Stacking velocities for different offsets in a medium with laterally inhomogeneous layers

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ABSTRACT

For small offsets, near-zero-offset stacking velocity can be expressed in terms of the second-order moveout derivative. Direct connection between the second-order moveout derivatives and wavefront curvatures allows us to use either of them (see Goldin, 1986, Krey and Hubral, 1980 and references in these books). We use the second-order derivatives to study near-zero-offset NMO velocities. According to Puzirjov and Goldin (Puzirjov, 1979, Goldin, 1986) we can call this stacking velocity "differential" and write it as V_{NMO}. For larger spreadlength, the stacking velocity for the layered medium depends on NMO geometry. We can name this velocity 'integral' and write it as V_{STACK}. For hyperbolic moveout, differential and integral velocities coincide. For the vertical inhomogeneous medium, V_{NMO} coincide with V_{RMS}. We can consider V_{RMS} as V_{NMO} velocity for the 1D velocity model along the vertical line through the CDP point.

If the medium contains lateral changes in the velocity, they can cause significant non-hyperbolic moveout. In this case, difference between differential and integral velocity increases and can influence interval velocity estimation.

The purpose of differentiating between integral and differential velocities is that from the velocity analysis we calculate integral stacking velocities, but for the time inversion (including Dix's formula), we need differential ones. Usage of integral velocities in Dix-type formula leads to systematic errors. This error depends on the vertical and lateral velocity changes and on the offset in CDP gathers.

To calculate the integral stacking velocity, we have to run raytracing, calculate CDP time arrivals for the specific (given) geometry and to approximate the obtained time arrival function with the hyperbola T(L):

$$T^2(L) = T_0^2 + L^2/V_{STACK}^2$$

Y. Riznichenko called V_{STACK} effective velocity (Riznichenko, 1946). We investigate the difference between the differential and integral stacking velocities using both an analytical approach and modeling. We consider velocity model

composed of the layers with small-dips curvilinear boundaries and with lateral variations of interval velocities. We assume that the lateral changes in the interval velocities are relatively small (10% - 15%) with respect to their absolute values. It is well known (Lavrentiev et al., 1969) that in this medium, we can linearize time with respect to the slowness changes and calculate traveltime along the straight ray. As shown in (Blias, 1988), we can also calculate zero-offset traveltime along the vertical ray because the influence of the small boundary dips is the second order with respect to depth changes.

We also analytically consider the behaviour of the stacking velocities in a medium with laterally changing interval velocities. W. Lynn and J. Claerbout (Lynn and J. Claerbout, 1982) obtained the formula for stacking velocity for the one layer model. They also considered the inverse problem using obtained second-order differential equation and its numerical solution. Gritsenko and Chernjak (2001) used another approach to solve this equation. Blias (1981, 1987, 1988,) derived a formula for stacking velocities in multilayered medium with gently curvilinear boundaries and lateral variable velocities in 2-D and 3-D models. Using a perturbation method, he obtained an explicit formula, connecting laterally changing interval velocities with differential stacking velocities. This formula allows us to analytically estimate the influence of the shallow inhomogeneous layers on the stacking velocities from deep horizontal reflectors. Here we may mention that, in layered medium, we can see several effects that cannot be seen in one-layer velocity model.

Analytical analysis of NMO velocities in medium with laterally inhomogeneous layers

Let us consider a velocity model composed of n horizontal laterally inhomogeneous layers. For near-zero-offset stacking velocity V_{NMO} (in our terminology - differential), we can derive the formula (Blias, 1981, 1987, 1988)

$$1/V_{NMO}^2 = 1/V_{RMS}^2 \left(1 + \sum_{k=1}^{n} h_k s_k'' b_k\right) \quad (1)$$

where

$$b_k = \left[\sum_{i=k+1}^{n} h_i v_i \sum_{i=k}^{n} h_k v_k + (1/3)\, h_k^2 v_k^2\right] / \left(\sum_{i=1}^{n} h_i v_i\right) \quad (2)$$

$$V_{RMS}^2 = \sum_{i=1}^{n} v_k h_k / \sum_{i=1}^{n} h_k / v_k$$

Here h_k is the thickness of the k-th layer, $v_k(x)$ is an interval velocity in this layer, $s_k(x) = 1/v_k(x)$ stands for slowness. Let us analyse this formula and make some conclusions about the influence of lateral changes in interval velocities on stacking velocities.

