Bayesian porosity inference using rock physics and geostatistical modeling

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Introduction

This is an investigation of the problem of constructing a porosity model from multiple sources of information, considering a certain degree of uncertainty associated with each one of these sources. A methodology for porosity inference is developed using fundamental concepts of the Bayesian methodology, which is the direct use of probability theory as a method of inference. The main goal in Bayesian methodology is to obtain the posterior probability density function (PDF) for parameter under investigation, which serves to answer all questions of inference. The posterior is the product of two functions: the prior PDF, which carries prior information about the unknown porosity, and the likelihood function, which carries data-fit information. In this work, we consider the combination of elastic seismic attributes and well log data in the process of obtaining the likelihood function.

Seismic data have been the main source of information for determining reservoir properties in the interwell region. The most frequently employed methodologies are based on multivariate regression, which treat the data as spatially independent, or geostatistics. The latter is highly dependent on a model of spatial variability constructed from the data.

In this work, we follow the local approach of Moraes and Scales (2000), to derive marginal probabilities for porosity at a particular cell of the reservoir instead of working with the joint distribution. The final solution is given by a set of posterior distributions for interval porosity given the well log and seismic attribute data. Questions of inference such as estimates or associated uncertainty is addressed to these posteriors.

The posterior distribution for porosity is obtained, considering only prior information about bounds on porosity variation and data generated from seismic attributes, composed by P and S wave velocities (V_p and V_s, respectively) and density (r), and well logs (porosity, sonics (P and S wave), density and gamma ray). Equations from rock physics are used to relate seismic attributes to porosity. We assume that the seismic attributes are provided by a generic seismic inversion program without any kind of uncertainty analysis.

The application of the methodology follows a two step procedure. First, we run a separated inversion in each well, following the work described by Loures and Moraes (1999). Next, we propagate the well information to the interwell region, using variability measures and their corresponding formula. This information and the information derived from seismic attributes are both represented in terms of likelihood functions. These functions are then combined with the prior distribution by simple use of Bayesian rules for deriving posterior probability distribution. A synthetic data example for reconstructing a slowly varing porosity model illustrates how the methodology works.

Methodology

Consider a reservoir model composed by a set of *N* block cells with average porosity $f \in R^M$. Our problem is to find the porosity of the *i*th cell f_i, $i=1,¼,M$, represented simply by f, using elastic seismic attributes V_p, V_s and r, respectively represented by $\mathbf{s}=(\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3)^T \in R^{3N}$, and additional data carrying information about the spatial variability of reservoir porosity, which is represented by $\mathbf{v} \in R^L$.

Our choice is to make $\mathbf{v}$ a set of experimental porosity-porosity variogram values computed from subsurface porosity information, after integration of multiple well log data sets by a 1-D Bayesian inference procedure of Loures and Moraes (1999). These authors have shown that the integration of different types of well log data provides a considerable reduction of systematic and random components of error, which may occur when deriving porosity estimates from a single type of well log data. The source of uncertainty is specific of each type of log, e.g. logging tool calibration processes. The 1-D Bayesian well log inversion methodology provides a porosity model for a set of depth intervals at multiple well locations and the associated uncertainty. This is represented in Figure 4 by a color images corresponding to 5 diferent well location. The color variation represents the probability density for porosity at a fixed depth along the well.

Bayesian formulation

According to the Bayesian methodology, the solution to this problem of inference is given by the posterior PDF for porosity given the data. This posterior distribution can be obtained by application of Bayes' Theorem which gives

$$p(f \mid \mathbf{v},\mathbf{s}) \propto l(\mathbf{v},\mathbf{s} \mid f)q(f)\,, \tag{1}$$

where $q(f)$ is the prior distribution, representing any information which are independent from data, and $l(\mathbf{v},\mathbf{s}|f)$ is the likelihood function. The latter corresponds to the data distribution, representing the uncertainty in the data and incorporating the relations between porosity and the data.

