

Velocity Model Building with 1-D Tomographic Residual Curvature Analysis

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ABSTRACT

3-D prestack depth migration is becoming a routine process in seismic data processing, hence the requirement for accurate velocity model building. For building an initial 3-D velocity model, the traditional Deregowski loop is often used. Based upon the assumptions of straight-ray and small-offset/depth ratio, it converts depth residual errors into time residuals and then applies traditional NMO velocity analysis. The interval velocities are then obtained by a layer-stripping process with the Dix formula. Since this process is based on the assumptions of small-offset/depth ratio and straight-rays, the results are often unsatisfactory, from both theoretical and practical standpoints. One notable shortcoming is the error accumulation in depth intrinsic to this process.

In this abstract, we propose a 1-D tomographic residual curvature analysis method. It applies ray tracing to recover the specular ray paths, therefore accommodating the ray-bending effect. Compared with the traditional Deregowski loop, this method completely eliminates the assumptions of straight-ray and small-offset/depth ratio. The interval velocities are obtained by globally solving a linear system of equations. Therefore, it partially solves the problem of error accumulation intrinsic to the layer-stripping process with the Deregowski loop and the Dix formula. Analyzing 1-D tomography in simple medium helps us to understand the behavior of more general 3-D tomographic residual analysis methods. It can also provide a more accurate initial velocity model or low-frequency velocity model for subsequent 3-D global tomography. We present algorithms for updating velocity for constant-gradient as well as blocky models. Numerical results will also be presented for discussion.

Introduction

3-D prestack depth migration positions reflected seismic events at their correct subsurface locations. Accurate knowledge of seismic interval velocities is essential for converting surface prestack reflection data into images of subsurface structures. Analysis of depth-migrated gathers, (e.g. common image

gathers, or CIGs), is the basis of most current interval velocity estimation techniques. These gathers should consist of flat events when the velocity used for the migration is correct. Inaccurate velocities will result in the residual moveouts in such gathers. So, the goal of interval velocity analysis is to flatten events in the gathers. The flattening of residual curvature by adjusting seismic velocities allows depth migration to be used as a powerful tool for velocity analysis.

Although 3-D full-scale tomography has been available in some oil companies, there are still some unknown mechanisms in the inversion process. Analyzing 1-D tomography in simple medium helps us to understand the behavior of more general 3-D tomographic residual analysis methods. It can also provide a more accurate initial velocity model or low-frequency velocity model for subsequent 3-D global tomography.

Traditional Deregowski loop [1] is often used for building initial velocity model. However, it is based upon the assumptions of straight-ray and small-offset/depth ratio. The interval velocities are obtained by layer-stripping process with the Dix formula, which may introduce accumulation errors during the solving process. In order to solve these problems in Deregowski loop, we propose a 1-D tomographic residual curvature analysis method. It applies ray tracing to recover the specular ray paths, therefore accommodating the ray-bending effect and completely eliminates the assumptions of straight-ray and small-offset/depth ratio. The interval velocities are obtained by globally solving a linear system of equations. This partially solves the problem of error accumulation intrinsic to the layer stripping process with the Dix formula.

Tomography, no matter whether it is 1D or 3D, relies heavily on raypaths traced through the initial velocity model. In general, this requires the knowledge of reference depth reflectors or time events. These locations are either guessed or estimated somehow (e.g. by picking events on the stacked image) before the tomography begins. In residual curvature analysis, specular raypaths from these reflector locations and the velocities the rays pass through provide all the information needed to build linear system of equations that convert the depth or time errors to velocity perturbations.

Depending on how the equations are built, the approaches can be categorized into two types: the floating-reflector method ([2,7,3]) and the fixed-time event method (van Trier, 1990; Zhou et al., 2001). Both are based on the principle that ray tracing (modeling) undoes migration whether the velocity in migration is accurate or not. The specular ray pairs are recovered by tracing a ray pair for a given offset from the migrated depth for that offset.

The floating-reflector method assumes that true reflector locations are known and they are used as reference reflectors. However, this assumption certainly is not

true at this tomographic stage, therefore, it has to be removed by some operations [2,7,3], resulting in the loss of accuracy.

The fixed-event method goes further than the floating-reflector method in correcting the error in reflector location. Instead of using true reflectors as its reference reflectors, this method uses true zero-offset events in the time (unmigrated) domain as its reference events. In this abstract, only the fixed-time event algorithm will be discussed in detail along with its applications to a synthetic example.

