Reflectivity amplitude restoration in Gabor deconvolution

V. Iliescu and G. F. Margrave - Department of Geology and Geophysics, University of Calgary*

Summary

A nonstationary deconvolution technique based on the Gabor transform was applied to a synthetic trace, attenuated by a constant *Q* quality factor. Gabor deconvolution compensates for the time-variant frequency attenuation, which is the main cause of the nonstationarity of seismic signals, as well as, inverts the source signature. To do this, the source signature and the *Q* attenuation surface must be estimated by smoothing the Gabor transform of a seismic trace. Among the many types of the smoother techniques, two alternate methods are compared in this work. In the first method of smoothing, a 2D boxcar is convolved with the magnitude spectrum of the Gabor transform of the trace. In the second case, the *Q* attenuation surface is estimated by smoothing the magnitude spectrum along hyperbolic contours of $f\tau = constant$ in the time-frequency plane. Then, the source signature is estimated by removing the attenuation surface and averaging over time. The result obtained from the hyperbolic smoother shows much more accurate amplitude restoration while the result of the 2D boxcar smoother is strongly whitened but has equalized amplitudes over time, much like an AGC.

Introduction

The Gabor deconvolution presented in this work is based on a particular case of the Gabor transform that uses a Gaussian function to achieve temporal localization. Multiplying the signal by a Gaussian centred at time τ, the result is a modified signal that is a function of two times, the fixed time, τ, and the running time, *t*. A suite of localized traces are generated by repeating this operation as the Gaussian function is translated along the signal. If at every fixed time τ, a Fourier transform is applied, the result is a time-frequency decomposition. Using a white reflectivity assumption, seismic deconvolution can be conducted in this 2D (time-frequency) plane by estimating the deconvolution operator through smoothing the magnitude of the Gabor spectrum of the signal. In this paper, the deconvolution operator is derived from the magnitude spectra of the Gabor transform in two ways, through a 2D convolution with a boxcar of certain dimensions in time and frequency or smoothing the magnitude spectrum on hyperbolic contours of $f\tau = constant$. The resulting deconvolution operator inverts both, the source signature and the attenuation surface. A minimum-phase assumption is used to provide the phase of the deconvolution operator.

Nonstationary convolutional model and the deconvolution algorithm

The nonstationary convolutional model (Margrave and Lamouroux, 2002) considers the earth attenuation effect acting on the source waveform. This model adjusts the well-known stationary convolutional model by introducing a constant *Q* attenuation function. Recovering an estimate of the reflectivity in the Gabor domain is a spectral factorization problem. The Gabor transform of the seismic signal must be factored into source signature, attenuation, and reflectivity components.

A detailed mathematical description of Gabor deconvolution is given by Margrave and Lamoureux (2002). Here a summary of the spectral factorization problem of the deconvolution in Gabor domain is presented. Taking the Gabor transform of a seismic trace and considering just the absolute values of the Gabor spectra denoted by the modulus symbol, we get

$$\left|\breve{S}(\tau,f)\right| = \left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|\left|\breve{R}(\tau,f)\right|, \qquad (1)$$

where $\left|\breve{S}(\tau,f)\right|$ is the magnitude of the Gabor spectrum of a trace, $\left|\hat{W}(f)\right|$ - the magnitude of the Fourier spectrum of the source signature (stationary), $\left|\alpha(\tau,f)\right|$ is the magnitude of the attenuation function, and $\left|\breve{R}(\tau,f)\right|$ is the magnitude of the Gabor spectrum of the reflectivity.

The relation between the attenuation α, in equation (1) and the quality factor *Q* is (Kjartansson, 1979)

$$\left|\alpha(\tau,f)\right| = e^{\frac{-\pi f\tau}{Q}}. \qquad (2)$$

In order to estimate the reflectivity from the trace, first, we estimate the source signature and the attenuation function. Assuming that the reflectivity series has the statistical properties of random white noise such that $\left|\overline{\breve{R}(\tau,f)}\right| = 1$ (where the overbar denotes smoothing), a smoothed version of the magnitude Gabor spectrum of the seismic signal will give an estimate of the embedded wavelet combined with the attenuation function. Two techniques of smoothing will be discussed next in this paper. First the boxcar smoother will be discussed and second, hyperbolic smoothing will be presented.

