Two-way-wave-equation migration: Overkill or Necessity

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ABSTRACT

Current production-grade-wave-equation migration technology is almost totally based on one-way methods. These methods are typically derived through an asymptotic approximation of the square root of a one-way propagator derived by factorization of some form of wave equation. Primary reasons for the dominance of one-way approaches are most certainly the relative ease of implementation and computational efficiency they afford. In contrast, two-way methods, while easy to implement, are not nearly so computationally friendly. One might argue that they are, in fact, computationally too intensive to consider for production style processing. One might also argue that their greater accuracy make them worthy of consideration when image quality and resolutions is of utmost importance. In this paper we compare one-way and two-way methods in an attempt to draw conclusions with regard to computational costs and image quality. The focus is on examples rather than on theoretical analysis, but necessary theory is explained in full.

Two-Way Shot-Domain Prestack Imaging

The full 2-way scalar wave equation for post-stack time data U(x,y,0,t) is given (Claerbout, 1985; Baysal et. al., 1983, 1984; Whitmore, 1984) by

$$\frac{\partial^2 U}{\partial t^2} = \rho v^2 \nabla \bullet \frac{1}{\rho} \nabla U \qquad (1)$$

where ρ is density and v is velocity. Viewing this as an ordinary differential equation in time allows one to incrementally solve for U(x,y,z,0) by using the reversed (Loewenthal, 1976) surface wavefield as the input source. That is, with the time-reversed recorded wavefield, F= U(x,y,0,T - t) (T is maximum time), as the source, one solves

$$\frac{\partial^2 U}{\partial t^2} - \rho v^2 \nabla \bullet \frac{1}{\rho} \nabla U = F \qquad (2)$$

for the migrated image U(x,y,z,0). Imaging is thus accomplished by back-propagating each reversed-time slice into the subsurface volume as a modeling exercise. The advantages of this methodology are clear. There is virtually no limit on velocity, dip, or wavefield type, and evanescent waves are handled correctly (Zhou et. al., 2002). The only issue is proper allowance for reflections, but this is easily accounted for by forcing the impedance, ρ(x,y,z)v(x,y,z) to be a constant and deriving an appropriate impedance-matched version of equation (1).

For prestack data, a shot domain imaging algorithm based on equation (1) or its impedance-matched version only requires the addition of a synthesized shot record and appropriate imaging condition.

One-Way Shot-Domain Prestack Imaging

Derivation of a one-way equation for postack data usually starts with the factorization of the constant-density versions of equation (1) into a term for upward and a term for downward traveling waves. The frequency-wavenumber - domain version of of the equation for upward traveling waves can be expressed in the form

$$\frac{\partial U}{\partial z} = i\sqrt{k^2 + (k_x^2 + k_y^2)}U \qquad (3)$$

where k = ω/v(x,y,z). Implementation of this equation and its prestack counterparts takes many forms (Jin et. al. 1998; Huang et. al., 1999), but basically the chief issue is the asymptotic approximation of the square root. Because of the possibility to perform most of the calculations in the frequency-wavenumber domain, this approach has the advantage of speed and accuracy. For prestack shot domain migration, one again back-propagates the reverse-time input wavefield, forward propagates a synthetic shot and applies an appropriate imaging condition to complete the process. Since equation (3) specifies the wavefields at each depth slice, it clearly has significant computational advantages over its two-way counterpart.

One-Way Survey Sinking

Utilization of the one-way-double-square-root equation

$$\frac{\partial U}{\partial z} = i\left[\sqrt{k^2 + \left\|\vec{k}_x - \vec{k}_h\right\|} + \sqrt{k^2 + \left\|\vec{k}_x + \vec{k}_h\right\|}\right]U \qquad (4)$$

where $\vec{k}_h$ and $\vec{k}_x$ are the offset and spatial wavenumber vectors, allows one to construct an algorithm for sinking the entire survey at once. This method can be shown to be at least theoretically equivalent (Biondo, 2002) to shot-domain imaging. Its chief drawback is that it can require enormous storage resources when input data sets are large. Its real advantage is its superior computational speed.

Advantages and Disadvantages

Each of the methods that spring from the equations described above have its own unique set of advantages and disadvantages. The two-way methodology is known to be computationally intensive. Improper implementation of the necessary derivative approximations can lead to extensive grid dispersion and migration artifacts that are difficult to remove. Because its really a modeling exercise in disguise, the two-way method demands much tighter spatial sampling and significantly more points per wavelength than its one-way cousins. Its great advantage is that when properly applied, it has virtually no velocity or dip limitations. Because it implicitly handles turning rays, it can image both sides of a reflector. It almost magically handles all amplitude and aliasing issues.