The model is assumed as piece-wise linear in this abstract, which is adequate for many regions such as the Gulf of Mexico. Numerical results will also be presented for discussion.

Algorithm

After transforming from time to depth by a 3-D prestack depth migration, a CIG is collected at a specific surface location. Each trace in the CIG represents the result of depth-migration. Given the picked image locations and their corresponding offsets, the linear system of tomographic equations are built with the following procedures:

1. Shooting the rays up from image location z to find the corresponding specular ray pairs associated with this location and the given offset.
2. Calculate the derivatives of traveltimes with respect to the velocity in each layer.
3. Build the linear system of equations with each row related to its corresponding specular ray pairs using the fixed-time event algorithm discussed below.
4. Solve the linear equation using least-squares method to obtain the solution of the model (i.e., velocity perturbation).
5. Obtain the updated image location.
6. Repeat the above procedure if necessary.

Model description

Tomographic methods tend to be characterized by the underlying model representation. An accurate and stable solution of the tomographic method can only be obtained if the model representation is selected according to the nature of the geology. In areas of limited complexity, a simple method can be efficient and produce a satisfactory solution, if the problems are defined appropriately. If the subsurface velocity is smoothly varying, then the velocity model can often be well represented by a smooth model. A typical example of this happens in the Gulf of Mexico outside of salt, where the sediment velocity is essentially smooth. In view of these aforementioned points, we simply represent velocity as a piece-

wise linear function with velocity defined at each knee-point. And the velocity field is parameterized independently of the reflector positions. In other words, the velocity field is decoupled from the reflector locations.

Algorithm (Fixed-time Event)

The fixed-time event method works by tracing normal incident rays from a given reflection event for all offsets besides finding their corresponding specular ray pairs. As the principle of ray tracing undoing migration implies, the true traveltimes can be recovered even though the true reflector locations remain unknown. In other words, even though the migration was performed with an incorrect velocity, the traveltime along the normal incidence ray from a migrated event on the zero-offset section is still one-half the correct zero-offset time for that event. However, if we shoot normal incident rays from corresponding imaging location in other nonzero-offset sections, the zero-offset time will not be the same if the velocity is incorrect. This difference can be used to invert for velocity perturbations. Doing this allows the back-projection operator to be more accurate than the floating-reflector method.

Let's suppose t_{h_0} represents the two-way traveltime for half-offset h_0 at location z_{h_0} . Then according to the demigration principle, t_{h_0} is theoretically accurate [8] even when the migration velocity is wrong. Now let $t_{h_0}^h$ represents the modeled two-way traveltime at location z_h . In principle, if the migration velocity is not the exact velocity, $t_{h_0} \neq t_{h_0}^h$. Therefore, the deviation between t_{h_0} and $t_{h_0}^h$ can be used to get the velocity information. The algorithm can be obtained by applying Taylor expansions

$$t_{h_0} = t_{h_0}^h + \sum_{i=1}^n \frac{\partial t_{h_0}^h}{\partial v_i} \Delta v_i + \frac{\partial t_{h_0}^h}{\partial z} \Delta z. \quad (1)$$

Here, indices $i = 1, \dots, n$ represents the velocity values the specular ray pairs pass. And

$$\Delta z = \sum_{i=1}^n \frac{\partial z_h}{\partial v_i} \Delta v_i, \quad (2)$$

where $\frac{\partial z_h}{\partial v_i}$ is given by equation (4).

Substituting equation (2) into equation (1) yields the following velocity updating equation

$$\Delta t = t_{h_0} - t_{h_0}^h = \left[\sum_{i=1}^n \frac{\partial t_{h_0}^h}{\partial v_i} \Delta v_i + \frac{\partial t_{h_0}^h}{\partial z} \sum_{i=1}^n \frac{\partial z_h}{\partial v_i} \Delta v_i \right]. \quad (3)$$

To look carefully at the *equation (3)*, we find that derivative information for $\frac{\partial z_h}{\partial v_i}$ has to be given to make a linear system of equations. According to zero-time imaging principle [6],

$$\frac{\partial z_h}{\partial v_i} = \frac{\partial t^h}{\partial v_i} \bigg/ \frac{\partial t^h}{\partial z_h}, \quad (4)$$

where

$$\frac{\partial t_h}{\partial z_h} = \frac{2 \cos \theta_h}{v_0} \quad (5)$$

Here, θ_h is the half-opening angle of the specular ray corresponding to the given half-offset h [3,8], and v_0 is the velocity at the current imaging location. Except for the traveltimes derivatives with respect to the velocities that will be discussed in the following section, it's clear that we already get all the information to build a linear system of equations.

