Inversion for Thomsen's parameters using Shifted Hyperbola NMO equation.

Pavan Elapavuluri*,
pavan@geo.ucalgary.ca

and

John Bancroft,
bancroft@geo.ucalgary.ca
CREWES, University of Calgary, Calgary, AB, Canada.

ABSTRACT

Introduction

The main hurdle in incorporating anisotropy into the processing algorithms is that the anisotropy parameters are often not known accurately.

Tsvankin and Thomsen (1994) described a NMO equation for TI media in terms of the Thomsen's parameters. Castle (1994) proposed shifted hyperbola NMO equation, which is a better approximation to the moveout than the Dix's NMO (Dix, 1955) equation. These two equations are used for the parameter estimation in this study. Shifted hyperbola is exact through fourth order in offset while Dix's NMO equation is only a second order approximation (Castle, 1994). He also showed that RMS velocities estimated using the shifted hyperbola are much more accurate than those estimated from the Dix's equation.

The estimation procedure consists of two steps. In the first step, the parameters for normal moveout correction, V_{NMO} and the *"shift parameter (S)"*, are determined using Monte-Carlo inversion from common scatter point (EO) gathers. In the next step, the anisotropic parameters are computed over the data. A relationship that describes their dependency on the S, V_{NMO} and V_0 (vertical velocity from well-logs/VSP surveys) is used.

In a related study Elapavuluri and Bancroft (2002) showed that both ε and δ could be estimated accurately over synthetic data using this method.

Equivalent Offset (EO) gathers

In this study, velocity analysis is performed on the Equivalent Offset (EO) gathers formally referred to as common scatter point (EO) gathers. An EO gather is a pre-stack migration gather that is a selection of traces containing energy from vertical array of scatter points. (Bancroft *et al*., 1998).

EO gathers are similar to common mid-point (CMP) gathers as both are driven by NMO correction and consequently stacking. However, the EO stack is a complete migrated stack describing the true sub-surface position of the seismic events whereas the CMP stack requires further post-stack migration. As the EO gathers contain all the traces within the migration aperture, they have a higher fold in the offset bins and larger offset range. This leads to a more accurate velocity analysis. (Bancroft et.al, 1996)

EO gathers are formed by mapping all input traces to an equivalent offset at every migration output location (Bancroft et al., 1998). The mapping is described as:

$$h_e^2 = x^2 + h^2 - \frac{4x^2h^2}{t^2 v_{mig}^2} \tag{1}$$

where h_e is the equivalent offset, x is the horizontal displacement between the CMP and the EO, h is half source-receiver offset, v_{mig} is the migration velocity and t is the traveltime.

Shifted hyperbola

The normal moveout equation used commonly to shift events at non-zero offsets to their equivalent zero offset time is given as:

$$t = \sqrt{t_0 + \frac{h^2}{V_{NMO}^2}} \tag{2}$$

where t is the traveltime at offset h, t_0 is the zero-offset (normal incidence) traveltime and V_{NMO} is the normal moveout velocity (Dix, 1955). V_{NMO}, and is essentially a parameter that yields the best stack, is commonly used as an approximation for the Root Mean Square (RMS) velocity.

Dix's NMO equation is an exact hyperbola, which is symmetric about the *t*-axis with the asymptotes that intersect at the origin (*x=0, t=0*). However, for a layered earth model, Dix's equation is only a small offset approximation. Castle (1994) derived a new approximation to the NMO equation using the principles of reciprocity, finite slowness and exact constant velocity limit. For *"reasonable"* (Castle, 1994) offsets, his approximation, termed as the shifted hyperbola equation, is given as:

$$t = t_0\left(1 - \frac{1}{S}\right) + \sqrt{\left(\frac{t_0}{S}\right)^2 + \frac{x^2}{SV_{NMO}^2}} \;. \tag{3}$$

In the above equation, the *shift parameter*, *S*, is a constant and is described as:

$$S = \frac{\mu_4}{\mu_2^2}, \tag{4}$$

where μ_2 and μ_4 are the second and fourth order moments of the velocity distribution.

Estimation of S and V_{nmo}

The shifted Hyperbola equation is a non-linear problem so linear inversion techniques e.g least square inversion fail. A random walk technique like Monte-Carlo inversion would serve the propose of inverting the moveout equation (2) for both '*S*' and V_{nmo}.

Monte-Carlo Inversion

The method can be described by the following equation for a model parameter set **m**(*S,* V_{nmo})

$$m_i^{new} = m_i^{\min} + (rn)\left[m_i^{\max} - m_i^{\min} \right] , \qquad (5)$$

where m_i is the model parameter m^{min} and m^{max} are the minimum and maximum values of the model parameter specified and '*rn*' is a random number drawn from a uniform distribution [0,1].

