A New Method of NMO and Stacking for Converted-Wave Processing

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Introduction

Converted-wave seismic exploration can provide improved subsuface images as well as give a meaure of subsurface S-wave properties relating to rock type and saturation (Stewart et al., 2002). It has been increasingly applied to the search for subsurface targets. Converted-wave data, however, are difficult to process because the traveltime curves of converted waves for a common conversion point are not symmetric or hyperbolic. Classical hyperbolic approximations are not valid for converted waves. Higher-order Taylor series expansion (Tessmer and Behle,1988; Thomsen, 1999; Tsvankin and Thomsen, 1994) incurs considerable mathematical complications and the results tend to become inaccurate with increasing offset. This paper presents a simple method of computing more accurately the Taylor series of converted-wave traveltimes and then performing the transform of velocity and offset. In the transformed system, the traveltime curves are hyperbolic and all standard processing procedures such as NMO and stacking can be carried out.

Taylor Series Expansion of t^2 –x^2 Curves

The traveltime curve of a P-P or S-S reflection from a horizontal reflector below a homogeneous isotropic layer is a hyperbola in the t-x domain:

$$t^2 = t_0^2 + \frac{x^2}{v^2} \qquad (1)$$

where t is the travel time from source to reflector and back to receiver, x is the source-receiver distance, i.e., offset, t_0 is the travel time at zero offset, and v is the velocity. Equation (1) is a straight line if t^2 is viewed as the function of x^2. t^2– x^2curves from seismic data are routinely fitted linearly to give the intercept and the slope, which are approximated as t_0^2 and $1/v^2$, respectively. For P-S reflections, the t-x relationship cannot be expressed in the same form as in equation (1), but from general function-theoretical considerations (Copsen 1935; Taner and Koehler, 1969) t^2 can still be seen as a function of x^2, i.e., $t^2 = f(x^2)$, which can be expanded in Taylor series about x^2=0:

$$t^2 = f(x^2) = f(0) + f'(0)x^2 + \frac{1}{2}f''(0)x^4 + ... \qquad (2)$$

Determination of the high-order derivatives from the third term in equation (2) is quite mathematically involved (Tessmer and Behle, 1988; Thomsen, 1999; Tsvankin and Thomsen, 1994). The attempt to approximate t^2 with more terms is not computationally efficient, and the approximations also become inaccurate with increasing offset because of truncation errors. In fact, t^2 can be expressed as the sum of only two terms according to Taylor series theorem. Equation (2) can be reformulated as:

$$t^2 = f(0) + f'(c)x^2 \qquad (3)$$

where c is some value between 0 and x^2, f(0) is t_0^2, f' is $d(t^2)/d(x^2)$. To evaluate equation (3), it is thus necessary to find f' and c. From Figure 1, the traveltimes and offset for P-S reflections are related as:

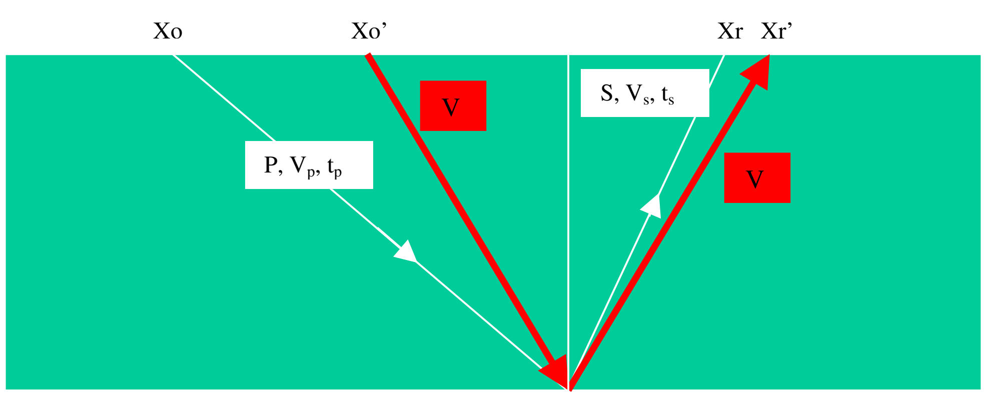

Figure 1. Converted wave propagation and transformation of velocity and offset

$$x = v_{px} t_p + v_{sx} t_s \tag{4}$$

where V_{px} , t_p, V_{sx} , t_s are the horizontal components of velocity and travel times for P and S waves, respectively. f' can be derived from equation (4) as follows (Tsvankin and Thomsen, 1994):

$$f' = \frac{d(t^2)}{d(x^2)} = \frac{t}{x}\frac{dt}{dx} = \frac{tp}{x} = \frac{t_p + t_s}{\frac{v_{px}}{p} t_p + \frac{v_{sx}}{p} t_s} = \frac{1}{v_p^2 + v_s^2 t_s/t_p} + \frac{1}{v_s^2 + v_p^2 t_p/t_s} \tag{5}$$

where p is the ray parameter. The evaluation of f' thus reduces to the evaluation of t_s/t_p or t_p/t_s. t_p/t_s depends on the V_p/V_s ratio and offset/depth ratio (x/h). Let $x/h=\alpha$, $t_p/t_s=\beta$ and $V_p/V_s=\gamma$. They are related in the following formula:

$$\beta^6\gamma^6 + \beta^4(\gamma^4 - \alpha^2\gamma^4) + \beta^2(\alpha^2\gamma^2 - \gamma^2) - 1 = 0 \tag{6}$$

t_p/t_s (β) can be solved analytically from equation (6) as a function of x/h (α) and V_p/V_s (γ). Beyer (1984) gives a detailed solution to this equation. For a CCP gather, V_p, V_s, V_p/V_s and h are fixed. So f' is a function of offset x only.