Traveltimes and Traveltime derivatives

According to Snell's law, the ray parameter p will be constant and satisfy

$$p = \frac{\sin \alpha_1}{v_1} = \frac{\sin \alpha_0}{v_0} \quad (6)$$

where α_0 and v_0 are the emergent angle and velocity at the top of the layer, respectively. And α_1 and v_1 are the incidence angle and velocity at the bottom of the layer.

In each layer, given the ray parameter p , which happens to be constant throughout the ray tracing, we have the following analytical results [4] and (Dr. Yu Zhang, private communication):

$$h = \frac{1}{pk} (\cos \alpha_0 - \cos \alpha_1) \quad (7)$$

$$t = \frac{1}{k} \operatorname{arcosh} \left(1 + \frac{k^2 (h^2 + z^2)}{2v_0 v} \right) \quad (8)$$

Here, specular ray pairs are assumed to travel from subsurface upward to the top of the layer and $k = (v - v_0)/(z - z_0)$ is the velocity gradient, having the unit of inverse time.

Therefore, the total traveltime and horizontal distance the ray travels can be represented as the summation of x and t in each layer is described as:

$$t = t_0 + \sum_{i=1}^{n-2} t_i + t_{n-1} \quad (9)$$

where t_i is the traveltimes for the ray in each layer with t_0 at the surface layer and t_{n-1} at the bottom layer.

Therefore, the traveltime derivative with respect to velocity at each knee-point can be denoted by

$$\frac{\partial t}{\partial v_0} = \frac{\partial t_0}{\partial v_0} \quad (10)$$

$$\frac{\partial t}{\partial v_i} = \frac{\partial t_{i-1}}{\partial v_i} + \frac{\partial t_i}{\partial v_i}, \quad i = 1 \cdots n - 2 \quad (11)$$

$$\frac{\partial t}{\partial v_{n-1}} = \frac{\partial t_{n-1}}{\partial v_{n-1}} \quad (12)$$

In this case, we assume that the first layer is a constant-velocity medium (which may correspond to the water layer). This leads to *equation (10)*.

In case when the velocity gradient is small ($k_i \ll 1$), the traveltime derivatives can be represented as

$$\frac{\partial t_{i-1}}{\partial v_{i-1}} \approx -\sqrt{B_{i-1}/2} \left[\frac{1}{v_{i-1}} + \frac{1}{\bar{v}} \left(1 - \frac{\Delta z}{\Delta z_{i-1}} \right) \right]. \quad (13)$$

and

$$\frac{\partial t_{i-1}}{\partial v_i} \approx -\frac{1}{\bar{v}} \frac{\Delta z}{\Delta z_{i-1}} \sqrt{\frac{B_{i-1}}{2}} \quad (14)$$

where

$$k_{i-1} = \frac{v_i - v_{i-1}}{\Delta z_{i-1}} \quad (15)$$

$$B_{i-1} = \frac{x_{i-1}^2 + \Delta z^2}{2v_{i-1}\bar{v}} \quad (16)$$

$$\bar{v} = v_{i-1} + \Delta z_{i-1}k_{i-1} = \left(1 - \frac{\Delta z}{\Delta z_{i-1}}\right)v_{i-1} + \frac{\Delta z}{\Delta z_{i-1}}v_i \quad (17)$$

Numerical examples

In this section, a 1-D example of tomographic velocity analysis has been tested. Since the reflectors are decoupled from velocity grids, we test the algorithm using both five-reflectors and nine-reflectors models. To simulate real case of the tomography, the curvatures of the imaging locations are picked manually instead of computed analytically. The true velocity model and initial velocity model are shown in *Table 1*. In this table, the columns 4 and 5 are the updated velocities for the nine-layer reflectors after 1st and 4th iterations, respectively. And columns 6 and 7 are the updated velocities for the five-layer reflectors after 1st and 4th iterations, respectively. The velocities updated from both five-layer reflectors model and nine-reflectors model show similar results. *Fig. 1* shows the residual curvatures (*Fig. 1a*) and semblance plot (*Fig. 1b*) of nine-reflectors model before tomographic updating and the corresponding results after tomographic updating (*Fig. 1c and Fig. 1d*). It's obvious that the tomographic algorithm discussed in this abstract indeed flattens the CIG gathers and recovers the true locations of reflectors.

