Sparse Spike Inversion and the Resolution Limit

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Summary

Through the model study of the linear programming sparse spike inversion we have demonstrated that the resolution limit given by Widess may also serve as a limit to uniquely recover a true geological model based on band limited seismic data and the layered earth assumption. Above this limit we can uniquely invert a model that represents the true model. The inversion failed once the thickness is below the limit.

Introduction

The vertical resolution of post stack seismic reflection data has been studied by many authors (Widess, 1973; Kallweit and Wood, 1982; de Voogd and den Rooijen, 1983; Chung and Lawton, 1995) and they all reached the similar conclusion: the resolvability of thin layers depends on the dominant frequency, or the highest terminal frequency if the wavelet has a broadband of white spectrum (Kallweit and Wood, 1982). Widess in his pioneer paper of the field "How thin is a thin bed?" derived a limit for resolvable thickness of $(1/8)\lambda_d$, where λ_d is the dominant wavelength. The composite wavelet stabilizes once the thickness of the thin bed is below the limit. There is no change in peak to trough time once the thickness of the bed below the limit, and for all practical concerns we cannot resolve the thickness.

Using Widess' definition, we have $\lambda_d = \tau V_b$, where τ is the predominant period of the wavelet and V_b is the velocity of the bed. Describing the thickness of the bed with the time for seismic wave travel through it, we have $\lambda_d/V_b = \tau = 1/f$. With the predominant frequency at 30Hz, and the seismic recording sample rate at 2ms, we have the limiting thickness of thin bed at 4ms, or 2 sample points.

Nevertheless, the seismic data does not seem to offer such high resolution as Widess resolution limit promised. It is our common knowledge that the resolution promised through a detecting wave is half of its wavelength rather than 1/8 of it. This leads to the Nyquist law of sampling the data at its half wavelength. Widess' high-resolution limit at $(1/8)\lambda_d$ is under an assumption of a specific model, i.e. a thin layer embedded in a homogenous medium. It is not a general conclusion of physics. Therefore we need to have a specific model in mind if we want to reach such a high resolution at $(1/8)\lambda_d$.

Sparse Spike Inversion

Interpreting seismic data with a specific model in mind is how interpreters incorporate the knowledge of geology into interpretation. This knowledge provides a prior information needed to compensate the inadequate of the seismic observation. Seismic impedance inversion is a tool that offers a quantitative way of incorporating the available a prior information into model building, and aid the interpretation of the seismic data. It may lead to a result that offers a high resolvability and a better link between seismic data and lithology (Li, 2001; Sacchi and Ulrych, 1995).

It is well known that for an input seismic data, there exists more than one model that its synthetic matches with the input seismic data. This may be better illustrated in Figure 1 that divides the frequency spectrum of an earth model into three distinct regions. The frequency spectrum of an input seismic data set generally only overlap with the middle frequency band, the seismic band. Lack of low and high frequency content in seismic data result in low resolution and non-uniqueness in the impedance inversion. To obtain an earth model that is any better than the seismic data we need to fill the gap of missing frequency band.

Mathematically, we may define the model with the probability $p(\mathbf{m}|\mathbf{d})$, the posterior probability of the model conditional on the observation $\mathbf{d}$. Using the Bayes' formula, we may express this probability as:

$$p(\mathbf{m} \mid \mathbf{d}) = \frac{p(\mathbf{d} \mid \mathbf{m})\, p(\mathbf{m})}{p(\mathbf{d})},$$

where $p(\mathbf{m})$ and $p(\mathbf{d})$ describes our prior knowledge on model and data respectively, and the probability $p(\mathbf{d}|\mathbf{m})$ described the discrepancy between the model and the observation. To build a model based on the probability $p(\mathbf{m}|\mathbf{d})$, we may seek a MAP solution that maximizes a posterior probability $p(\mathbf{m}|\mathbf{d})$. In linear programming sparse spike inversion, we minimize the l_1-norm of the reflectivity $\Sigma|r_n|$ as the constraint, where r_n is the reflection coefficients of the model.

To recover an impedance model with a full spectrum band, we may carefully choose our solution that honors well log data for the low frequency band, and honors high frequency band with sparse reflectivity. The latter is done by, among all the compatible models that honor seismic observations, recovering a model of sparse reflectivity through maximize a posterior probability. It yields an earth model with the fewest number nonzero reflection coefficients, and hence a model that honors the assumption of layered geology. Such algorithm would pick out the major features in the acoustic impedance structure; recover an earth model having the smallest number of layers (Oldenburg et al. 1983; Levy and Fullagar 1981).