Formula (1) shows that nonlinear variation of the interval velocities creates a bigger influence than the gradient. Gradient of the interval velocities fits into stacking velocities in the second power. Since we took into account only linear changes, *equation (1)* does not include the first derivatives of the interval velocities.

The second multiple in the right side of (1) (the difference between this multiple and 1) shows the difference between the stacking and RMS velocities. The value of the dimensionless sum within the brackets exactly affects the difference between the RMS and stacking velocities – the larger the sum the bigger is the difference. This sum represents all laterally inhomogeneous layers. From (1) it follows, that the larger the sickness h_k the bigger is the influence of the k-th inhomogeneous layer on the stacking velocity V_{NMO}

Coefficient b_k reflects the influence of the k-th inhomogeneous layer – the larger this coefficient the more is the influence of this layer on the stacking velocity. The value of this coefficient depends on the position of this layer in the ground. Let us consider the k-th layer and its influence on the difference between RMS and stacking velocities with deeper reflection boundaries – i.e. with "n" increasing. For b_k we can write an approximate formula:

$$b_k \approx \left(\sum_{i=k}^{n} h_i v_i\right)^2 / \left(\sum_{i=1}^{n} h_i v_i\right) \qquad (3)$$

This formula shows that with n increasing the numerator increases as second power of the sum and denominator only as a first power that is much slower. It implies that the influence of the inhomogeneous layer increases with greater reflector depth and with a decrease in the depth of the inhomogeneous layer. Often the biggest lateral velocity changes, we can see at the shallow part of the section.

Taking this into account, let us consider a layered medium with all laterally homogeneous layers except the first layer. Then the formulas (1) and (2) can be written in the way:

$$1/V_{NMO}^2 = 1/V_{RMS}^2 \left(1 + h_1 s_1'' \sum_{i=2}^{n} h_i v_i\right) \qquad (4)$$

In terms of velocity, this formula can be rewritten as:

$$1/V_{NMO}^2 = 1/V_{RMS}^2 \left(1 - h_1 v_1'' \sum_{i=2}^{n} h_i v_i / v_1^2\right) \qquad (5)$$

Formulas (4) and (5) show, that stacking (NMO) velocity can be either less or bigger than RMS velocity. It depends on the sign of the slowness second derivative s_1''. It’s obvious that the deeper is reflector (the bigger is the some in the round brackets) the more is the difference between RMS and stacking velocities. From (4′) it follows, that, for deep reflectors, the stacking velocity function repeats the behavior of the second derivative of the velocity anomalies.

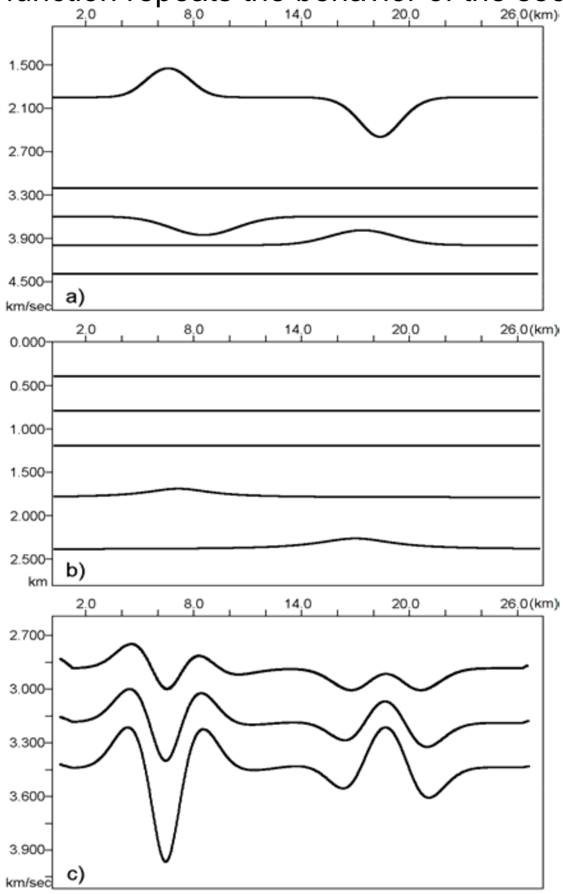

Fig.1 Velocity model with shallow inhomogeneous layer
a - Interval velocities
b - boundaries
c - Stacking velocities for the model with inhomogeneous layer