Strict one-way approaches generally cannot handle dips higher than 90 degrees. The approximations necessary to implement the square root of the operator in equation (3) usually limit the ability of the resulting methodology to image steep dips. It is rare for a one-way equation to go beyond 60 or 70 degrees (Zhou et. al. 2002). It is also not unusual for the one-way methodology to exhibit sensitivity to strong lateral velocity variation. In cases where velocity contrasts are on the order of 3:1, many one-way implementations break down completely. On the other hand, one-way methods are very efficient and generally produce high quality images. They handle multi-arrivals well and can be implemented so as to be classifiable as true-amplitude methods. They are certainly to be preferred over single arrival Kirchhoff approaches.

Examples

Figure 1 demonstrates that it is possible to achieve excellent results with one-way methods. The top part of this figure shows a salt body inside sediments that are typical of the structural styles that might be expected in the Gulf of Mexico. The bottom half of Figure 1 provides an example of the type of imaging that might be expected from a generalized phase screen implementation of a one-way-shot-domain-prestack-migration method. In spite of several 3:1 velocity contrasts, the imaging method is quite good. It has even managed to image the steeply dipping left-hand face of the salt structure.

Figure 2 is an example of a two-way migration of a Gulf of Mexico salt structure. The white lines on this figure define the velocity boundaries of the macro model used in the migration. In this case, the macro model was actually derived via the two-way equation. That is, the velocity boundaries, including those that defined the salt structure, were interpreted from two-way migrations of the input data. Close examination of this graphic shows a sharp image of the salt-sediment interface. Identification of the source of this reflection reveals that it was imaged partially from within the salt itself and simultaneously from turning ray energy that propagated primarily only through the sediments. This is clearly not something a one-way method can accomplish, but is a crucial aspect of model development and velocity analysis. The ability to image both sides of a steeply dipping reflector is an exceptionally strong advantage of the two-way technology. When reflections from both sides of an event are present, one-way methods must, of necessity, turn one into unacceptable migration artifacts.

Figure 3 compares a one-way migration to the two-way migration in Figure 2. The top half is the two-way and the bottom half is the one-way migration. Note that the steeply dipping energy defining the salt-sediment interface is not visible on the one-way image. Although the sediments are imaged quite well in both figures, the two-way method clearly provides the additional information necessary to properly image the salt-sediment interface. Definition of the salt-sediment interface in this case is possible only when the two-way image is available.

Figures 4 and 5 compare a maximum-amplitude-Kirchhoff migration to a full two-way-reverse-time migration of a South Texas land data set. The objective in this case was to resolve steeply dipping beds below the shadow of the large central fault system. As is evident in Figure 4 the Kirchhoff approach did an acceptable job of imaging the structure previously hidden by the fault. However, the two-way image in Figure 5 suggests that the increased accuracy of the reverse-time method has provided much more detail than the Kirchhoff approach. The two-way migration not only does a much better job of imaging the fault themselves it provides much higher resolution of the data set and reveals a previously invisible fault set.

Figure 6 and Figure 7 show, respectively, the velocity slice corresponding to crossline 360 of the SEG/EAGE synthetic salt data set and the full 3D image of this cross line. Note the excellent subsalt image. Subtle subsalt faults are imaged quite nicely and the overall quality of the image is excellent. Empirical evidence suggests that this image is one of the best possible.

Computational Issues

Two-way prestack migration is an expensive process. In general, any back of the envelope calculation will demonstrate that the two-way method should be at minimum and order n^5 method. In contrast, the one-way approach will be much closer to n^4. By carefully controlling in-model propagation distances and using pseudo-spectral methods to minimize the points-per-wavelength necessary to achieve an acceptable result, it is possible to reduce the computational resources to a reasonable level. The 3D migration of the SEG/EAGE salt data set as partially illustrated in cross-sectional view in Figure 7 required approximately one month on 48 1.5 Gigaflop CPU's. This translates into approximately two-weeks on 2.8 Gigahertz INTEL Pentium 4 processors and less than one week on 100 such CPU's.