The boxcar smoother

Smoothing the Gabor magnitude spectrum of the seismic trace through a 2D convolution with a 2D boxcar tends to suppress the reflectivity information and will therefore, estimate the source signature of the spectrum times the attenuation function. In this case, the attenuation and the source signature are estimated as a single entity and the size of the boxcar in time and frequency significantly affects the result. The frequency dimension of the smoothing window determines the number of the points to be smoothed along the frequency axis and controls the temporal size of the assumed source signature estimate. Shorter source signatures have smoother spectra. The time dimension of the smoothing operator determines the number of spectral points to be smoothed in time. This parameter controls the nonstationarity of the deconvolution. The longer this value, the more stationary the deconvolution becomes. Consequently, smoothing the magnitude Gabor spectrum will yield a combined estimate of the source signature and the attenuation function

$$\left|\overline{\breve{S}(\tau,f)}\right| \cong \{\left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|\}_{est}. \qquad (3)$$

Thus, the magnitude spectrum of the deconvolution operator will have the form

$$\left|D(\tau,f)\right| = \left(\{\left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|\}_{est} + \varepsilon A_{\max}\right)^{-1}, \qquad (4)$$

where ε is a small real number and A_{max} is the maximum value of the smoothed Gabor spectrum (equation (3)) introduced to avoid any division by zero.

The phase information is calculated from the amplitude spectrum of the deconvolution operator with the Hilbert transform, assuming the minimum-phase condition,

$$D(\tau,f)=\frac{e^{-iH\left(\ln\left|D(\tau,f)\right|\right)}}{\left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|+\varepsilon A_{\max}}, \quad (5)$$

and the deconvolution equation in the Gabor domain has the form

$$\breve{R}(\tau,f)_{est}=D(\tau,f)\,\breve{S}(\tau,f). \quad (6)$$

This estimate of the Gabor transform of the reflectivity can be inverse transformed (Margrave and Lamouroux, 2002) to give the reflectivity in the time domain.

The hyperbolic smoother

The attenuation function described in equation (2) is constant for $\tau f =$ *constant*, that is, along a hyperbola in the time–frequency plane. Therefore, an average of equation (1) along such a hyperbolic contour will estimate the magnitude of the attenuation function provided that the reflectivity and source signature terms average to unity along the contours. Without further justification, we assume this is nearly the case and explore the consequences of hyperbolic smoothing.

Hyperbolic smoothing is achieved by calculating the average of the Gabor magnitude spectrum on constant time-frequency hyperbolae contours. Let $\sigma=\tau f$ be a hyperbolic contour of $\tau f =$ *constant*. Then, let $\tau(f)=\sigma/f$, and the hyperbolically smoothed Gabor spectrum is given by

$$\left|\overline{\breve{S}(\tau,f)}\right|_{Hyp}=\frac{\int\left|\breve{S}(\tau(f),f)\right|df}{\int\tau(f)df}\equiv\left|\alpha(\tau,f)\right|_{est}. \quad (7)$$

In equation (7), $\left|\alpha(\tau,f)\right|_{est}$ denotes the hyperbolically smoothed spectrum, an estimate of the attenuation surface. Smoothing the magnitude spectrum on hyperbolic contours means that, along every hyperbola on the time-frequency plane, an average value is computed.

Dividing the Gabor magnitude spectrum by the hyperbolically smoothed spectrum, the attenuation information is removed and the source signature can now be estimated. Let $\mu(\tau,f)$ denote this non-attenuated spectrum,

$$\mu(\tau,f)=\frac{\left|\breve{S}(\tau,f)\right|}{\left|\alpha(\tau,f)\right|_{est}}. \quad (8)$$

The (stationary) source signature can be estimated after averaging $\mu(\tau,f)$ over time as

$$\left|\hat{W}(f)\right|_{est}=\frac{\int_{0}^{\tau_{\max}}\mu(\tau,f)\,d\tau}{\tau_{\max}}. \quad (9)$$

Smoothing this result by convolution with a frequency boxcar will improve the source signature estimate.