Assuming that the only relevant prior information comes from lower and upper bounds on porosity (f_l and f_u), an uniform distribution can be assigned for the prior. In this case, a standard procedure in Bayesian inference is to incorporate the prior into a proportionality constant. So the posterior becomes a normalized product of two data distributions:

$$p(f \mid \mathbf{v},\mathbf{s}) \propto l_p(\mathbf{v} \mid f) l_s(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3 \mid f), \quad \mathit{for}\ f_l \le f \le f_u. \tag{2}$$

In this work, we assume lack of information regarding the method used to obtain seismic attributes making it an independent process. This is often the situation when using commercially available software to generate attribute volumes. Although we admit that correlations and higher order moments exist, they are not known. In this case, assuming an uncorrelated model is a more conservative decision than adopting any ad hoc correlation model which may bias the estimates.

Assigning probabilities

Next task is to define mathematical forms for the likelihood functions $l_p(\mathbf{v}|f)$ and $l_s(\mathbf{s}|f)$. To do that, it is first necessary to specify the relations between data and the unknown porosity. In our case, assuming additive errors in the data, we may write

$$\mathbf{s}_i = \mathbf{f}_i(f) + \mathbf{e}_i, \quad i = 1,\ldots,3 \tag{3}$$

and

$$\mathbf{v}_i = \mathbf{f}_4(f) + \mathbf{e}_4, \tag{4}$$

In the first case (f_i, i=1,2 and 3), we seek for equations of seismic attributes as a function of porosity. These are available in the rock physics literature. For instance, Eberhart-Phillips et al. (1989) derive empirical formulas for V_p and V_s, which are given by

$$f_1 \equiv V_p = 5.77 - 6.94f - 1.73\sqrt{c} + 0.446(Pe - e^{16.7Pe}) \tag{5}$$

and

$$f_2 \equiv V_s = 3.70 - 4.94f - 1.57\sqrt{c} + 0.361(Pe - e^{16.7Pe}), \tag{6}$$

where P_e is effective pressure and c is the clay content. For density, we use

$$f_3 \equiv r = (1-f)r_m + fr_f\,, \tag{7}$$

where r_m is the matrix density and r_f is the pore fluid density.

In the case of modeling operator f_4, our choice of the experimental variogram data $\mathbf{v}$ as the measure of spatial variability of porosity makes f_4 the variogram function. This function involves pairs of well log porosity values f and the unknown cell porosity, which is given by

$$g(h) \equiv f_4(f) = \frac{1}{2NP}\sum_{i=1}^{NP}\left[\hat{f}(\mathbf{r}_i) - f(\mathbf{r}_i + \mathbf{h})\right]^2, \tag{8}$$

where $\mathbf{r}_i$ and $\mathbf{r}_{i+h}$ represent two different locations separated by a lag vector $\mathbf{h}$ of size h and NP is the number of pairs.

Next, we select the normal PDFs to describe the errors in equations (3) and (4). According to the maximum entropy criterion (Jaynes, 1978), this implies that first and second order moments are, for our purposes, appropriate to describe the uncertainty in the data.

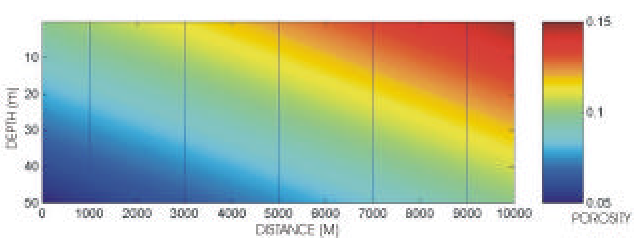
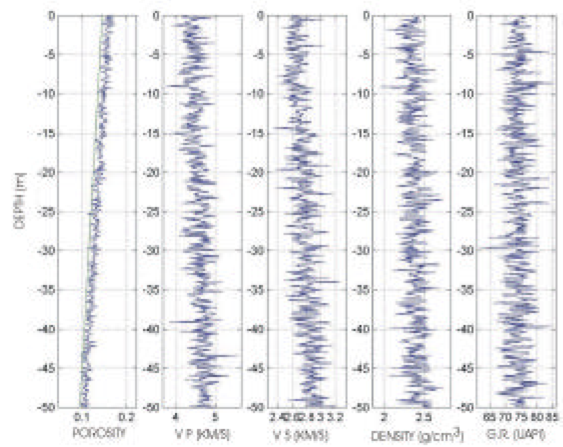


Figure 1. Image representing the true porosity model used in the synthetic data example. Vertical lines show the locations of 5 wells distributed across the model.

