Elastic Inversion for Lamé Parameters

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Abstract
A new, direct method for the extraction of the fundamental rock properties expressed by Lamé's parameters, Lamé's constant (λ) and shear rigidity (μ), from pre-stack seismic data is proposed. It will be shown that this new method is more stable and less ambiguous than the method currently used to extract these parameters from seismic data.

Introduction
It is easier to understand the connection of reservoir properties to fundamental rock properties such as compressibility and rigidity, than it is to understand their connection to traditional seismic attributes, like amplitude and velocity (Gray and Andersen, 2000). Goodway et al (1997) proposed a method to extract rock properties $\lambda\rho$ and $\mu\rho$; where λ, μ and ρ are Lamé's parameters: Lamé's constant (closely related to incompressibility), shear modulus and density, respectively. Lamé's parameters are often considered to be fundamental elastic constants. Goodway's method has been shown by Gray and Andersen (2000) and Soldo et al (2001) to be generally applicable for exploration and development of reservoirs in various geological settings throughout the world and by Chen et al (1998) to be useful for detailed reservoir characterization.

This paper proposes an improvement to Goodway's method, that is based on the Gray et al (1999) re-expression in Lamé's parameters of Aki and Richards (1980) approximation to the Zoeppritz equation, and post-stack inversion methods. This new method extracts λ and μ without the ambiguity introduced by the density parameter, ρ, in $\lambda\rho$ and $\mu\rho$. Significantly, the new method should also be more stable statistically. Therefore, if this new method can successfully extract these rock properties, it is an improvement on Goodway's method, which has already been used successfully in many reservoirs.

Method
Gray et al (1999) re-expressed Aki and Richards (1980) approximation of the Zoeppritz equations in terms of the parameters $\Delta\lambda/\lambda$, $\Delta\mu/\mu$ and $\Delta\rho/\rho$; that is, the reflectivity of Lamé's constant, the shear modulus reflectivity and density reflectivity, respectively. Amplitude versus Offset (AVO) analysis using this equation allows $\Delta\lambda/\lambda$ and $\Delta\mu/\mu$ to be extracted from conventional, pre-stack, P-wave seismic data. It is proposed that the reflectivity of Lamé's constant and the shear modulus reflectivity, extracted by this AVO analysis, can be inverted using post-stack inversion to derive the individual, fundamental rock properties λ and μ from conventional seismic data. Since it is possible to solve for the individual parameters using this new method, their interpretation is less ambiguous than that of $\lambda\rho$ and $\mu\rho$. This is because $\lambda\rho$ and $\mu\rho$ suffer from additional ambiguity caused by the density term, ρ.

Goodway's method calculates $\lambda\rho$ from the squares of the P-impedance (I_p) and the S-impedance (I_s) using subtraction. These impedances are generated from seismic data and are therefore subject to measurement error. If it is assumed that the measurements of these impedances have a normal distribution, then it can be shown that squaring them introduces a bias into the results $\lambda\rho$ and $\mu\rho$ that is approximately equal in magnitude to the variance of I_s. In addition, taking the square of these measurements approximately halves the signal to noise ratio. Since the squares of the impedances are positive, subtracting them to calculate $\lambda\rho$ increases its potential error. In fact, it can be shown that the error associated with $\lambda\rho$ is greater than two times that associated with $\mu\rho$ and therefore about four times greater than the error associated with I_s. Mathematical proof for these assertions is given in the Appendix.

The new approach derives λ by inverting for it directly from the $\Delta\lambda/\lambda$ derived from Gray's AVO equation. The same procedure is followed for the calculation of μ from $\Delta\mu/\mu$. Since squaring is no longer required, then there is no bias in the result and the signal to noise (S/N) ratio does not get worse. Since no subtraction is required to calculate λ, its potential error should be less than Goodway's $\lambda\rho$.

This presentation shows Gray's result inverted directly for λ and μ for the synthetic data used in Gray et al (1999). Comparisons of the inversions to correct values of λ and μ derived from the logs are shown for these data. The new method is also tested on real seismic data containing both clastic and carbonate sequences from Erskine, Alberta, Canada. For these data, the new method is compared to Goodway's to determine which method has a better S/N ratio and the results are compared to λ and μ logs calculated for wells in this reservoir to test it for accuracy.

Results
The most striking observation is the comparison between Goodway's $\lambda\rho$ and λ calculated using the new method (Figure 1). Here it is clear that $\lambda\rho$ is much noisier than λ derived by the new method. This is a visual confirmation of the statistical result, derived in the Appendix, showing that the variance of $\lambda\rho$ should be about four times that of I_s; the variance of λ should be close to that of I_s. Additional benefits accrue from not having to deal with the density term, ρ, and from not having a bias in the answer for real seismic data.