Formula (5) shows that two magnitudes affect NMO velocity: RMS velocity (first multiple) and laterally inhomogeneous overburden layer. If the reflector is shallow, than the second multiple is close to 1 and NMO velocity repeats the behavior of RMS velocity. For the reflector depth increasing, the sum in the brackets is growing and its influence on NMO velocity values is increasing. For deep reflector, the stacking velocity repeats the behavior the second-order derivative of the shallow velocity $v_1(x)$. This is very well seen on the *Fig. 1*, where the interval velocities (a), boundaries (b) and near-zero-offset (NMO interval 800 m) stacking velocities (c) are shown. The stacking velocity behavior (c) is very close to that of the second-order derivative (with the scalar from (5)) of the first interval velocity. The same quality holds for the curvilinear boundary (Blias, 1981, 1987, 1988, 2002). Because of deep reflector, the stacking velocity behavior just reflects the behavior of the second-order derivative of the curvilinear shallow boundary. This fact explains why we should not expect stacking velocity to repeat average (or RMS) velocity behavior when we have strong nonlinear lateral velocity changes in the shallow part of the section.

Calculation of near-zero-offset NMO velocities

To calculate the differential stacking velocity, we may calculate the second derivative of the moveout time at the point L = 0. For moveout t(L) where L is the offset, we can write the Taylor series presentation:

$$T^2(L) = T_0^2 + L^2/ V_{NMO}^2 + \dots \qquad (6)$$

Finding the second-order derivative at the point L=0, we obtain the connection between differential velocity and moveout derivative:

$$V_{\mathrm{NMO}} = \sqrt{1/(t_0 \cdot t_{LL})} \qquad (7)$$

where t(L) – moveout, L – offset, the second derivative t_{LL} of the function t(L) is considered at the zero-offset point L = 0, t_0 = t(0) – normal incident time. We may also think of the differential velocity as a velocity found through ray tracing and moveout hyperbolic approximation for a very small offset.

From (2) it follows that differential NMO velocity is directly connected with the second-order moveout derivative t_{LL} at the zero-offset ray. First of all, as shown by S. Gritsenko and V. Chernjak (Gritsenko and Chernjak, 1979), this derivative is equal to one half of the second order derivative of the one-way traveltime from the zero-offset reflection point. This second-order derivative we will calculate using an approach developed by E. Blias, S. Gritsenko and V. Chernjak (Blias et al. 1984). In this paper, an algorithm for all traveltime derivative layerwise calculation has been suggested. We are interested only in a layerwise calculation of the derivative with respect to receiver coordinates.

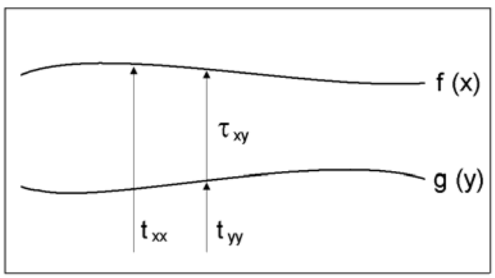

Fig. 2 Diagram for the second order derivative calculation through the boundary g(y) to the boundary f(x)

Let us use these notations: t(y) is traveltime from the zero-offset reflection point to the boundary g(y); t(x) is traveltime from the zero-offset reflection point to the next boundary f(x) along the ray, *Fig. 2*. If we know the second-order derivative t_{yy} along the boundary g(y) (as if the receiver is on this boundary), then the second-order derivative t_{xx} along the next boundary f(x) along the ray can be calculated according to the formula:

$$t_{xx} = -\frac{\tau_{xy}^2}{t_{yy} + \tau_{yy}} + \tau_{xx} \qquad (8)$$

Here τ is the time in the layer between the points g(y) and f(x). To use formula (8), we need to find the second-order derivatives of the vertical time in the layer with laterally changing velocity. Formulas for these derivatives were obtained in (Gritsenko and Chernjak, 2001):

$$\begin{aligned} \tau_{xx} &= \frac{n}{s} + s \cdot n_{xx}/3 - f_{xx\cdot} \cdot n - f_x n_x \\ \tau_{xy} &= -\frac{n}{s} + s \cdot n_{xx}/6 + (g_x - f_x) \cdot n_x/2 \\ \tau_{yy} &= \frac{n}{s} + s \cdot n_{xx}/3 + g_{yy} \cdot n + g_y n_x , \end{aligned} \qquad (9)$$