Figure 2 Synthetic well log data for the first well represented in the model of Figure 1. These are, from left to right, neutron porosity, sonic logs (P and S waves), density and gamma ray. All log data are corrupted with pseudo random gaussian noise. In addition, the porosity neutron log has a shift of 10 % of the true porosity model to simulate a calibration error. The green line in the porosity log plot represents the true porosity.



Posterior probabilities

Mathematically, normal distributions depend on two parameters, mean and variance. Consequently, choosing normal distributions to describe errors in the data introduces new parameters into the posterior distributions, the data variances. These are unknown which we have no direct interest in infering. Marginalization of the posterior distribution is a standard tool in Bayesian inference to eliminate parameters such as these. This marginalization process consists of integrating the posterior distribution with respect to data variances. Integration of normal distributions with respect to the variance yields t-student distributions.

If the above procedure is carried out in our porosity inference, the final porosity posterior distribution is a t-student distribution. Useful inferences can be taken from a measure of a central tendency for the estimates of porosity (e.g. mean or mode) and the length of an interval of a fixed probability for a measure of the associated uncertainty.

Implementation

Using a moving window, running across the reservoir, a distribution $l_s(\mathbf{s}|f)$ is calculated for center position of each window (the data vector $\mathbf{s}$ is the seismic attributes from cells falling inside the window). The Fresnel Zone can be considered for defining the dimension of the window, allowing it to vary across the reservoir. In the same way, $l_p(\mathbf{v}|f)$ is also computed for each cell position. Finally, both distributions are combined by the application of Equation (2) to yield one posterior distribution for each cell of the reservoir.

The final results are represented by two images of the discretized reservoir. One image shows the mode of the posterior distributions, representing the final estimated porosity model, and another image shows the length of 0.95 posterior probability centered at the mode, representing the associated model uncertainty.

A synthetic data example is presented to show how this methodology works. Three different tests are performed to evaluate the importance of each set of data $\mathbf{v}$ and $\mathbf{s}$ in increasing the confidence of porosity estimates: using only data set $\mathbf{s}$, which gives the posterior $p(f|\mathbf{s}) \propto l(\mathbf{s}|f)q(f)$, using only the data set $\mathbf{v}$, which gives $p(f|\mathbf{v}) \propto l(\mathbf{v}|f)q(f)$, and using both data sets ($p(f|\mathbf{v,s}) \propto l(\mathbf{v,s}|f)q(f)$).

Synthetic data example

Using a 2-D model of vertical and lateral changes in porosity (Figure 1) and equations (5), (6) and (7), we simulated all required data: neutron porosity, P and S-wave sonics, density and gamma ray logs and seismic attributes of V_p, V_s and r. The gamma ray log is used to obtain information about clay content of the medium, which is required in equations (5) and (6). The clay content c of this model is constant and is equal to 0.5.

All simulated well log data are corrupted with pseudo-random gaussian noise with zero mean. Additionally, a systematic error component of 10 % is included in the neutron porosity log. The noise corrupted logs are shown in Figure 2. In the case of seismic attributes, the standard deviation of the noise is defined based on examples of elastic inversion available at literature (e.g., Debski and Tarantoa (1995)), respectively 10 %; 20 % and 30 % for V_p, V_s and r. The noise corrupted seismic attributes, which are shown in Figure 3, constitutes data vectors $\mathbf{s}_i$, for $i=1,2,3$.