One of the benefits of the removal of the density term is that the results of the new method are isolated elastic constants. Therefore, other elastic constants can be calculated from them. In Figure 2, synthetic pre-stack P-wave data are inverted using the new method. On the left is an inversion for λ compared to λ calculated directly from the logs. On the right is a compressibility section derived from the reciprocal of the bulk modulus derived using the new method from its reflectivity calculated from Gray et al's (1999) Equation 1.

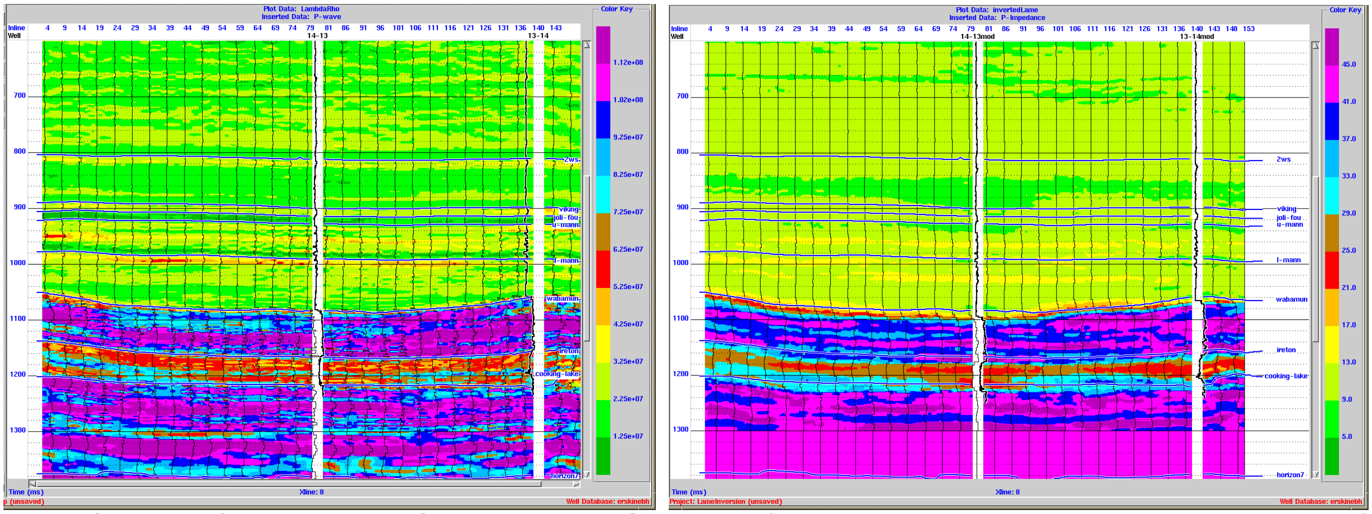

Figure 1: On the left is λρ calculated using Goodway's method. On the right is λ calculated using the new direct method. As expected, the λρ plot is noisier than the new inversion for λ. Inserted in the sections are λρ and λ logs at two well locations for comparison.