Here n(x) is a slowness, s stands for the distance between the points f(x) and g(y), that is s = |g(y) – f(x)|. The scheme for the calculation as follows: We calculate the second-order derivative on the top of deepest layer using the first *formula (9)*. Then, knowing this derivative, we use *formulas (8)* (here t_{yy} is known from the previous step) and *formulas (9)* to recalculate the second order derivatives on the top of the shallower layer. Then we go to the next upper layer along the vertical ray and so on, until we reach the measurement surface. If we linearize (at each step) the formulas with respect to the second-order boundary and interval velocity derivatives, we obtain formulas derived by one of the authors (Blias, 1981, 1988, 2002). Actually, here we have recurring way of these formulas, if we linearize them.

Velocity model and geometry affect the integral NMO velocity, while the differential velocity depends only on a velocity model. Because the integral stacking velocity is finding on the spreadlength (offset interval), it depends on the source and receiver locations in this interval. As was stated above, the less the interval length, the closer is integral velocity to the differential velocity. On the other hand, the less the spreadlength the bigger is the influence of the source-receiver position on the integral velocity. This influence is very big if the source interval is bigger than the receiver one.

Usually the difference between the differential and integral velocities is connected with non-hyperbolic moveout. Indeed, if moveout is exactly hyperbolic, the integral and differential velocities are the same. Usually the larger the offset interval the more non-hyperbolic moveout is and the more difference between differential and integral velocities would me. However, when we have lateral velocity changes, the difference between differential and integral velocities can increase even when we have a very small difference between moveout and its hyperbolic approximation. It means that this almost hyperbolic moveout can change significantly because of lateral velocity changes within the increasing measurement interval.

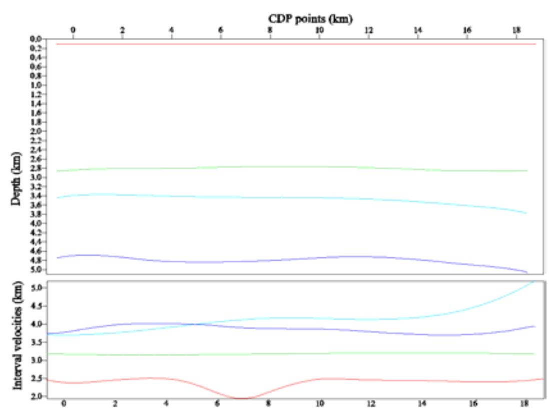

Fig. 3 Velocity model with lateral velocity variations

To illustrate this we created a four-layer model with permafrost thaw, *Fig. 3*. This model reflects the geology of Tymir district in northern Siberia. First let us consider the geometric influence on the integral velocity V_{STACK}. *Fig. 4* shows the stacking velocity for the deepest reflector for the different interval lengths. This plot shows that the smaller the NMO interval the bigger is the integral velocity oscillation. For the shortest interval (850 m) stacking velocity changes are up to 50m/s while for the longest they are less

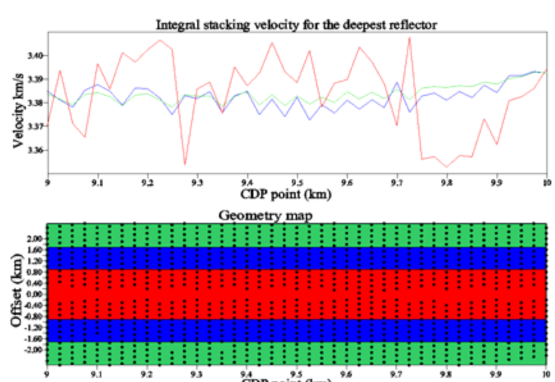

Fig. 4 Integral stacking velocity for different NMO interval

than 5m/s. In velocity analysis we find integral NMO velocity and for the shallow reflectors lateral velocity changes can be significant because of muting. *Fig. 5* shows differential (red line) and integral (blue) velocities with 850m NMO interval. Because integral velocity was calculated for the short NMO interval, the high-frequency difference between these velocities was caused by geometry changes.