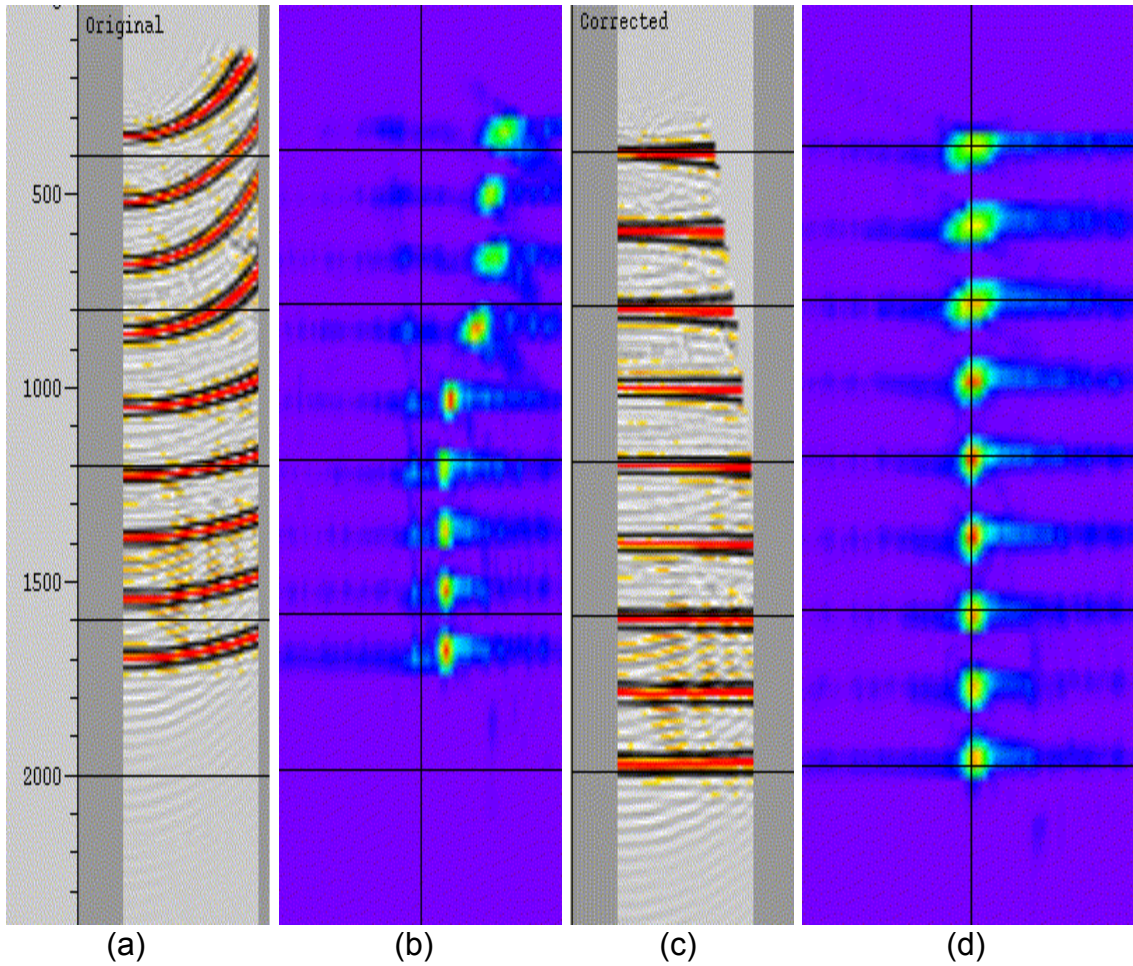


Fig. 1. Common image gathers and semblance plots.

Depth	True velocity	Initial velocity	Using nine-layer reflectors		Using five-layer reflectors	
			Velocity after 1 st iteration	Velocity after 4 th iteration	Velocity after 1 st iteration	Velocity after 4 th iteration
400	1700	1500	1677.88	1688.64	1678.04	1680.77
800	2500	2200	2415.00	2453.06	2421.73	2490.31
1200	2000	1800	1958.02	1974.89	1995.30	2032.59
1600	2500	2200	2259.51	2491.51	2290.08	2509.47
2000	3000	2700	2700.59	2735.92	2701.21	2728.44

Table 1: Results of tomographic curvature analysis.

Conclusions

In this paper, we discussed a type of 1D tomography that updates the velocity model using the residual curvature in common image gathers. Linear system of tomographic equations are built and solved to obtain velocity perturbations. A numerical example is provided to show the effectiveness of the method.

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