Next, the deconvolution operator is derived by multiplying the source signature (equation 9) at all times with the hyperbolically smoothed spectrum, estimated attenuation described by equation (7), and inverting the result,

$$\left|D(\tau,f)\right|=\left(\left|\hat{W}(f)\right|_{est}\left|\alpha(\tau,f)\right|_{est}+\varepsilon A_{\max}\right)^{-1}, \quad (10)$$

where εA_{max} is the same as in equation (4).

Next, the minimum-phase information and the estimated reflectivity can be calculated in the same way as in the boxcar smoother case (equations 5 and 6).

Example

In Figure 1A is illustrated a random reflectivity that has a low amplitude interval located between 1.5 – 3 seconds. The minimum-phase source signature convolved with this reflectivity model was attenuated with a constant *Q* filter (Q=25). This represents the input trace, before deconvolution (Figure 1E). In Figure 1B, is illustrated the reflectivity after a minimum-phase bandpass filter has been applied in the Gabor domain. The high-cut value of the filter was set to a maximum 0.7 of the Nyquist frequency, this value decreasing hyperbolically (on a contour of $f\tau =$ *constant*) for longer times (Figure 2). This bandlimited reflectivity will be compared with our deconvolution results. Since the frequency bandlimit tracks along a hyperbolic path in time-frequency domain, it corresponds to some constant power level in the attenuation function. Given a constant power background noise, we expect the signal to drop below noise level along such contour. The deconvolution results are also bandpass filtered with the same filter. Figure 3 is the Gabor magnitude spectrum of the attenuated trace (Figure 1E). Figure 4 is the magnitude Gabor spectrum of the filtered reflectivity (Figure1B). Comparing figures 3 and 4 the effect of the constant *Q* attenuation function can be clearly seen. In Figure 5 is illustrated the product $\{\left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|\}_{est}$, as described by equation (3). This is the smoothed magnitude spectrum in the boxcar case. The time dimension of the boxcar was set to 0.5 seconds and the frequency dimension was 10 Hz.

The result of the deconvolved trace using the boxcar smoother is illustrated in Figure 6. The spectrum of Figure 6 is obtained by dividing the spectrum of Figure 3 by that of Figure 5. Analyzing these figures in the Gabor domain (Figures 4, 5 and 6), as well as the traces in the time domain (Figures 1B, 1C and 1D), it is apparent that the boxcar has equalized the amplitudes -in the weak and strong reflectivity zones. It can be concluded that the boxcar has an effect similar to an AGC operator applied to the trace while, as will be discussed below, the hyperbolic smoother restored more accurately the relative amplitudes.

In Figure 7 is the result of the product $\left|\hat{W}(f)\right|_{est}\left|\alpha(\tau,f)\right|_{est}$, as described by equation (10) before inversion. In Figure 8 is the result of the hyperbolic smoother (Gabor domain), the trace at position 1D in Figure 1. The spectrum of Figure 8 was obtained by dividing the spectrum of Figure 3 by that of Figure 7.

A physically valid estimate of $\left|\hat{W}(f)\right|\left|\alpha(\tau,f)\right|$ must show steadily decreasing power with increasing time. In particular, the estimate of Figure 5 cannot be physically correct because the power increases abruptly at about 3 seconds. The low power zone from approximately 1.5 seconds to 3 seconds is a residual imprint of the reflectivity that was not removed by the boxcar smoother. The estimate of Figure 7 is much more plausible though we cannot prove that hyperbolic smoothing will always give a correct result.

In Figure 9 at position A is the Fourier magnitude spectrum of the entire trace deconvolved with the boxcar (Figure 1C). At position B is the Fourier magnitude spectrum of the same trace windowed between 0 to 1.5 seconds and at position C, windowed between 1,5 to 3 seconds. The whitening achieved by the boxcar filter is excellent in all three cases. In the case of the hyperbolic smoother, the whitening level achieved after deconvolution is illustrated in Figure 9, positions D, E, and F. At position D is the Fourier magnitude spectrum of the entire trace that was deconvolved with the hyperbolic smoother, while at position E and F are the windowed intervals between 0 to 1.5 seconds and 1.5 to 3 seconds, respectively. In the hyperbolic smoother case, the magnitude spectrum is slightly less white than in the case of the boxcar smoother.