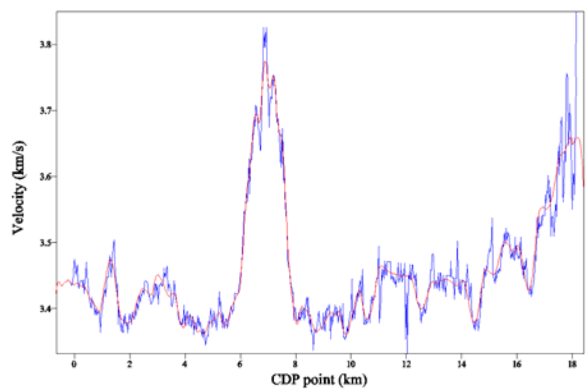

Fig. 5 Differential and integral stacking velocities for the deepest reflector. Maximum offset 850m

Integral stacking velocity   Differential stacking velocity

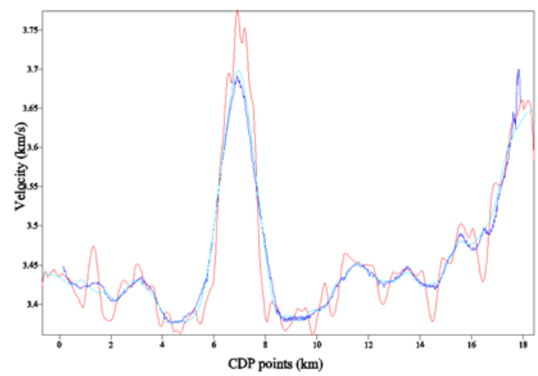

Fig. 6 Differential and integral stacking velocities for the deepest reflector. Offset 2550 m

Integral stacking velocity   Differential stacking velocity

Differential velocity smoothed with 2.5 km window

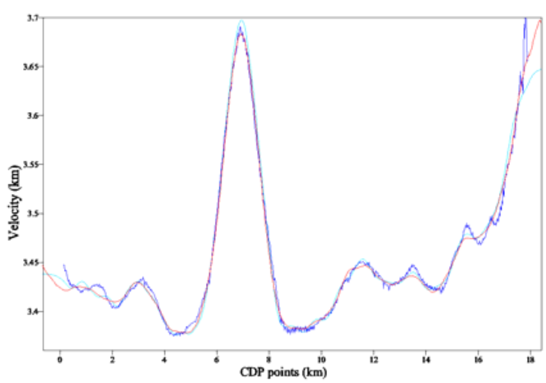

Fig. 7 Differential and integral stacking velocity for the deepest reflector. Offset 2550 m

Integral stacking velocity   Differential stacking velocity with optimum approximation interval

Smoothed differential velocity. Smoothing window 2.5 km

Fig. 6 shows differential (red line) and integral (blue) velocities with 2550m NMO interval. This plot shows that for big NMO interval velocity can be considered as smoothing differential velocity. On the long NMO measurement interval, integral velocity slightly depends on geometry (set of the offsets), so it has less oscillation than differential stacking velocity. At the same time, Dix's type inversion formulas require differential velocity that cannot be covered from the integral. The difference between integral (obtained from real data) and differential (calculated fro the second-order NMO derivative) can lead to the error in interval velocity estimation.