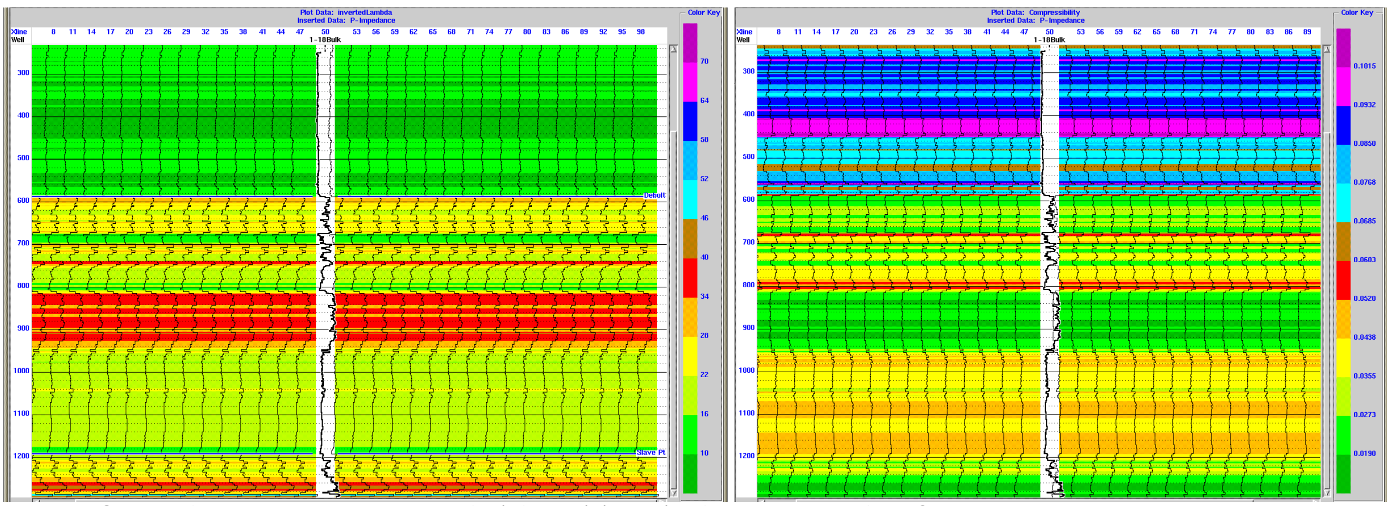

Figure 2: On the left is the new direct inversion for λ from Δλ/λ, the λ reflectivity derived from Gray's equation on synthetic data. Inserted is an exact λ log calculated from the P-wave sonic, S-wave sonic and density curves for this well. The new inversion for λ captures all the details in the λ log. On the right is an example of one of the possibilities derived from using this method, an inversion for compressibility, which may be of interest to Reservoir Engineers.

Conclusions

A new method of extracting the fundamental rock properties, Lamé's parameters, λ and μ, by post-stack inversion of their reflectivities derived from conventional, pre-stack, P-wave seismic data using Gray's AVO equation is proposed. This new method successfully predicts λ and μ, producing results that are similar to λ and μ logs. It produces more stable results than can currently be achieved from the method of Goodway, improving the S/N by a factor of two for μ and four for λ. It also avoids ambiguity caused by the density component, ρ, in λρ and μρ. The absence of the density component allows other elastic parameters, such as compressibility, to be calculated from the results of the new method. As a result, this new method should be considered as an important extension of Goodway's method.

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Appendix

Using Goodway et al's (1997) notation:

$$\mu\rho = I_s^2 \quad \lambda\rho = I_p^2 - 2\mu\rho$$

Assuming that the measurements of the P- and S-wave impedances, Îp and Îs, follow normal distributions, $N(I_p,\sigma_p^2)$ and $N(I_s,\sigma_s^2)$, then their distributions can be represented as follows:

$$\hat{I}_p = I_p + e_p \quad e_p \sim N(0,\sigma_p^2) \quad E(\hat{I}_p) = I_p$$
$$\hat{I}_s = I_s + e_s \quad e_s \sim N(0,\sigma_s^2) \quad E(\hat{I}_s) = I_s$$

Therefore their expectations are:

$$E(\mu\rho) = E(\hat{I}_s^2) = I_s^2 + \sigma_s^2$$
$$E(\lambda\rho) = E\left(\hat{I}_p^2 - 2\mu\rho\right) = I_p^2 - 2I_s^2 + \left(\sigma_p^2 - 2\sigma_s^2\right)$$

If $\sigma_p \approx \sigma_s$, then:

$$E(\lambda\rho) \approx I_p^2 - 2I_s^2 - \sigma_s^2$$

Using the Moment Generating Function for a random variable, *x*, distributed by a Normal Distribution, $N(0,\sigma^2)$ (Hogg and Craig, 1978), then the expectations of powers of x are:

$$E(x^{2k}) = \frac{(2k)!\cdot\sigma^{2k}}{k!2^k}; \quad E(x^{2k+1}) = 0$$

Since $e_s \sim N(0,\sigma_s^2)$ and $e_p \sim N(0,\sigma_p^2)$, then the variances of $\mu\rho$ and $\lambda\rho$ can be calculated:

$$V(\mu\rho) = V(\hat{I}_s^2) = E\left[(I_s + e_s)^4\right] - E^2\left[\hat{I}_s^2\right] = 4I_s^2\sigma_s^2 + 2\sigma_s^4$$
$$V(\hat{I}_p^2) = 4I_p^2\sigma_p^2 + 2\sigma_p^4$$

Assuming that I_p^2 and I_s^2 are independent random variables:

$$V(\lambda\rho) = V\left(\hat{I}_p^2 - 2\mu\rho\right) = V\left(\hat{I}_p^2\right) + 4V(\mu\rho) = 4I_p^2\sigma_p^2 + 2\sigma_p^4 + 16I_s^2\sigma_s^2 + 8\sigma_s^4$$

If $\sigma_p \approx \sigma_s$, then

$$V(\lambda\rho) \approx 20I_s^2\sigma_s^2 + 10\sigma_s^4 = 5V(\mu\rho)$$