Conclusions

Two-way migration methods require significantly greater computational resources than their one-way counterparts. Their great advantage is that they have virtually no dip limitations and are algorithmically robust with regard to strong velocity gradients. That is, they respond to such gradients in the proper manner. When compared to even the best one-way methods, two-way approaches provide the additional information necessary to produce optimal macro models and associated subsurface images. In addition, they appear to provide superior resolution as far as fault definition and clarity of reflector strength is concerned. This enhanced resolution is actually not surprising. Migration is a lateral deconvolution process, and as such the more accurate two-way method should be expected to produce better lateral resolution. Since reverse-time migration makes no inherent approximations that must be *repaired* to control amplitude, handle or avoid evanescent propagation, and correct for problems arising from caustics, it should be expected to produce superior results in all cases.

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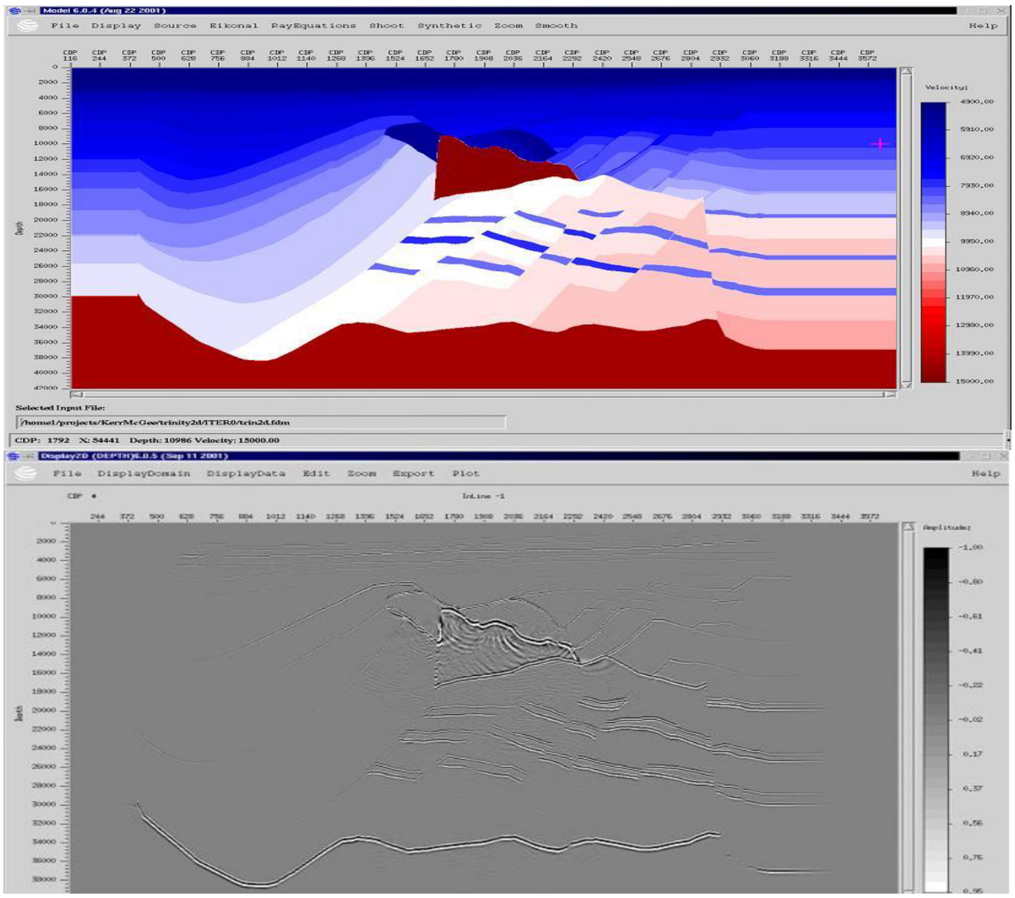

Fig. 1: Top is a velocity model of a Gulf of Mexico Salt Structure. Bottom is a one-way migration of synthetic data calculated from the model.