The second step is to find c, which is some value between 0 and x^2. The evaluation of f' in the interval of (0, x^2) (x^2 as the variable) in equation (3) is equivalent to doing that in the interval of (0, x) (x as the variable). At a given offset x, we need to find c between 0 and x and bring it into equation (5) to compute the exact value of the second term in equation (3). The degree of difficulty in the search for c in the interval (0, x) depends on the behavior of the function f'. As shown in Figure 2, $1/f'$ exhibits a smooth monotonic property, which implies that c may be related to x and $f'(c)$ may be estimated from $f'(x)$.

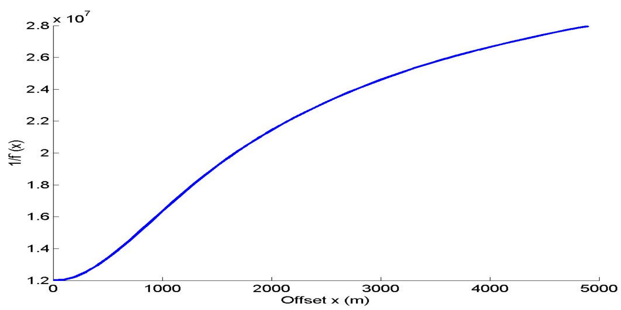

Figure 2. Plot of offset (x) versus 1/f'(x)

In order to explore the possibility of predicting $f'(c)$ from $f'(x)$, ray tracing was used to calculate the exact $f'(c)$ at a series of offsets, which were then compared with $f'(x)$ to see if any good quantitative relationship exists between them. Figure 3 is the crossplot of $1/f'(c)$ (computed from ray tracing) and $1/f'(x)$ [computed at x from equations (5) and (6)]. Their relationship can be modeled as a straight line, which may then used for the $f'(c)$ calculation.

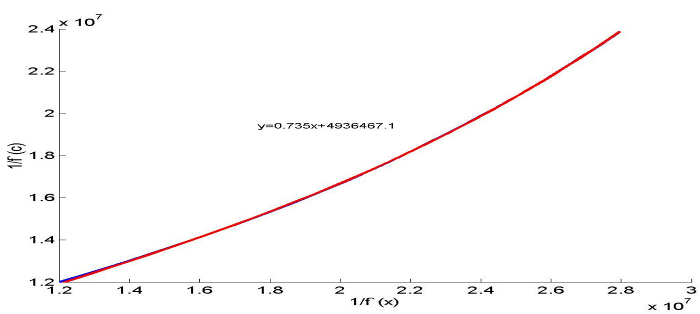

Figure 3. Crossplot of 1/f'(x) (calculated at x) versus 1/f'(c) (from ray tracing)

NMO and Stack for synthetic data

The direct benefits from the accurate estimation of the second term in equation (3) are NMO and stacking enhancement. In fact, the results for the second term can be applied directly to NMO and stacking. Considering the industry convention, it is better to convert P-S nonhyperbolic curves to hyperbolae for standard seismic data processing. As stated previously, $f'(c)$ is offset dependent. Direct t^2-x^2 curve fitting for NMO and stacking is bound to give poor quality, especially at far offsets, because $f'(c)$ is not constant. In order to generate a constant coefficient for the second term and make the result hyperbolic, the original offsets have to be adjusted. The scheme for offset modification is to assume that a wave propagates at constant velocity from a new source to the CCP and then reflects back to a new receiver, and that the time it takes is equated with that for the P-S reflection (Figure 1). According to this principle, the fabricated constant velocity is the harmonic average of Vp and Vs so that equating the traveltimes can be achieved at zero offset. The transformation of velocity and offsets is expressed as:

$$\frac{1}{V} = \frac{1}{2}\left(\frac{1}{V_p} + \frac{1}{V_s}\right) \qquad (7)$$

$$t_0^2 + f'(c)x^2 = t_0^2 + \frac{X^2}{V^2} \Rightarrow \quad X = \sqrt{f'(c)x^2V^2} \qquad (8)$$

where V is the new velocity, X is the new offsets and x is the original offsets. The transformation in equation (8) through travel time equating amounts to factorizing $f'(c)\,x^2$ into a new offset variable times a constant. As indicated in equations (9) and (10), the transformed velocity and offsets create hyperbolic travel time curves. Standard NMO, stacking and other processing procedures can be then carried out.