As described above, the first step in the application of the proposed methodology is to proceed with the inversion of well log data. Figure 4 shows the resulting porosity PDFs for each depth interval along the wells. The mode of these PDFs are estimates for interval porosities at the wells. Data vector $\mathbf{v}$ is generated from the well porosity estimates using experimental horizontal variogram calculated with a lag spacing of 2 km.

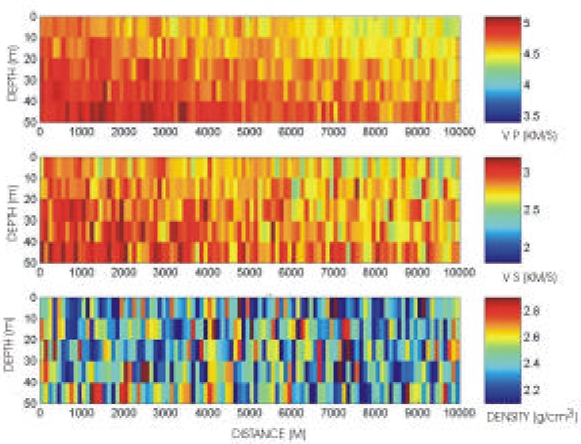


Figure 3: Seismic attributes V_p, V_s and r respectively the images from top, meddle and bottom calculated from the geologic model (Figure 1), using the petrophysical models given by equations (5), (6) and (7) and corrupted with pseudo random gaussian noise with mean zero.

The next step is the evaluation of both functions $l_s(\mathbf{s}|f)$ and $l_p(\mathbf{v}|f)$ to compute the posterior as their product. To do that, we use an interpretative model composed of cells 100 m wide by 10 m thick. A moving 2-D window, covering three cells (10 x 300 m), is used to obtain the likelihood in each cell of the reservoir. As explained before, to evaluate the individual contributions of well information and seismic attributes data, the posterior distribution is computed using three different data combinations: using well and attribute data individually ($p(f|\mathbf{s})$ and $p(f|\mathbf{v})$) and both data sets combined ($p(f|\mathbf{v,s})$).

The porosity models obtained by the mode of posterior distributions are shown in Figure 5. At the top (Figure 5a), one can find the result obtained just from the use of variogram data ($\mathbf{v}$). The middle figure (Figure 5b) shows the result obtained just from the use of seismic attribute data ($\mathbf{s}$). Finally, Figure 5c shows the result of using both data types ($\mathbf{v}$ and $\mathbf{s}$). Reasonable models of porosity are obtained. The model shown by Figure 5a has a horizontal variation pattern characterized by step variations. The positions of these steps are related to the lag limits and well positions. The model shown by Figure 5b has high frequency horizontal variations derived from the random noise in the seismic attribute data. The Figure 5c shows a porosity model that has a more slowly varying porosity than the model from Figure 5a and no high frequency variations around the well position, where the data $\mathbf{v}$ has more influence on the estimates.

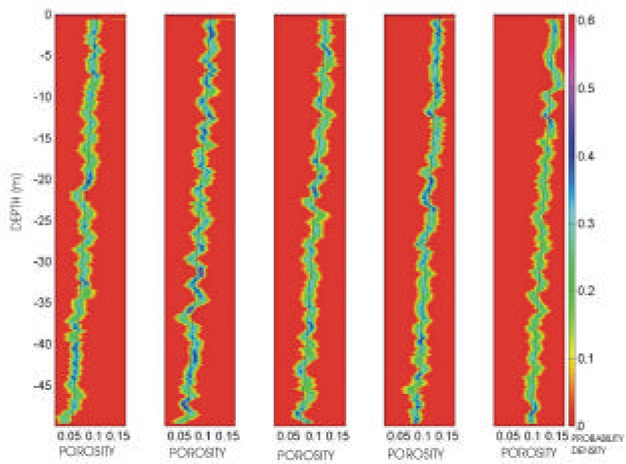


Figure 4: Images representing the distributions for interval porosity for each well from the model (Figure 2), as the results of the application of the inversion procedure by Loures and Moraes (1999). For a fixed depth interval, the color scale gives the posterior probability density distribution for porosity. Porosity estimates are taken from the mode of the posterior at each depth interval. The spread of the distribution around the mode gives an idea of the associated uncertainty.