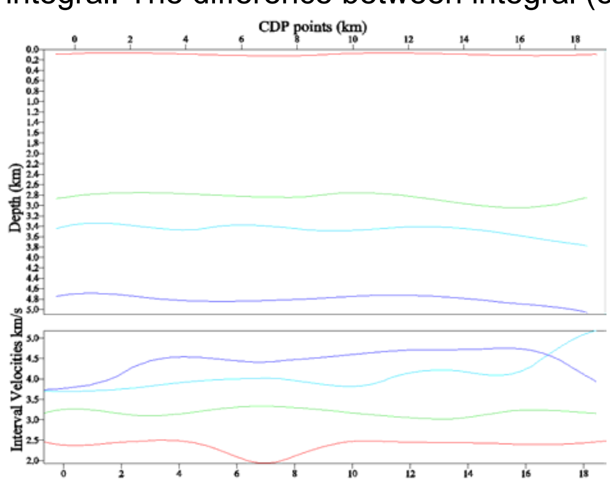

Fig. 8 Velocity model with lateral velocity changes

Permafrost bottom   Second layer   Third layer   Fourth layer

Let us consider an approach that allows us to find integral NMO without an expensive raytracing procedure. While calculating differential velocities with the use of *formulas (8) and (9)* we have to calculate the first and second-order derivatives of the boundaries and interval velocities along the zero-offset ray. To find these derivatives, we use parabolic approximations of interval velocities and boundaries around the vertical ray. The length of the interval for parabolic approximation affects the value of these derivatives, which, in their turn, influence the value of the differential velocities. The larger the approximation interval the less is the oscillation of the second derivatives and the less is the oscillation of the differential NMO velocity. We can find the length of the approximation interval for which the difference between differential and integral velocities is the least. *Fig. 7* shows differential velocity calculated for the optimum approximation interval 2550m (--), integral stacking velocity for 2550 NMO interval (--) and differential velocity (--) smoothed with the a smoothing window of 2.5 km. This plot shows that to find integral stacking velocity, instead of smoothing differential velocity, we can choose a proper approximation interval to calculate interval velocity and boundary derivatives.

This fact shows that we can find integral velocities using differential velocities and an appropriate approximation interval for the derivative calculation. The question is how this optimum approximation interval depends on the model. To test it, we ran calculations on the more complex model (*Fig. 8*) and we have got the same result *(Fig. 9)*. It's interesting that smoothing of the differential velocity gives worse result than using optimum approximation interval for the derivative calculations. A very important feature of this is that we can use Dix's type inversion for the differential velocities (which are very close to the integral) and this inversion will give us a better result than if we use a small (non optimum) approximation interval for the derivative calculations.

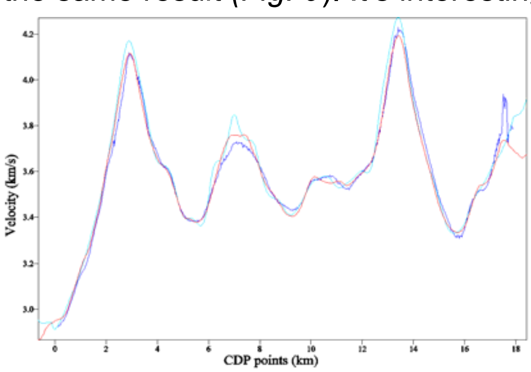

Fig. 9 Differential and integral stacking velocities for the deepest reflector. Offset 2550 m

Integral stacking velocity — Differential stacking velocity with optimum approximation interval for derivative calculation — Differential velocity smoothed with 2. 5 km window

Let us go into some detail about optimum approximation intervals for the boundary and interval velocity derivative calculation. *Fig. 10* shows maps of the standard deviation between differential and integral velocities. The optimum (dark brown) area is big enough so we can choose the approximation interval in a wide range. This means that the length of the optimum interval slightly depends on the velocity model. For two models (*Fig. 3 and Fig. 8*) we can choose the same approximation intervals (one for the interval velocity and another for the boundary). Optimum approximation interval for the interval velocities is little less then the NMO measurement interval. For the boundaries, this interval is a little less than half of the boundary depth and it can be chosen from a wider range than for the velocity.

Choosing optimum approximation intervals for the derivative calculations we can calculate integral velocities only through zero-offset raytracing and *formulas (8)* *and (9)*. Moreover, we can use Dix's type of inversion and apply them to the integral velocities.