Limit of Resolution

Consider a thin wedge bed embedded in an infinitely homogeneous medium (Figure 2). The thickness of the wedge is 2 ms on the left of the model and increases 2 ms/trace across the model to the right. The synthetic seismic is created with a Ricker wavelet of central frequency at 30Hz. At the 2ms sample rate, this wavelet has a predominant period of 16 samples. Widess' resolution limit is given as the thickness of 2 samples, which corresponds to the thickness located at the trace 2.

Below the resolution limit the waveform tends to stabilize, the amplitude decreases as the wedge getting thinner. Applying the linear programming sparse spike (LPSS) inversion, we obtain the inversion result displayed in Figure 3. The most part of the model is recovered faithfully. The inversion result starts to deteriorate at the trace 2 where the limit of resolvability is reached. Figure 4 takes a close look at the inversion results for trace 1 to 3. The true models are shown as black lines, the red lines indicate the inverted models. As we can see for the trace 3 where the thickness of the model is 3 samples, the inverted model matches with the true model perfectly. For trace 2 and 1, the inverted models cannot match the true models, however, the error between the synthetic and the seismic data is less than 1 percent within the bandwidth of the wavelet.

The discrepancy between the inverted models and the true model at trace 1 and 2 shows the nonuniqueness of inversion. The reason for the non-uniqueness of the inversion can be clearly explained with the plot of power spectrum distribution shown in figure 1. The input seismic data has a limited bandwidth, while the model we are seeking for has a much broader band of spectrum. Any energy that falls outside of seismic frequency band will not cause any discrepancy between the synthetic and the input data, but it will lead to a different model.

Since the seismic data is honored in the inversion, there isn't a nonuniqueness problem within the seismic frequency band. How could the model be determined out side of seismic frequency band will depend on the inversion methods that incorporate prior information. For a deterministic method once the criterion is set we will arrive at a unique solution. The problem of nonuniqueness becomes to what extent could the inverted model faithfully represent the true model. Figure 5 shows an example of power spectrum distribution of an inverted model. The power spectrum within the seismic band is faithfully recovered. The energy outside of the seismic band is added based on the prior information of the geological model.

The results shown in figure 3 and 4 indicate that the resolution limit given by Widess may also serve as a limit for uniquely recover a true geological model based on band limited seismic data and the layered earth assumption. Above this limit we can uniquely invert a model that represents the true model. The inversion failed once the thickness is below the limit.

Figure 6 shows a more realistic model created with a well log displayed as the black curve. The red curve shows the inverted model. The red seismic curve is the synthetic generated with the inverted model using a Ricker wavelet of center frequency at 30Hz. The synthetic overlaps on top of the black seismic curve generated with the true model. The blue line on top of the display shows the error of synthetics. As it is shown in the display that the inverted model is much simpler than the true model, but it catches the major characteristics of the true model. The synthetic seismic trace generated from the inverted model matches that of true model perfectly. This indicates that the inverted model honors all the information contained in the seismic data. In addition to seismic frequency band, it has a much broader bandwidth that yields the high-resolution blocky solution of the model (figure 5).

As we can see the discrepancy mainly shown at the time 1060ms, where the true model constitutes many thin layers below the resolution limit. It is not surprise that we cannot resolve the true model below the limit. But if the true model has all its layers larger than the resolution limit, can the inverted model faithfully represent the true model?

The P-wave log displayed in figure 6 is about 5000m/s at deeper part of the log. With the predominant frequency at 30Hz, we have the thickness of the resolvability $(1/8)\lambda_d$ at about 20m. We blocked the log with a 20 meters thickness interval and use it as the true model to test if we could recover it with LPSS inversion. Figure 7 shows the inversion result. The synthetic matches with the input perfectly as indicated by the traces plotted in the middle and the error plotted on the top of the figure. The inverted impedance model (red curve) is plotted at the bottom of the figure on top of the true model (black curve) for comparison.

Conclusion

Through the model study of the linear programming sparse spike inversion we have demonstrated that the inversion can help us to incorporate prior knowledge of the earth model and retrieve much of the information hidden within the seismic data. The model study indicates that the resolution limit given by Widess may also serve as a limit for uniquely recover a true geological model based on band limited seismic data and the layered earth assumption. Above this limit we can uniquely invert a model that represents the true model. The inversion failed once the thickness is below the limit. This practically gives us a resolution at 2 sample points for a recorded seismic data at 2ms sample rate and a predominant frequency at 30Hz.

References

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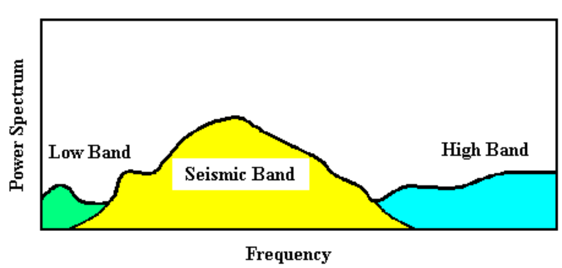

Figure 1. The frequency spectrum of the earth model versus seismic data.

