Neural Networks and AVO

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Introduction

In this paper we will discuss how a neural network can solve a simple AVO problem. In doing so, we will shed light on several important questions: why are some neural networks only able to solve linear problems, whereas others can solve nonlinear problems, and how can neural networks be trained to do these tasks? The type of neural network that we will use is the multi-layer perceptron (MLP), which is sometimes called the multi-layer feed forward network (MLFN) (Hampson et al, 2001). The AVO problem that we will train the network to address is the recognition of a Class 3 anomaly on an AVO attribute crossplot.

The AVO Problem

The basic AVO interpretation problem that we will study is differentiating between the AVO responses of the two reservoirs in Figure 1. Figure 1(a) shows a wet sand encased between two shale layers, and Figure 1(b) shows a gas sand encased between the same two shales. The P-wave velocity (V_P), S-wave velocity (V_S), and density (ρ) for each layer are shown in each figure. We will assume that the far angle of incidence is small enough (i.e. approximately 30°) that we can ignore the third term in the Aki-Richards equation and write the reflectivity as a function of angle of incidence θ as

$$R(\theta) = A + B \, sin^2 \, \theta \; , \tag{1}$$

where A is the AVO intercept, and B is the AVO gradient.
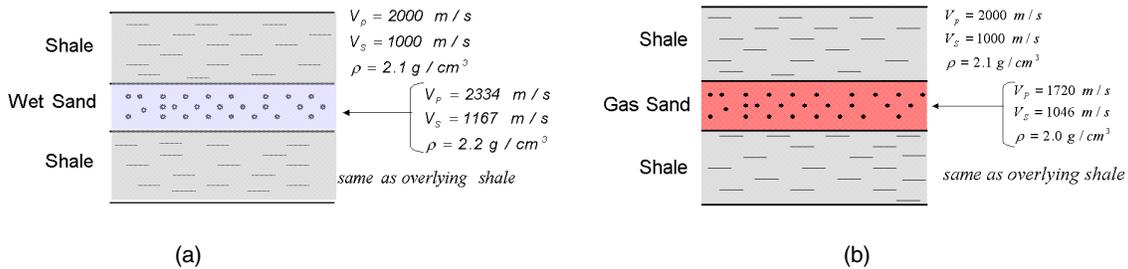
Figure 1: Two simple geological models where (a) shows a wet sand between two shales and (b) shows a gas sand between the same shales.

Using the values for V_P, V_S, and density shown in Figure 1, we can now work out the values for the AVO intercept and gradient for the wet and gas sands. This gives

$$A_{TOP\_WET} = B_{BASE\_WET} = +0.1, \;\; A_{BASE\_WET} = B_{TOP\_WET} = -0.1, \; A_{TOP\_GAS} = B_{TOP\_GAS} = -0.1, \; A_{BASE\_GAS} = B_{BASE\_GAS} = +0.1 \, .$$

Although simple, these values do fall within a reasonable petrophysical range encountered for typical sands and shales. After scaling each of the values of A and B by a factor of 10 (to give values of +1 and -1) they have been put on an A-B crossplot, as shown in Figure 2.
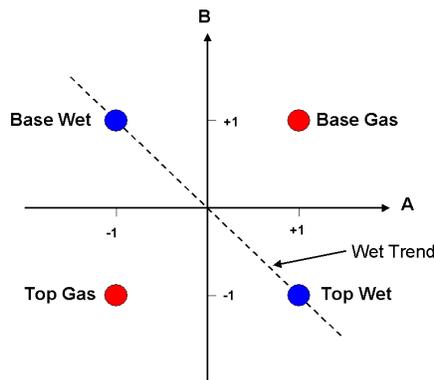
Figure 2. Intercept vs gradient crossplot from the A and B values from the wet and gas models of Figure 1, after being scaled by a factor of 10.

In our example, the wet points (shown as solid blue circles) establish the wet sand-shale trend, and the top and base gas (shown as solid red circles) plot in the other two quadrants of the A-B crossplot. This is a typical Class 3 AVO anomaly (Rutherford and Williams, 1989), caused by gas saturation reducing the sand impedance and the V_P/V_s ratio of the sand encased in the shale.

Despite the simplicity of the model, Figure 2 shows us what is expected in the noise-free AVO crossplot. Identifying the wet trend and the outlying two points is a trivial problem for the eye to interpret. As we will see in the next section, the early neural networks could not properly address situations as simple as this, and technology improvements (often through trial and error) were required to eventually solve the problem.

The Perceptron

The classic model of the neuron is called the perceptron (McCulloch and Pitts, 1943) and is illustrated in Figure 3(a). The perceptron accepts N inputs $a_1,\ldots a_N$, and produces a single output. Mathematically, the perceptron has two separate stages. First, the inputs are weighted and summed according to the equation:

$$x = w_1 a_1 + w_2 a_2 + \mathrm{K} + w_N a_N + b \tag{2}$$

The last weight, b, is called the bias. Next, a threshold function $f(x)$ is applied to the intermediate output x to product the final output y. We will discuss a typical function shortly.

