

Minimum DFT-weighted norm interpolation of seismic data using FFT

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Abstract

Seismic data interpolation problem can be posed as an inverse problem where from inadequate and incomplete data one attempts to recover the complete band-limited seismic wavefield. The problem is often ill posed due to the factors such as inaccurate knowledge of bandwidth and noise. In this case, regularization can be used to obtain a unique and stable solution. In this abstract, we formulate band-limited data interpolation as a minimum norm least squares type problem. An adaptive DFT-weighted norm regularization term is used to constrain solutions. In particular, the regularization term is updated iteratively using a smoothed version of the periodogram of the estimated data. The technique allows for adaptive incorporation of prior knowledge of the data such as the spectrum support and the shape of the spectrum.

Introduction

The problem of interpolation/resampling seismic data from its incomplete observations arises in many processing steps that requires a regular sampling. Many methods has been proposed, for examples, prediction error filtering based method (Claerbout, 1992; Splitz, 1991), dip moveout based interpolation (Biondi, 1999), Fourier based interpolation (Cary, 1997; Duijndam et al., 1999; Hindriks et al, 1997). Among those methods, Fourier based reconstructions do not make geological (geophysical) assumptions other than the data to be reconstructed are spatially band-limited. The methods start by posing the interpolation/resampling problem as an inversion problem where from inadequate and incomplete data one attempts to recover Fourier coefficients of complete seismic wavefield. However, the problem is often effectively under-determined which, as is well known, can be satisfied by many solutions. In this case, a regularized solution can be used where the regularizer (weighting function) serves to impose a particular feature on the solution. The criteria to choose a suitable weighting function have been discussed by several researchers (Cabrera and Thomas, 1991; Duijndam et al., 1999; Hindriks et al., 1997; Sacchi and Ulrych, 1998; Zwartjes and Duijndam, 2000). The essential ideal is to incorporate as much limited prior knowledge as possible. e.g. a regularizer imposed by the Cauchy distribution can be used to obtain an estimation of Fourier transform with sparse distribution of spectral amplitudes in Fourier domain (Sachhi and Ulrych, 1998); a weighting function based on the distance between samples or the area surrounding samples is effective in nonuniform Fourier reconstruction of irregular sampled data along one or two spatial coordinates (Duijndam et al., 1999; Hindriks et al., 1997).

In this abstract, a band-limited seismic data interpolation method is developed in space domain. The method is equivalent to a Fourier domain reconstruction when discrete Fourier transform is used to obtain the solution (Liu and Sacchi, 2001). In particular, we have modified the least squares approach to include an adaptive DFT-weighted norm regularization term which incorporates a priori knowledge of energy distribution in wavenumber domain that deals with the ill posed band-limited signal interpolation problem. And, unlike conventional method, the bandwidth of the data is not assumed to be known in our approach. An adaptive frequency weighted norm scheme has been proposed by Cabrera and Parks (1991) to extrapolate time series. In their approach, the method of modified periodiom is used to obtain adaptive weights from a previous estimation of the time series. In our approach, the weighting function is updated through iterations using a smoothed version of the periodogram. The smoothing was done by convolving a suitable function which is useful as a mean to reduce to irregularity in the spectrum introduced by missing samples. In addition, we show that the iterative regularization can be done very efficiently using FFT and preconditioned conjugate gradient algorithm. The new method can be applied to seismic data in any domains with one or two spatial coordinates. Finally, examples illustrate effectiveness of the method for 3D real seismic data interpolation.

Adaptive weighted regularization

We will denote y the length- M vector of observations and x the length- N vector of unknowns such that

$$y = \mathbf{T}x \quad (1)$$

where \mathbf{T} is the $M \times N$ sampling matrix of the problem. The interpolation problem can, therefore, be posed as an inverse problem where from inadequate and incomplete data y one attempts to recover complete data x . Note that the problem is rank deficient. The uniqueness of solution of the problem can be imposed by defining a regularized solution by solving the problem which is often expressed by

$$J(x) = \mu^2 \|Lx\|_2^2 + \|\mathbf{T}x - y\|_2^2, \quad (2)$$

where $\|\cdot\|_2$ stands L_2 norm, μ is a specified weighting factor controls the trade off between the data norm and misfit of observations. In this abstract, we have modified the regularization in (2) using DFT-weighted norm, in which case the particular object function to be minimized is

$$J_{wr}(x) = \mu^2 \|x\|_p^2 + \|\mathbf{T}x - y\|_2^2. \quad (3)$$

Here $\|\cdot\|_p$ denotes DFT-weighted norm defined by

$$\|x\|_p^2 = \sum_{k \in \Omega} \frac{X_k^* X_k}{P_k^2} \quad (4)$$

where X_k is DFT of x , k is the DFT index, Ω denotes the pass-band of the data and P_k^2 is a positive function which is similar in shape to the power spectrum of the unknown data. In equation (3), taking derivatives of $J_{wr}(x)$ and equating to zero yields the following result

$$\hat{x}_{wr} = (\mathbf{T}^T \mathbf{T} + \mu \mathbf{Q}^{-1})^{-1} \mathbf{T}^T y. \quad (5)$$

Where \mathbf{Q} is a circulant matrix which can be decomposed into

$$\mathbf{Q} = \mathbf{F}^H \mathbf{P}^2 \mathbf{F} \quad (6)$$

and \mathbf{P}^2 is a diagonal matrix with diagonal entries equals to P_k^2 .