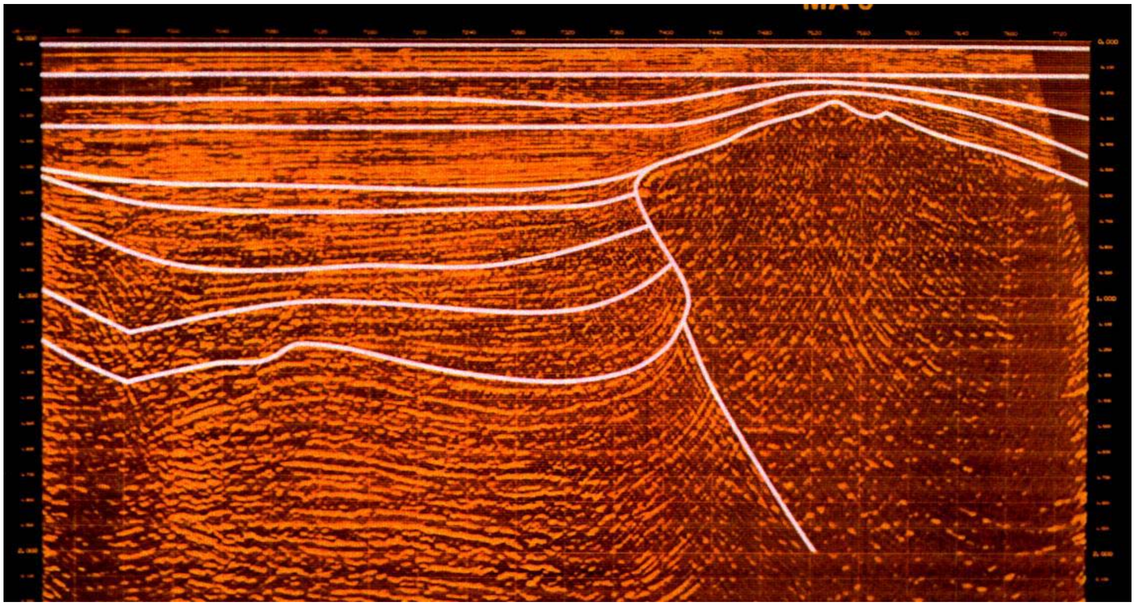

Fig. 2: Two-way migration of a Gulf of Mexico salt structure.