Conclusions and future work

The relative amplitudes are more correctly restored by the Gabor deconvolution performed with a hyperbolic smoother. In the boxcar smoother case the restoration of the relative amplitudes is poor and this constitutes a major drawback of this type of smoother. The temporal size of the smoother, as well as the length in frequency are important parameters in designing the deconvolution operator but in any case, the result of the boxcar smoother will be limited by the fact that it cannot provide simultaneously a satisfactory whitening level and amplitude restoration. In Figure 9, all magnitude spectra show good whitening in local time intervals but the amplitude restoration is similar to an AGC. The low amplitude interval between 1.5 to 3 seconds was severely distorted by the deconvolution (compare trace C with B in Figure 1). In the time interval from 0 to 1.5 seconds, or 3 to 4 seconds, the relative ratio of the amplitudes between the reference trace, B, and the output trace, C, was better preserved. When the length of the boxcar smoother in time is smaller than a critical interval (the low amplitude interval from 1.5 to 3 seconds in this example) the smoother fails to restore the relative amplitudes and provides a biased result.

The superiority of the hyperbolic smoother comes from the form of the *Q* attenuation function in equation (2). Figure 7 illustrates an estimate of the attenuation combined with the source signature of the trace in the time-frequency plane obtained from the input trace, equation (10), before inversion. Physically this estimate is more plausible than the estimate provided by the boxcar smoother (Figure 5). When analyzing the Fourier magnitude spectrum of the deconvolved trace on different time intervals, in the hyperbolic smoother case, (see Figure 9, D, E and F) it can be noticed that the degree of the whitening is not at the same level as in the boxcar smoother case. For all three intervals analyzed (the entire trace in example 9D, from 0 to 1.5 seconds in example 9E, and from 1.5 to 3 seconds in example 9F) the degree of decaying of the power in frequency domain is relatively constant in all cases.

In a future work, a different method of estimating the source signature can be applied. There are many possibilities to estimate the wavelet, and a weighted average can replace the simple average in equation (9).

Acknowledgements

Our work was sponsored by CREWES (consortium for Research in Elastic Wave Exploration Seismology), NSERC (Natural Sciences and Engineering Research Council), MITACS (Mathematics of Information Technology and Complex Systems), PIMS (Pacific Institute for the Mathematical Sciences), and Imperial Oil. We are grateful to all of these organizations.

References

Kjartansson, E., 1979, Constant Q-Wave Propagation and Attenuation: Journal of Geophysical Research, **84**, 4737-4748.

Margrave, G. F. and Lamoureux, M.P., 2002, Gabor deconvolution: CSEG Convention Expanded Abstracts.

Figures

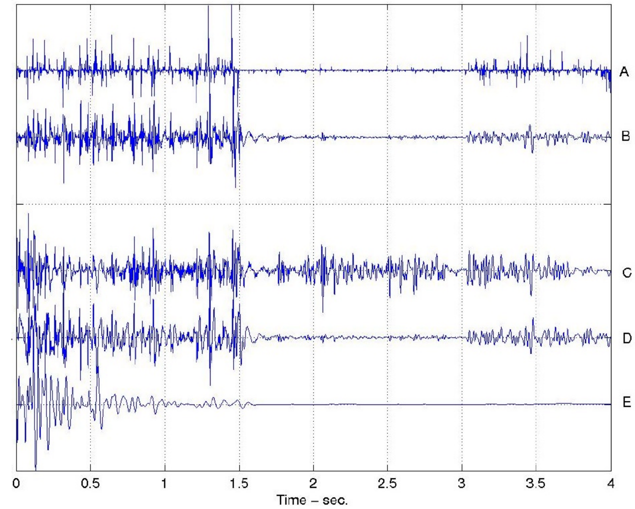

FIG. 1. Time domain results. 1A is the exact reflectivity trace, 1B is the bandpass filtered version of trace 1A(using the filter of Figure 2). Trace 1C is the result of the deconvolution using a boxcar smoother. Trace 1D is the deconvolution result from the hyperbolic smoother algorithm. Trace 1E is the attenuated trace (Q=25) that was input to the deconvolutions.