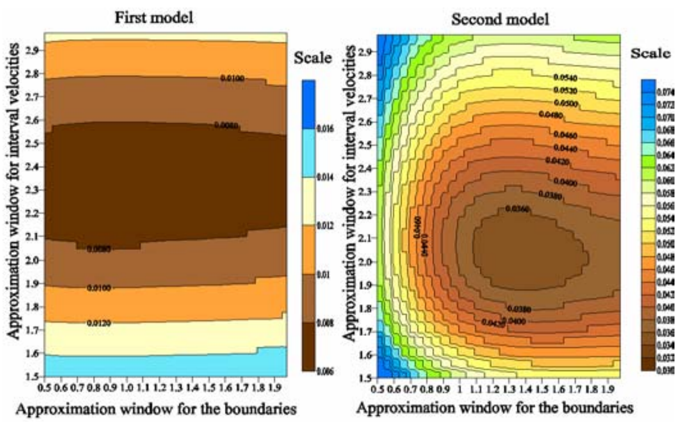

Fig. 10 Standart deviation maps between differential and integral velocities for the deepest reflector. Offset 2550 m

In conclusion, let us consider stacking velocities below the permafrost thaw. *Fig. 11* shows interval velocities (b) and stacking velocities for all reflectors. The extremum of the shallow stacking velocity has the same sign as the interval velocity in the first layer. For deeper reflectors, extremum has the opposite sign and the oscillations become bigger. This behaviour can be explained through the *formulas (1), (2)*. For the shallow reflections, the second scalar in (1) is close to unit so the stacking velocity is close to the RMS velocity, which is kind of average velocity. Zone of increasing first layer velocity corresponds with the same zone of RMS velocity.

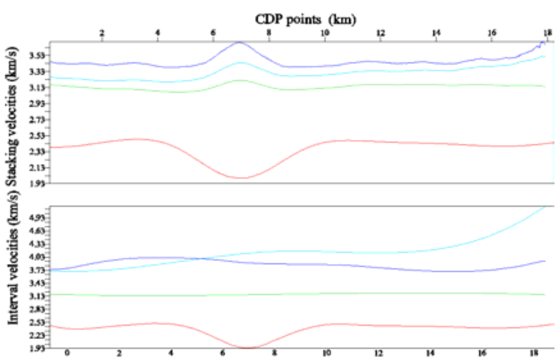

Fig. 11 Interval and integral stacking velocities for the first velocity model

Permafrost bottom   Second layer   Third layer   Fourth layer

When we go to deeper reflectors, the absolute value of the sum in the brackets becomes bigger and starts to play a significant role in stacking velocity behaviour. In the permafrost thaw zone, the first layer has the biggest lateral velocity changes, so we can neglect the other lateral velocity changes and use *formula (5)*. This formula can be rewritten in the way:

$$V_{NMO} = V_{RMS} \left(1 + \frac{1}{2} h_1 v_1'' \sum_{i=2}^{n} h_i v_i / v_1^2\right) \qquad (10)$$

This formula shows that, for deep reflectors (for big values of the second term in the brackets), the behaviour of stacking velocity repeats the behaviour of the second derivative of the velocity in the first layer, with the scalar $h_1v_1 + h_2v_2 + \ldots + h_nv_n$, that increases with the depth of the reflection boundary. At the same time, for shallow reflectors, the second multiple in (10) is close to unit and the behaviour of stacking velocity repeats the behaviour of RMS velocity that is close to average velocity. This is what we see on the *Fig. 11*, which shows stacking velocities for all boundaries.

Conclusions

We considered two types of velocities: near-zero-offset NMO (V_{NMO}) and stacking velocity for a long NMO interval (V_{STACK}). Stacking velocities depend not only on velocity model but also on spreadlength and NMO geometry. V_{NMO} depends on the smoothing interval for the second-order derivative calculation. We can adjust the smoothing interval in such a way that the velocities, connected with the second moveout derivatives (V_{NMO}), would be close to the spreadlength stacking velocities V_{STACK}. To calculate near-zero-offset NMO velocities, we use the second-order moveout derivatives.

We also analytically investigated the influence of the overburden velocity anomalies on the near-zero-offset NMO velocities. It was shown that, for a deep reflector, stacking velocity repeats the behavior of the second-order derivative of the shallow velocity $v_1(x)$.

Acknowledgements

The authors are grateful to Vladimir Chernjak for useful discussions on the subject matter. They also would like to thank Michael Burianyk for his help.

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