Figure 6 shows the length of centered interval having 0.95 probability, corresponding to each one of the estimates in Figure 5. This gives a measure of the spread of the posterior and the resolution for porosity of each cell of the reservoir. The Figure 6a shows that the data $\mathbf{v}$ is more informative for cells near the well than for cells away from the wells. The Figure 6b shows that the information about porosity contained in the seismic attributes is homogeneously distributed across the model, yielding high frequency variations on the estimates. The Figure 6c shows us an improvement of the resolution of a model when the porosity information from both well log and seismic attributes data are integrated by the bayesian formulation.

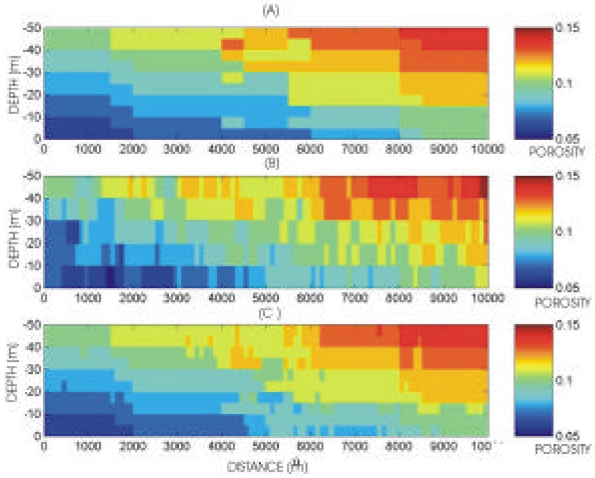


Figure 5: Images representing the modes of the posterior distributions by the use of variogram data (A), seismic attributes (B) and both data sets (C).

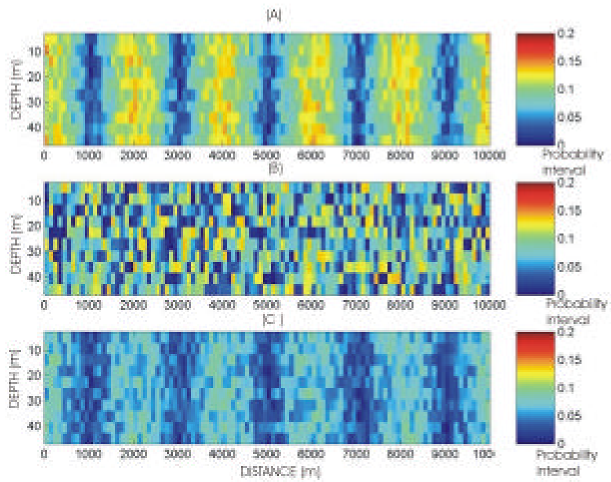


Figure 6: Length of the 0.95 probability interval of the posterior distributions obtained from the inversion of variogran values (A), seismic attributes data (B) and both data sets (C).

Conclusion

We presented an approach to reservoir characterization fully based on the inversion theory, which is capable of integrating multiple data sets in a straight forward way. The commonly employed formulations of the mathematical physics relating data and model parameters is replaced by empirical formulas of experimental rock physics. Geostatistics is also integrated through the experimental variogram and the corresponding formula, both used in the context of inversion theory. A synthetic data test using a slowly varying model demonstrated the consistency of the proposed methodology.

Analysis of results show reasonable reconstructions of the true porosity model obtained from the mode of the posteriors. The associated uncertainty, represented by the length of 0.95 probability intervals, consistently vary depending on the amount of information available. Higher resolution is obtained at the wells. The variogram fitting procedure allowed to describe the information about the porosity from the wells at interwell location. For the inversion of seismic attributes alone the level of uncertainty varies homogeneously across the model. When combining variogram and attribute data, we observe that the overall uncertainty is reduced and the porosity model is better reconstructed.

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