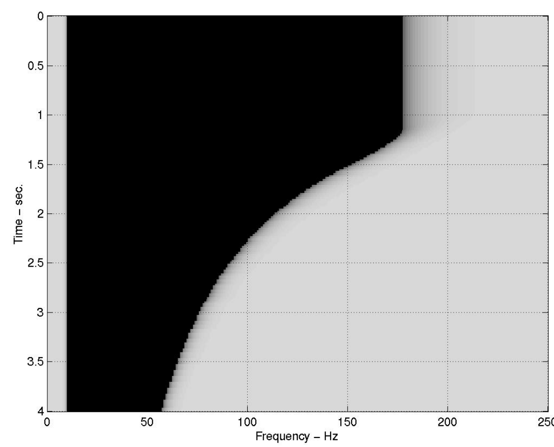

FIG. 2. Nonstationary bandpass filter.

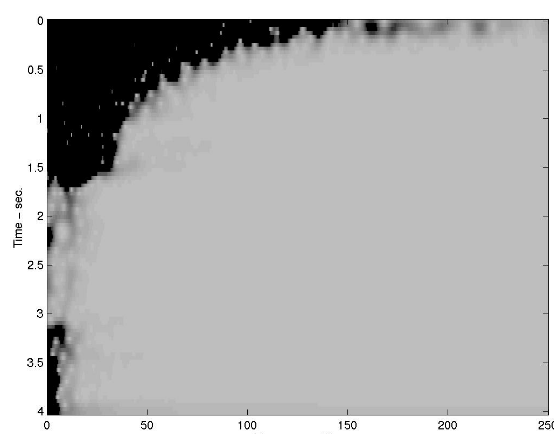

FIG. 3. Gabor magnitude spectrum of the attenuated trace (Figure 1E). This is displayed with a high gain to show subtle detail but the amplitude roll-off below 10 Hz is suppressed.

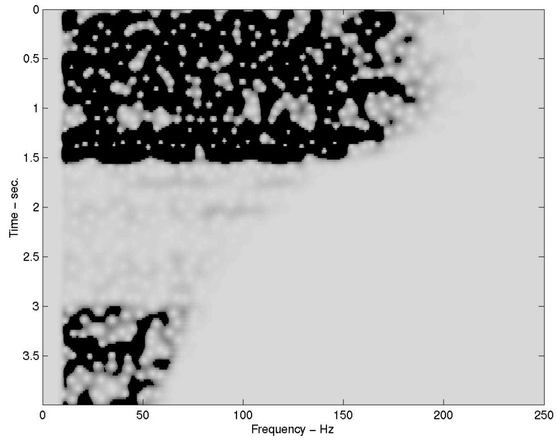

FIG. 4. Gabor magnitude spectrum of the filtered reflectivity (Figure 1B).

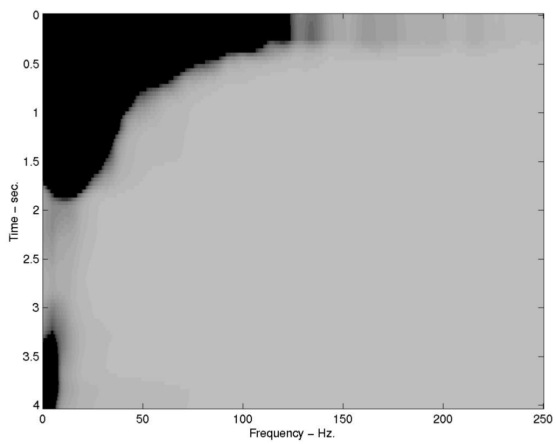

FIG. 5 Smoothed Gabor magnitude spectrum in the boxcar case (equation 3). This is displayed with a high gain to show subtle detail but the amplitude roll-off below 10 Hz is suppressed.