Iterative solution

To obtain the above regularization solutions, in practice, one should know the power spectrum of the data to obtain the weighting function \mathbf{Q} . Unfortunately, x is the unknown of our problem. The latter can be overcome by defining an iterative $\mathbf{Q}^{(n)}$ using a smoothed version of the periodogram of perviously estimate of data $x^{(n-1)}$. Furthermore, let $g = \mathbf{Q}^{-1/2} x$, equation (5) can be solved using a $\mathbf{Q}^{1/2}$ preconditioned CG

$$\begin{pmatrix} \mathbf{T} \mathbf{Q}^{1/2} \\ \mu^{1/2} \end{pmatrix} g = \begin{pmatrix} y \\ 0 \end{pmatrix}. \quad (7)$$

Note that $\mathbf{Q}^{1/2} = \mathbf{F}^H \mathbf{P}$, CG iterations can be implemented very efficiently using FFT. Finally, the estimation of x can be obtained using $x = \mathbf{Q}^{1/2} g$. The extension of above iterative algorithm for 2D interpolation is straightforward in which case CG is implemented using 2D FFT.

Examples

In Figure 1, we show an example of 3-D seismic data interpolation along two spatial coordinates using above algorithm. Figure 1a) shows a post-stack data cube include 51 lines in the y-direction and 31 lines in the x-direction. Figure 1b) shows an incomplete data cube where approximately 50% lines in the y-direction are randomly removed from the original data cube and Figure 1d) shows an incomplete data cube where approximately 50% traces of the data cube (in Figure 1a)) are randomly removed. The incomplete data cubes are used as the input to test our interpolation algorithm. The interpolation is carried along the x and y directions simutanously. The results are shown in Figure 1c) and Figure 1e) respectively. In both case, the interpolation method yields a good estimation of unknown data with small error.

Conclusions

In this abstract, we formulate band-limited interpolation as minimum norm least squares type problem where an adaptive DFT-weighted norm regularization term is used to constrain solutions. The method enables us to incoorprate both band-width and spectrum shape of the data as a prior knowledge into the band-limited data interpolation problem. And unlike conventional band-limited data interpolation, the bandwidth of seismic data is not assumed to be known in the weighted norm interpolation. The adaptive regularization algorithm is implemented using a preconditioned CG and FFT. It is very efficient and can be applied to seismic data in any domains with one or two spatial coordinates.

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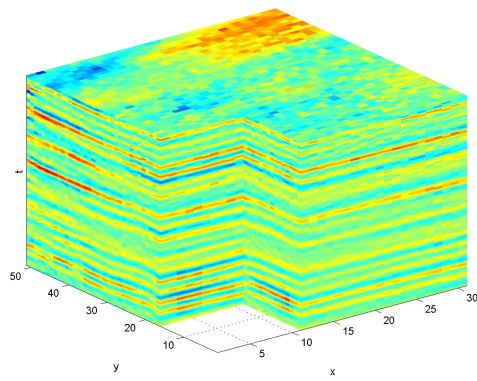
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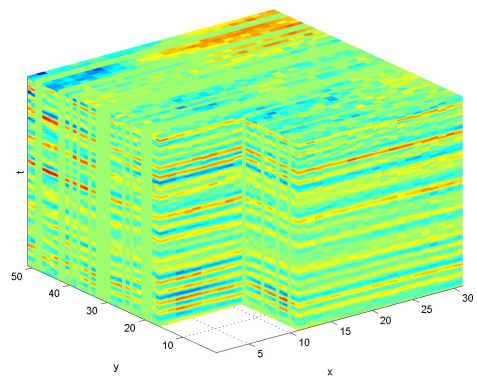
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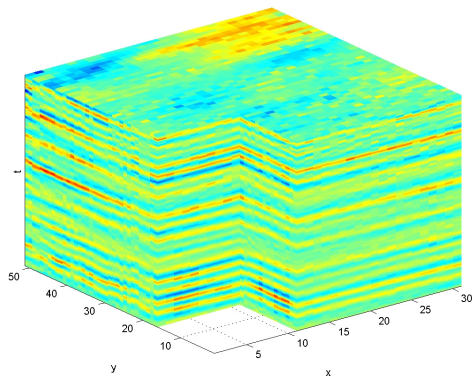
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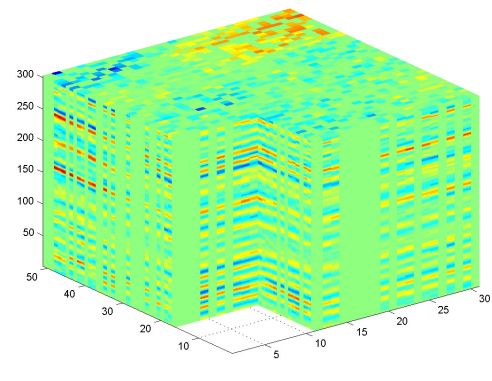
(a)



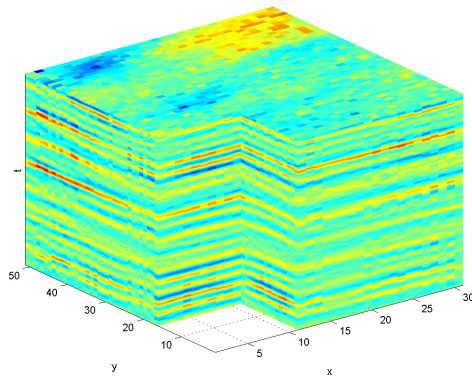
(b)



(c)



(d)



(e)

Fig. 1. 3-D seismic data interpolation along two spatial coordinates. (a) A post-stack data cube. (b) Incomplete data where 50% lines in the y direction are randomly removed from the original data. (c) Result of interpolation from (b). (d) Incomplete data where 50% traces are randomly removed from the original data. (e) Result of interpolation from (d).