The generated models $\mathbf{m}^{new}$ are tested iteratively. The generated model that best fits the data with a minimum misfit is accepted.

Methodology used in the current study

Taner and Koelhler (1969) gave the following generalized equation for NMO:

$$t^2 = c_1 + c_2 x^2 + c_3 x^4 + ... \qquad (6)$$

Conventional NMO of Dix truncates the above series to the second power of *x* (source-receiver offset) whereas Castle's algorithm extends to the fourth power in *x*. Castle's NMO equation (3), can be re-written in the from of Taner and Koelher's equation (6). The coefficients are as follows

$$c_1^S = t_0^2 , \qquad (7)$$

$$c_2^S = \frac{1}{V_{nmo}^2} , \text{ and} \qquad (8)$$

$$c_3^S = \frac{1}{4} \frac{(1-S)}{t_0^2 V_{NMO}^2} . \qquad (9)$$

Tsvankin and Thomsen (1994) described a NMO equation for TI media in terms of the Thomsen's parameters. Their equation can be re-written in the form of Taner and Koehler (1969), as in *equation (6)*, to yield the following Taylor series coefficients (denoted with a superscript *T*):

$$c_1^T = t_0^2 , \qquad (10)$$

$$c_2^T = \frac{1}{V_0^2 (1+2\delta)} , \text{ and} \qquad (11)$$

$$c_3^T = \frac{-2(\varepsilon-\delta)}{t_0^2 V_0^2}\left[\frac{1+\dfrac{2\delta}{1-k}}{(1+2\delta)^4}\right]. \tag{12}$$

where V_0 is the vertical velocity and k is ratio V_p/V_S
Given that co-efficient c_2^S (8) is equal to c_2^T (11), the relationship for δ can be written as:

$$\delta_n = \frac{1}{2}\left(\frac{V_{NMOn}^2}{V_{0n}^2} - 1\right) \quad . \tag{13}$$

Similarly, an expression for ε can be computed from the coefficients c_3^S (9) and c_3^T (12) given by equation (14)

$$\varepsilon_n = \delta_n - \frac{1}{2}\left(C_3^i * \frac{\left(1+2\delta_n\right)^4}{\left(1+{2\delta_n}/{1-k^2}\right)} * t^2 * V_{0,n}^4 \right). \tag{14}$$

C_3^i, H^i, and $F(N)$ can be calculated as given by Tsvankin and Thomsen (1994) where

$$C_3^i = \frac{H_i}{4t_o^2 V_{rms}^8}, \tag{15}$$

$$H^i = \frac{F(N)t_0(N) - F(N-1)t_0(N-1)}{t_0(N) - t_0(N-1)} - V_2^4, \tag{16}$$

$$F(N) = V_2^4(N)\left[1 - 4c_3(N)t_0^2(N)V_2^4(N)\right], \tag{17}$$

and $c_3(N)$ is

$$c_3(N) = \frac{1}{4}\frac{(1-S)}{t_0^2 V_{NMO}^2} \tag{18}$$

Field data

This method was applied over the P-wave seismic data collected over the Blackfoot field. Blackfoot field is near Strathmore, Alberta and is operated by PanCanadian petroleum. 3C 3D data was acquired on this field by CREWES consortium in 1997. The stack section is shown in Figure 1.

Figure 1 Position of the well 09-08 on the stacked section .

The geology of Blackfoot field has been discussed in detail by Miller et. al (1995). A brief review of the lithology of of interest to this work describes the reservoir rocks in this field as Glauconitic incised valleys in Lower Manville group of lower Cretaceous. Coals, Viking formation and Base of fish scales shales overlie these reservoir rocks. The abbreviations used for the units used in this study are given *Table 1*.

Haase(1998) showed that the non hyperbolic moveout observed with the flat reflectors in the plains data is due to the transverse isotropy (even though the individual layers are isotropic).

A 2D line numbered '*20M vertical*' with a well (#09-08) located very near to it was chosen to test this method. An EO gather was formed at CDP 149, which is nearest to the well location.

Monte-Carlo inversion was applied on major formations of interest in this field. The results of Monte-Carlo inversion are listed in *Table 2* and the values of ε and δ estimated in *Table 3*.