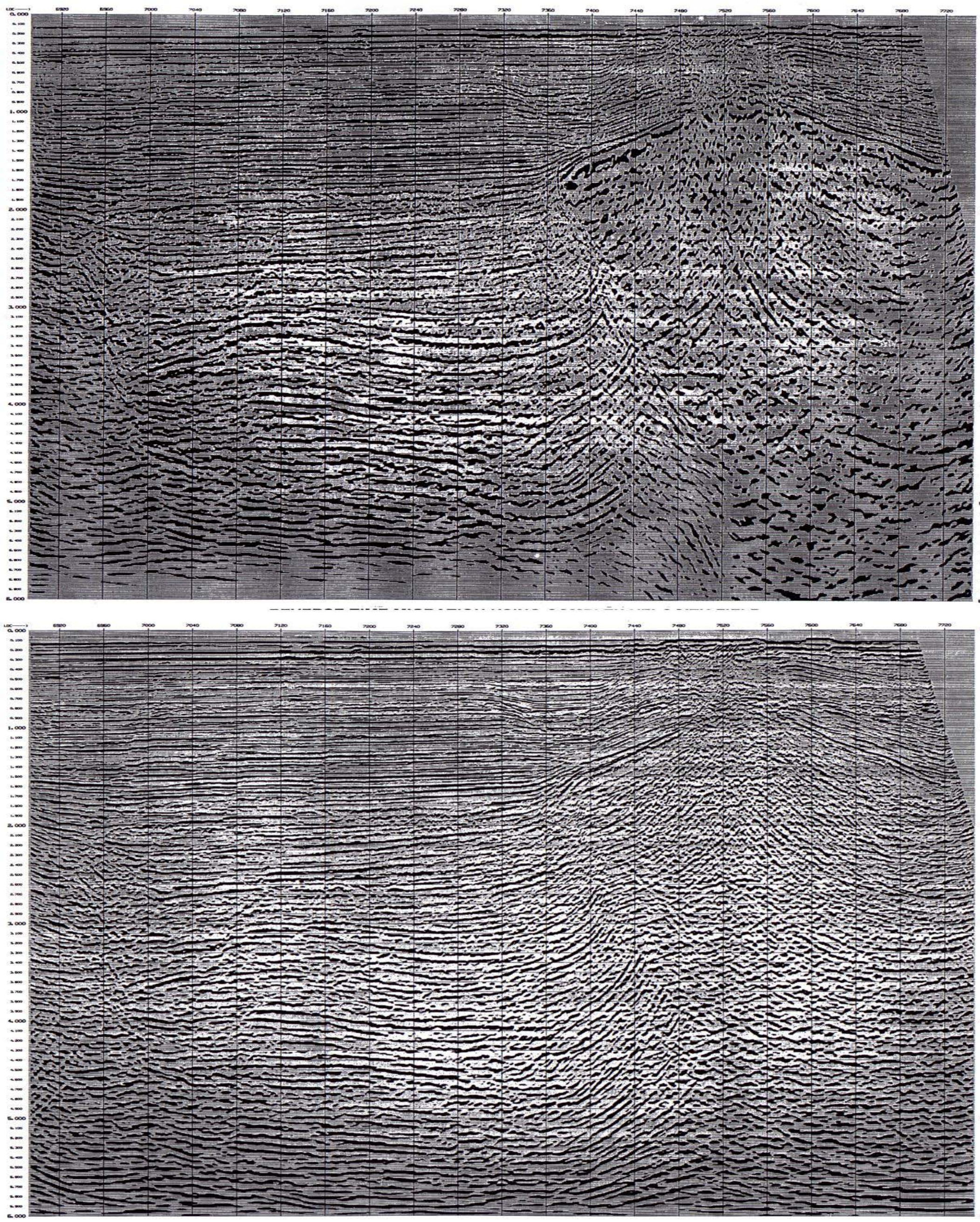

Fig. 3: Two-way vs. one-way migration of a Gulf of Mexico salt structure. Top is a full two-way migration. Bottom is a one-way steep dip migration of the same structure. Note: top is the same image as in Figure 2.

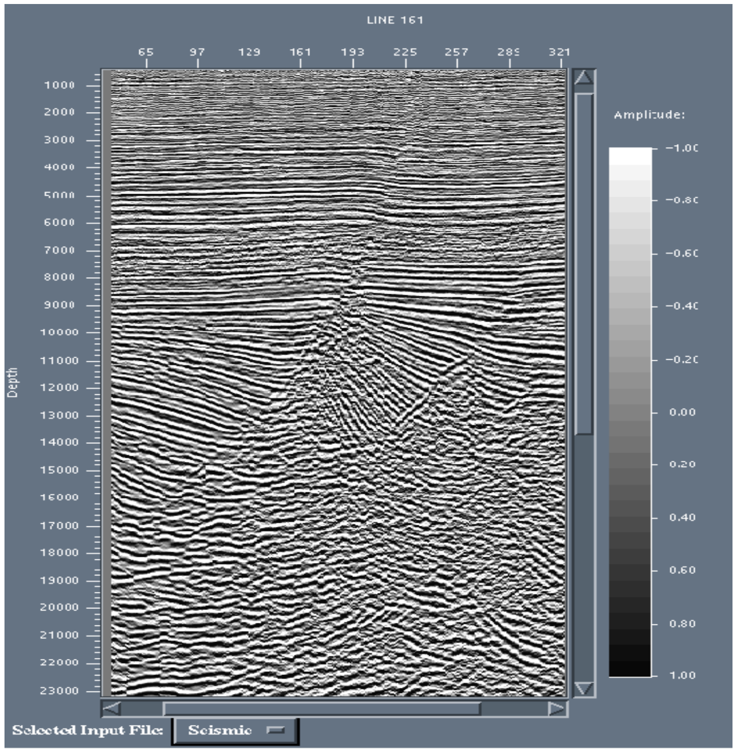

Fig. 4: One-way maximum amplitude Kirchhoff migration of a South Texas land data set. This line demonstrates the resolution of a typical fault shadow problem.