$$t^2 = t_0^2 + \frac{X^2}{V^2} \qquad (9)$$

$$NMO = t - t_0 = t - \sqrt{t^2 - \frac{X^2}{V^2}} = t\left(1 - \sqrt{1 - \frac{X^2}{t^2V^2}}\right) \qquad (10)$$

The right hand side of Figure 4 is a CCP gather for a reflector buried at 500m. t^2-x^2 is not hyperbolic. The conventional t^2-x^2 fitting gets inaccurate t_0 and stacking velocity from intercept and slope, respectively. Consequently the stacked section on the left side of Figure 4 has poor resolution, as evidenced by two wavelets generated from one reflector. The right hand side of Figure 5 is a CCP gather with transformed offsets. The shortened offsets from transformation can be observed. t^2-x^2 is hyperbolic on the new CCP gather. The conventional processing procedures can be carried out with excellent results. The left hand side of Figure 5 shows a strong reflection with only one wavelet.

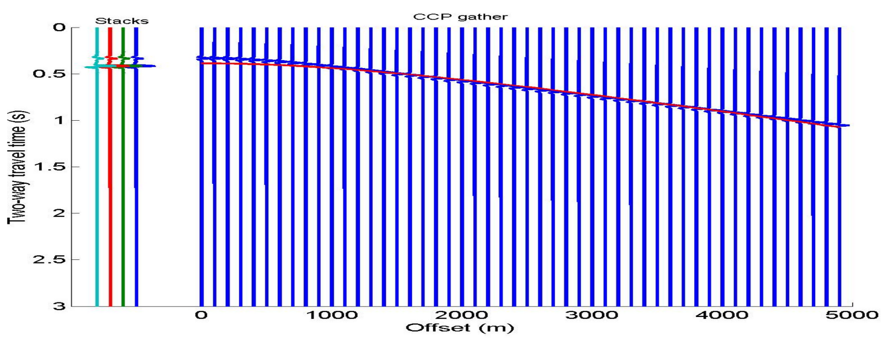

Figure 4. Hyperbolic fitting and stacking using conventional processing procedures for a CCP gather

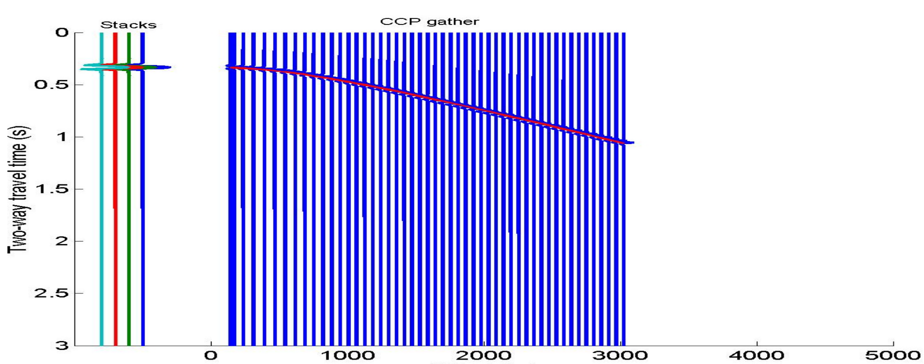

Figure 5. Hyperbolic fitting and stacking using conventional processing procedures for a CCP gather with transformed offsets.

The result in Figure 5 is based on the accurate depth. In reality the depth is roughly estimated. In order to test the validity of the method, the depth errors are artificially added into the whole process of NMO and stacking. The result in Figure 6 was produced with 30% depth error. As shown, the stacking is still very good with high resolution.

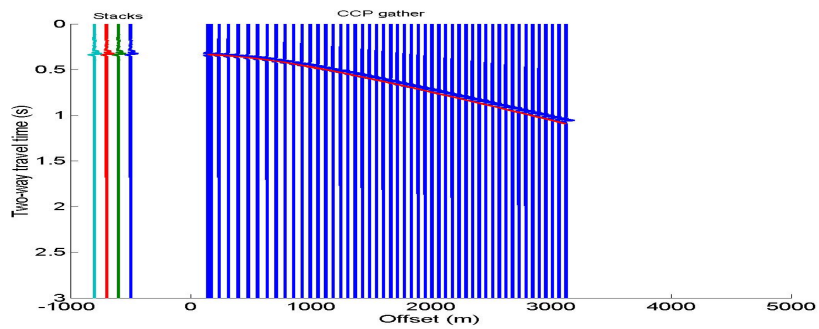

Figure 6. Hyperbolic fitting and stacks using conventional procedures for a CCP gather with transformed offsets (depth error=30%)

Conclusions

The P-S travel time curves for converted waves are not hyperbolic, but $t^2 - x^2$ can be expanded in a Taylor series about $x^2 = 0$ retaining only two terms. The coefficient of the second term can be accurately estimated through a quantitative model. The second term can be factorized into a squared constant and a squared variable. This is equivalent to the transformation of velocity and offset into a new system, in which $t^2 - x^2$ is hyperbolic and standard processing procedures can be carried out. Synthetic seismic data indicate that the method improves NMO and stacking quality considerably unless the percentage error of depth estimation exceeds 30%.

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