(a)   (b)
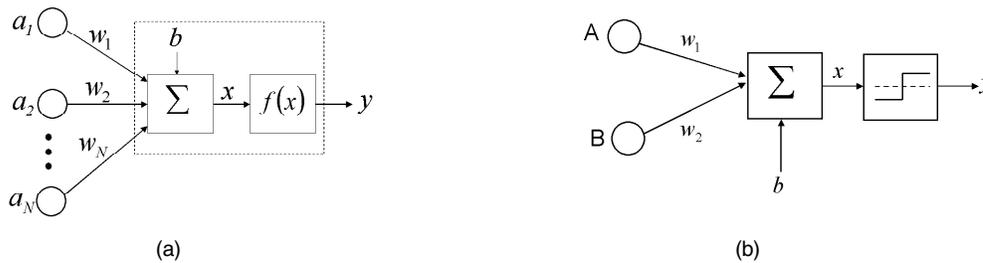
Figure 3. The perceptron neural network where (a) shows the general case, and (b) shows the simplified case for our AVO model.

Now, let's consider how to adapt the perceptron to our AVO problem, as shown by the neural network graph in Figure 3(b). We have reduced the problem to two inputs, the intercept (A) and gradient (B), and are using is the symmetric step function to compute the final output, which is given mathematically by the equation:

$$f(x) = \begin{cases} +1, & x \geq 0 \\ -1, & x < 0 \end{cases} \tag{3}$$

The step function is a logical and convenient choice because A and B have scaled values of ±1. However, the interpretation of the output from $f(x)$ will be different from the input. A value of +1 will indicate the presence of a gas sand and a value of −1 will indicate the presence of a wet sand. Notice that the equation for intermediate output x is now:

$$x = w_1 A + w_2 B + b \tag{4}$$

The key question is: how do we determine the weights w_1, w_2, and b? To answer this, let us take an intuitive look at what these weights mean. From equation (3), it is obvious we are interested in the separation between $x < 0$ and $x > 0$, which happens when $x = 0$. This is called the decision boundary. To find out where this boundary crosses the A and B axes, we simply need to successively set A and B to zero, to get $B = -b/w_2$ and $A = -b/w_1$. Let us calculate the actual values for our AVO problem, as shown in Figure 4. First of all, notice that we will not be able to separate both the top and base of the gas sand from the wet trend with a single decision plane, since this is a nonlinear separation problem. Thus, we must choose to separate either the top of the gas sand, as shown in Figure 4(a), or the base of the gas sand, as shown in Figure 4(b). To solve for the weights for the top of the gas sand, notice that $A = B = -b/w_1 = -b/w_2 = -1$. Although b, w_1, and w_2 can be scaled by any value, it is best to choose the simplest values, which are $w_1 = w_2 = -1$, and therefore $b = -1$. Next, to solve for the weights for the base of gas sand, notice that $A = B = -b/w_1 = -b/w_2 = +1$. Therefore, the weights are $w_1 = w_2 = +1$, and therefore $b = -1$.

(a)   (b)
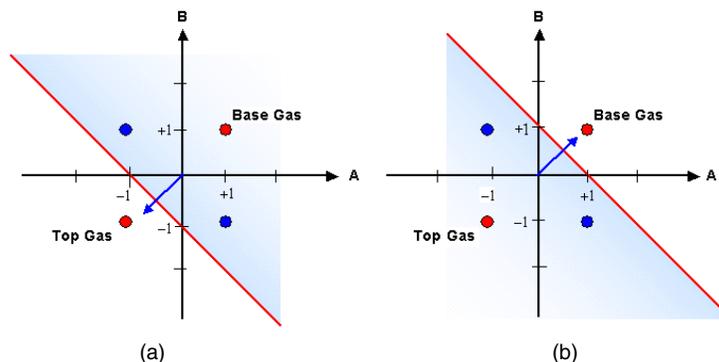
Figure 4: The AVO problem from Figure 2 with decision boundaries, where (a) shows separation of the base of the gas sand and (b) shows separation of the top of the gas sand.

Table 1 shows that these values do indeed solve the problem for the four possible cases.

	Inputs		Perceptron 1		Perceptron 2	
Sand	A	B	x_1	y_1	x_2	y_2
Top Gas	-1	-1	+1	+1	-3	-1
Base Wet	-1	+1	-1	-1	-1	-1
Top Wet	+1	-1	-1	-1	-1	-1
Base Gas	+1	+1	-3	-1	+1	+1

Table 1: The table above shows the outputs from the perceptron models of Figure 4.

Although we have solved for the top and bottom of the gas sand individually we have still not solved the complete problem, which is to separate the gas sand responses from the wet sand responses. This requires a multi-layer perceptron.

The Multi-layer Perceptron (MLP)

The problem that we encountered in the last section, that perceptrons can only solve linearly separable problems, was also encountered by the early neural network researchers. This dealt a severe blow to the enthusiasm for neural networks. Although the solution now appears quite straightforward (since hindsight is 20/20) it took almost forty years from the initial development of the perceptron for this new technique to be widely introduced (McClelland and Rumelhart, 1981). The solution is to add or cascade a second layer of perceptrons. To understand why a second layer of perceptrons will work, we need to recast the AVO problem of Figure 2 as a multi-layer perceptron, as shown in Figure 5. In this neural network, the inputs are still the intercept (A) and gradient (B), but now they are both interconnected, via the weights, to the two perceptrons.