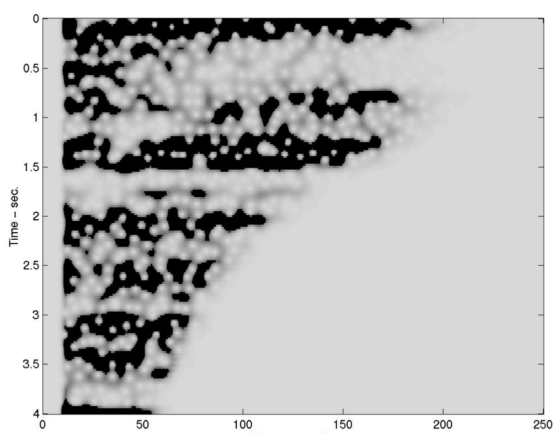

FIG. 6. Gabor magnitude spectrum of the deconvolved trace, boxcar smoother, size 0.5 sec. x 10 Hz in time domain trace at position 1C in Figure 1).

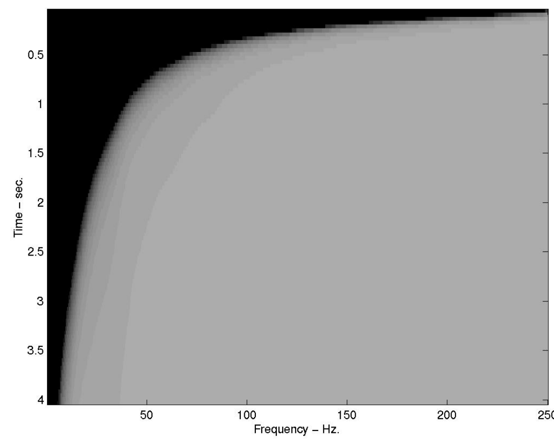

FIG. 7. Smoothed Gabor magnitude spectrum, hyperbolic smoother case. Equation (10) before inversion. This is displayed with a high gain to show subtle detail but the amplitude roll-off below 10 Hz is suppressed.

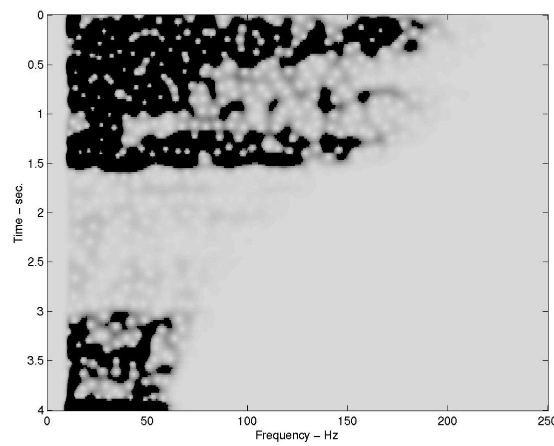

FIG. 8. Gabor magnitude spectrum of the deconvolved trace, hyperbolic smoother.

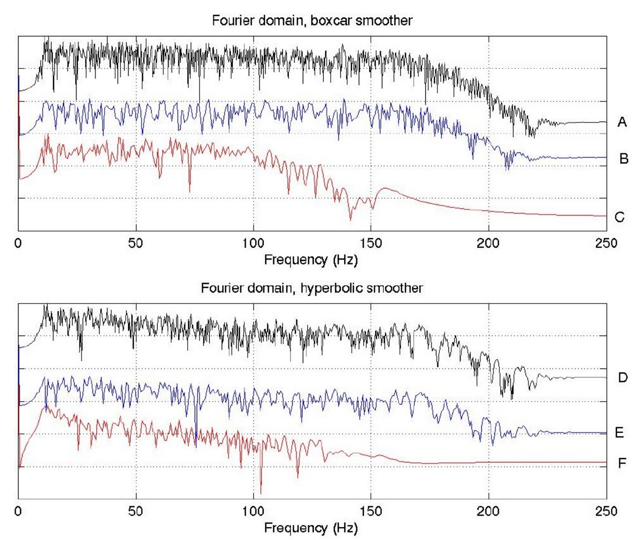

FIG. 9. Comparison of the Fourier spectra of the two smoothers.
A, B, C – Fourier magnitude spectrum of the trace deconvolved with a boxcar. A – whole trace, B - windowed between 0-1.5 seconds, C - windowed between 1.5-3 seconds.
D, E, F – Fourier magnitude spectrum of the trace deconvolved with the hyperbolic smoother. D – whole trace, E - windowed between 0-1.5 seconds, F - windowed between 1.5-3 seconds.