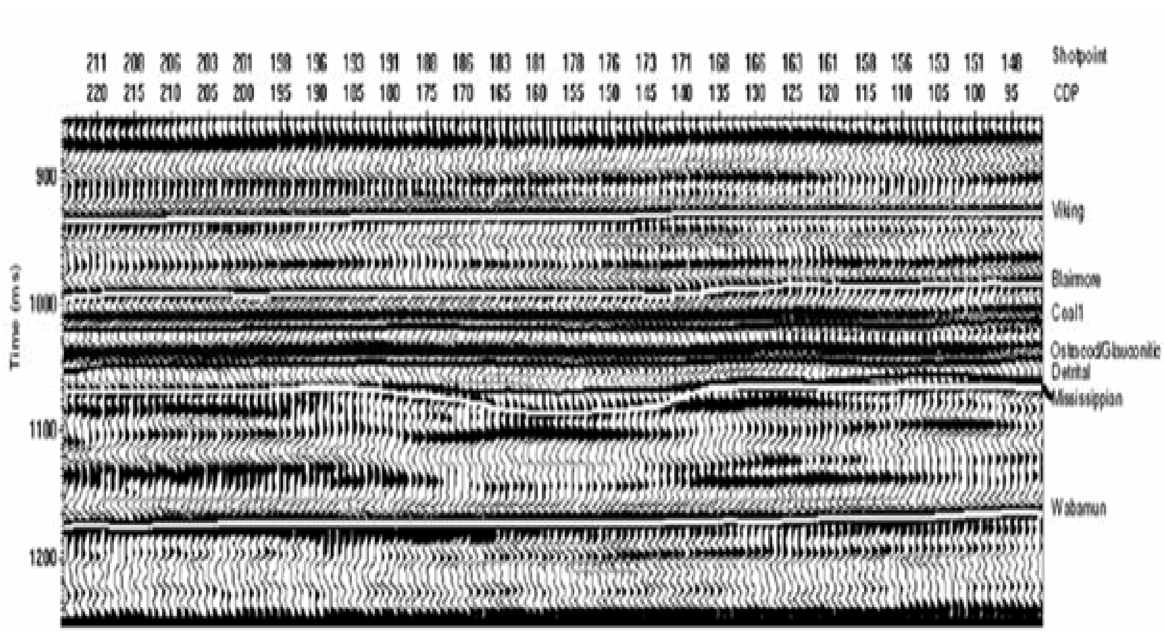

Fig. 1 .The seismic section with important horizons marked after Miller et al *(1995)*

Table 2: Formation naming convention

Formation	V_{nmo}	Shift (S)	$V_{0,i}$
BFS	4002	0.6987	3300
MANN	4148	0.9388	3990
COAL	4755	0.6145	3900
GLCTOP	4460	0.8604	3860
MISS	5998	0.7256	6000

Table 3: δ and ε values estimated

Formation	δ (estimated)	ε (estimated)
BFS	0.23	0.06
MANN	0.04	0.008
COAL	0.24	0.12
GLCTOP	0.06	0.006
MISS	0.00	0.001

Conclusions

In this paper, a method to estimate the Thomsen's parameters (ε and δ) for TI media is described. An inversion technique for the estimation of '*S*' is also proposed.

Extending this analysis to the real field data, we found that the shales and coals show very significant anisotropy. The vertical velocities estimated from a sonic log were used in this study. The velocities from sonic log are greater than the seismic velocities. Vertical interval velocities estimated from VSP data would give more accurate estimates in this area. The inversion can be made more robust.

References

Bancroft, J.C., 1996, Velocity sensitivity for equivalent offset prestack migration , Ann. Mtg: Can. Soc. Of Expl Geophys.

Bancroft, J.C., Geiger, H.D. and Margrave, G.F., 1998, The equivalent offset method of prestack time migration: Geophysics, Soc. Of Expl. Geophys., **63,** 2041-2053.

Castle, R.J., 1994, Theory of normal moveout: Geophysics, Soc. Of Expl. Geophys., **59,** 983-999.

Dix, C.H., 1955, Seismic velocities from surface measurements: Geophysics, Soc. Of Expl. Geophys., **20,** 68-86.

Elapavuluri, P., and Bancroft, J.C., 2002, Estimation of anisotropy parameters using shifted hyperbola NMO equation. , 64th Ann. Mtg: EAGE.

Taner, M.T. and Koehler, F., 1969, Velocity spectra - Digital computer derivation and applications of velocity functions : Geophysics, Soc. Of Expl. Geophys., **34,** 859-881.

Haase , Armin. B., 1998, Non hyperbolic moveout in plains data and the anisotropy question: Recorder, Can. Soc. of Expl Geophys. Nov, 1998, 21-33.

Miller, S.L.M., Aydemir, E.O. and Margrave, G.F., 1995, Preliminary interpretation of P-P and P-S seismic data from Blackfoot broad-band survey: Crewes Research Report 1995, Ch 42.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, Soc. Of Expl. Geophys., **51,** 1954-1966.

Tsvankin, I. and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, Soc. Of Expl. Geophys., **59,** 1290-1304.