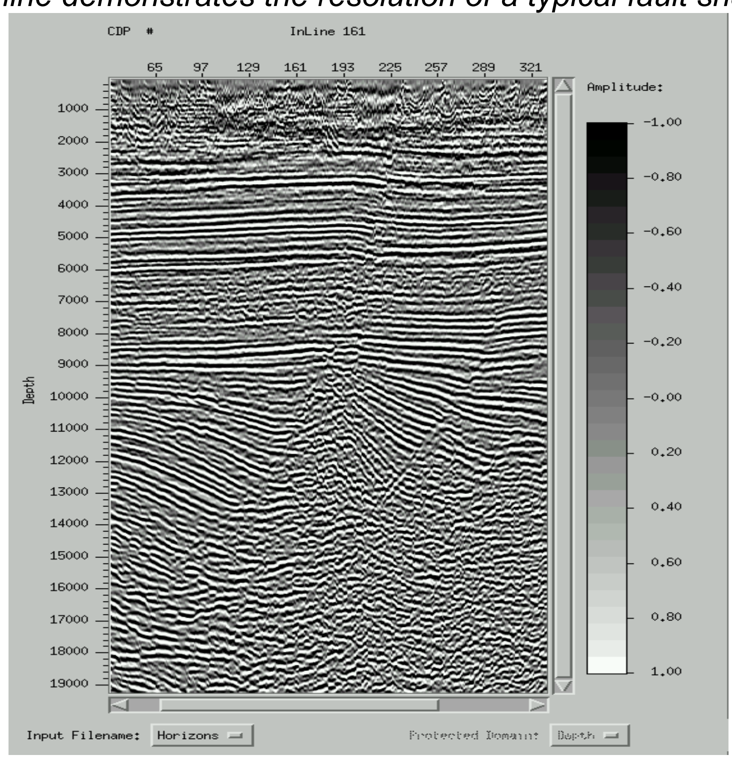

Fig. 5: Two-way migration of a South Texas land data set. The velocity model used for this migration was identical to the one used to obtain Fig. 4.

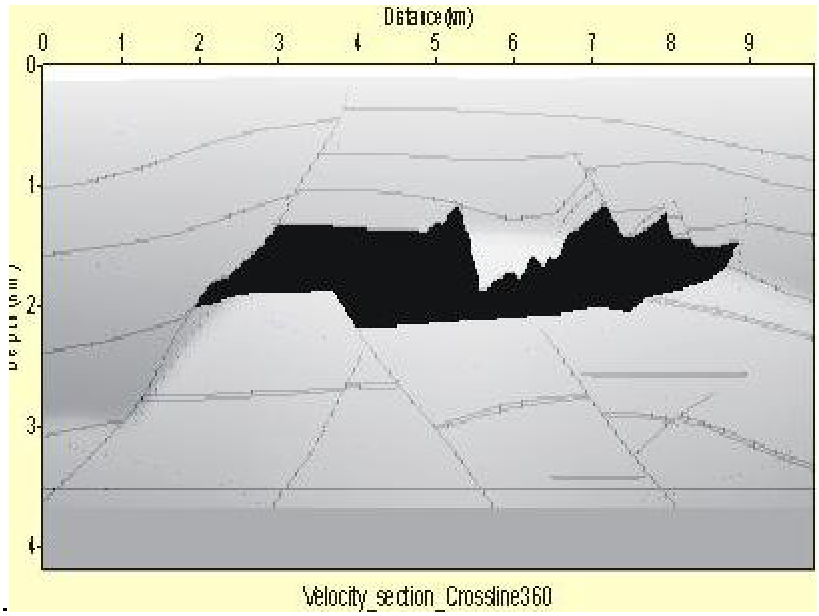

Fig. 6: Velocity crossline 360 from the SEG/EAGE salt data set. This slice is for comparison with Figure 7 below.

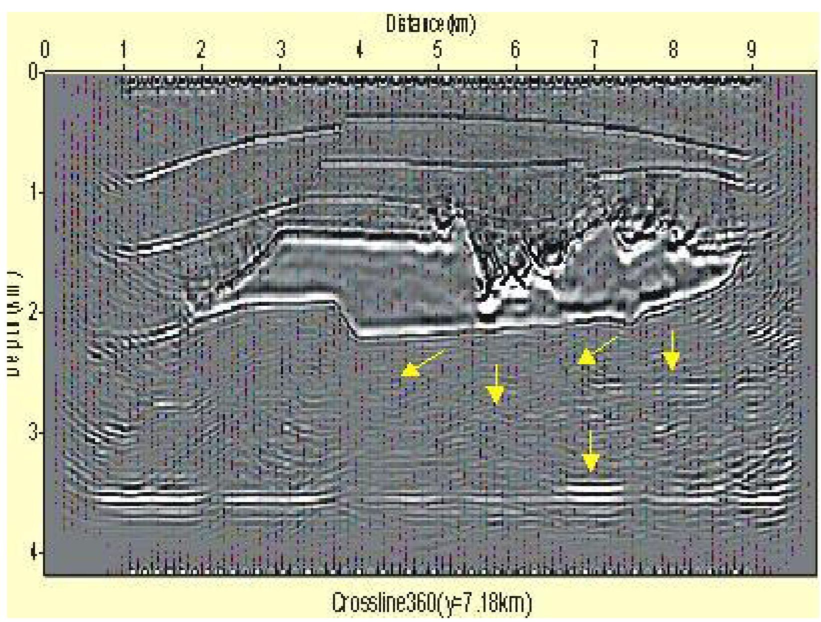

Fig. 7: Crossline 360 from a full 3D reverse-time depth image of the SEG/EAGE salt data set.