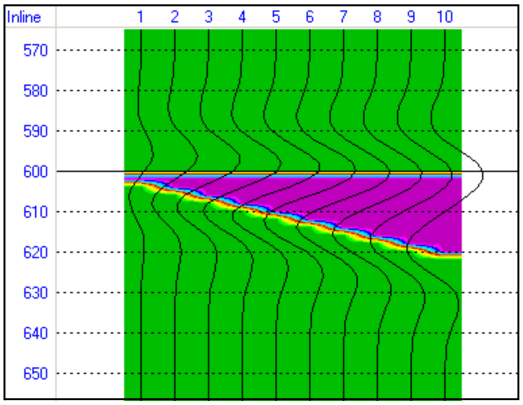

Figure 2. Synthetic traces generated from a wedge model.

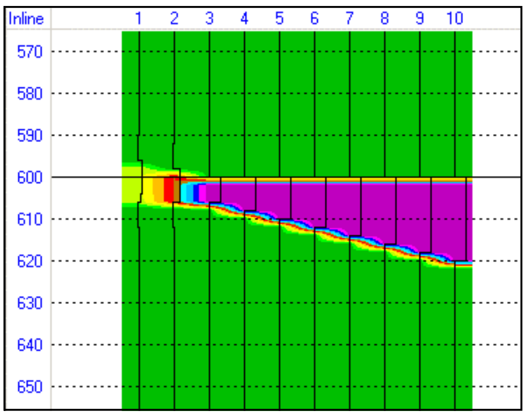

Figure 3. LPSS inversion result of the data in figure 2.

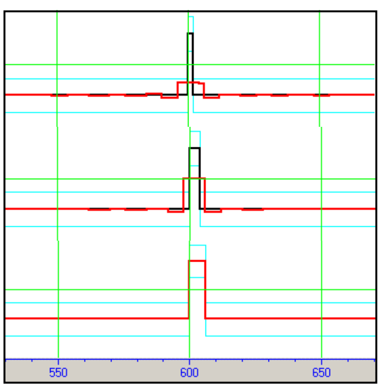

Figure 4. A close look at the inversion result near the resolution limit. The black line shows the true model, the red shows the inverted.

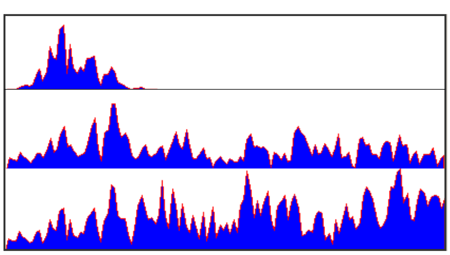

Figure 5. The displays of the power spectrum of the original data (top), that of the reflectivity of the inverted model (middle) and of the true model (bottom).

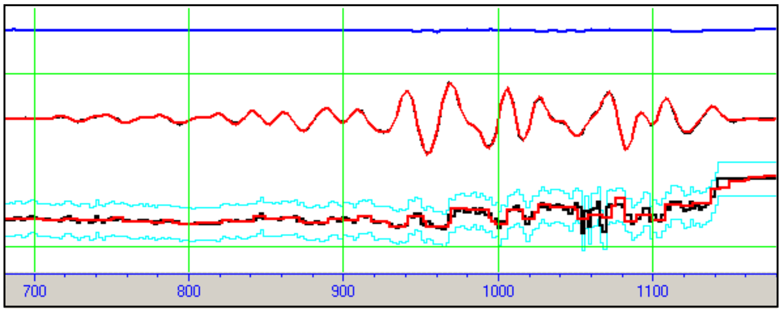

Figure 6. The top curve indicates the error of the synthetics. The middle curves are the synthetics generated from the model. The bottom curve shows the true model (black) and the inverted model (red).

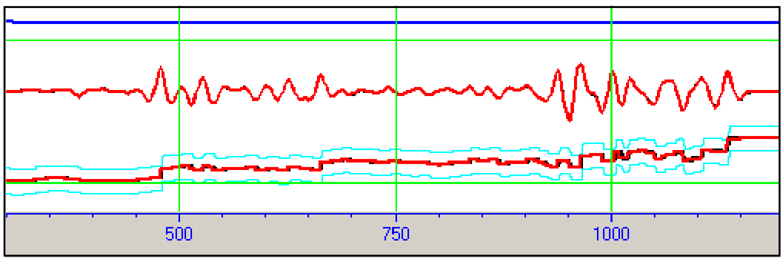

Figure 7. The top curve shows the error between the input and the synthetic. The red middle curve displays the synthetic overlap with the input (black). The bottom curve shows the true model (black) and the inverted model (red).