle of incidence that is incurred when we assume flat dip. Note that the error less than 3^o for a layer with 30^o dip when encountered by a wavefront at 34^o incident angle (34^o incident angle is a typical maximum far angle for AVO analyses).

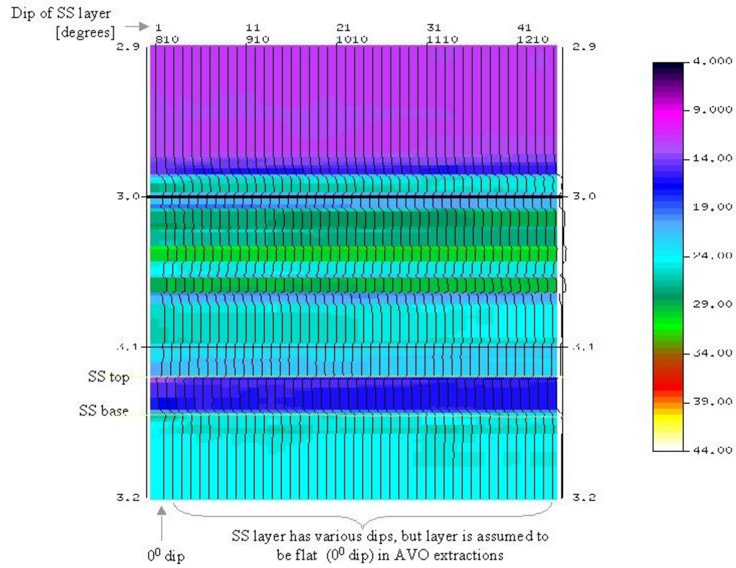

Figure 5: Modeled LambdaRho values for various dips.

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