(a)   (b)
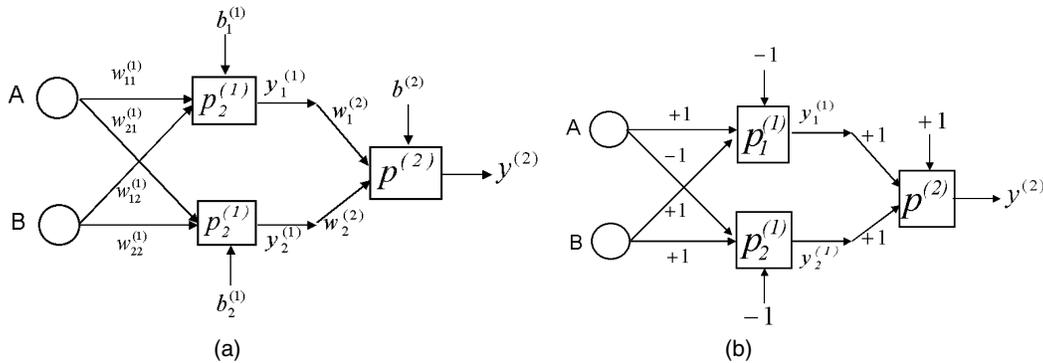
Figure 5: The multi-layer perceptron model, where (a) shows the general model and (b) show the actual weights for the gas-water sand model of Figure 2.

Mathematically, the multi-layer perceptron is written as a matrix equation. However, rather than inundate the reader with matrix mathematics, we will present an intuitive development of what happens in a multi-layer network. What we have done is simply connect two perceptrons to produce two separate outputs. These outputs now represent the input to a new perceptron. If we use the two perceptrons from Table 1 (the top and base of gas perceptrons) as our two first layer perceptrons, we can crossplot their outputs $y_1^{(1)}$ and $y_2^{(1)}$, and, and design a new perceptron, $p^{(2)}$, to separate them. This is shown in Figure 6.

The base gas and top gas points have moved to what was the wet trend on the input points, but both of the wet sands have moved to the point which was the top gas value for the input. This new linearly separable problem is graphically shown by the decision boundary on Figure 6. Thus, we can derive the weights for perceptron $p^{(2)}$ in Figure 5(a), which are shown in figure 5(b) and are given as:

$$ -\frac{b^{(2)}}{w_1^{(2)}} = -\frac{b^{(2)}}{w_2^{(2)}} = -1, \text{ or } w_1^{(2)} = w_2^{(2)} = b^{(2)} = +1. $$

To verify that this indeed is the solution, Table 2 then shows the outputs for all possible inputs. As we can see in Table 2, the correct output is given for all four input cases. Thus, we have solved the problem of how to separate a simple Class 3 gas sand from its equivalent wet sand.
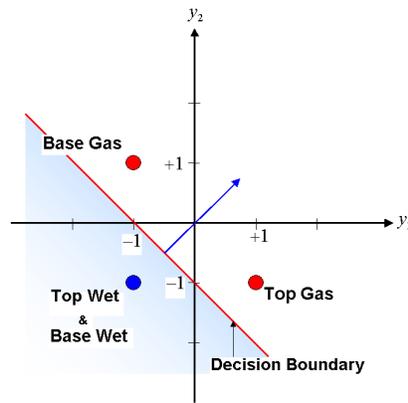
Figure 6: A crossplot of the outputs of the two perceptrons from Table 1.

	Inputs		Perceptron $p_1^{(1)}$	Perceptron $p_2^{(1)}$	Perceptron $p^{(2)}$
Sand	**A**	**B**	$y_1^{(1)}$	$Y_2^{(1)}$	$y^{(2)}$
Top Gas	-1	-1	+1	-1	+1
Base Wet	-1	+1	-1	-1	-1
Top Wet	+1	-1	-1	-1	-1
Base Gas	+1	+1	-1	+1	+1

Table 2: The computed output values from the multi-layer perceptron in Figure 14 for the four input models of Figure 3, where $y^{(2)}$ shows that the gas sand values (+1) have been separated from the wet sand values (-1).

Conclusions

In this paper, we have shown how a neural network can be used to solve a simple Class 3 AVO crossplot problem. The simplicity of the model allowed us to derive the weights by hand for the neural network. In more complex problems, the weights are derived using a technique called back propagation, in which we try to reduce the error between the known output and the output created by a set of initial weights by backward propagation of the errors. While this is a necessary procedure on a computer, the fact that we were able to intuitively derive the weights in this problem allowed us to discover several very important features about neural networks. First, a single layer perceptron can only solve a linearly separable problem, such as separating the top of a gas sand from the top of the wet sand encased in shale. Second, by adding a second layer of weights, we can transform a nonlinear problem, such as isolating both the top and base of a gas sand, into a linear problem, and thus find a non-linear solution using a two-layer perceptron.

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