Intrinsic anisotropy of shales

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ABSTRACT

Knowledge of the anisotropic properties of shales is becoming increasingly important as this information in needed in developing velocity models for seismic imaging in sedimentary basins. Incorporating elastic anisotropy into migration algorithms allows proper positioning of reflectors.

In this contribution modelling of the intrinsic elasticity of shales was based on information about the texture of shales with knowledge of elastic properties of the constituent minerals. The textural properties of shales were quantitatively described by an orientation distribution function (ODF). The method of volumetrically weighted averaging by the ODF was used to calculate the elastic constants of textured aggregate. Aggregate properties were determined on the basis of the Voigt, Reuss and Hill assumptions. Taking into account significant anisotropy of single crystal and, as result of it, wide separation of the Voigt and the Reuss (upper and lower) bounds the method of geometric mean was employed to further refine intrinsic elasticity.

Resulted from the geometric mean averaging elastic constants were used to calculate phase velocities of intrinsically anisotropic shale. Calculated velocities are in good agreement with reported in literature *P* and *S* velocities of shales obtained from experimental ultrasonic measurements.

Introduction

Since the original paper of Kaarsberg (1959) shales were extensively investigated both experimentally and theoretically (Jones and Wang, 1981; Vernik and Nur, 1992; Hornby et al., 1994; Sayers, 1994; Johnston and Christensen, 1995; Schoenberg et al., 1996; Hornby, 1998; Sayers, 1999). As a result of these studies anisotropy is a widely recognized feature of the seismic wave propagation in the majority of shales.

Despite substantial advances in the understanding of shale elasticity based on numerous laboratory ultrasonic measurements and significant progress in the modelling of shale elasticity, ambiguity related to the in situ causes of seismic anisotropy still exists. The origin of seismic anisotropy in shales is non-unique and may be attributed to several factors, including preferred orientation (texture) of clay platelets (Kaarsberg, 1959; Tosaya, 1982; Sayers, 1994; Johnston and Christensen, 1995), alternation of fluid filled collinear cracks with clay platelets (Vernik and Nur, 1992; Hornby et al., 1994), microcracks (Vernik, 1993; Vernik and Liu, 1997), fine layering (Schoenberg et al., 1996), fluid filled porosity (Hornby, 1998) and stress-induced anisotropy (Sayers, 1999).

In this contribution the authors address the issue of the origin of seismic anisotropy in shales by modelling intrinsic elastic properties of shales using concept of the geometric mean and the orientation distribution function (ODF) $f(g)$ (Viglin, 1960) averaging technique. 'Intrinsic' implies elastic anisotropy due solely to the elastic properties of the constituent minerals and their mutual orientation within the aggregate, i.e. the elasticity of the shale solid matrix. Method of the averaging by the ODF has been initially applied to shales by (Sayers, 1993, 1994) to identify texture parameters responsible for anisotropic velocity variations in shales. Sayers (1993, 1994) suggested that only two of the expansion coefficients affect seismic anisotropy of shales with vertical transversely isotropic (VTI) symmetry. By adjusting the values of these two parameters, he investigated anelliptic shale anisotropy. Results of his modelling show that shales might develop strong anelliptic anisotropy due to intrinsic textural properties.

To carry out ODF averaging information on the volumetric fraction of corresponding mineral phases, textural orientation distribution and elasticity of shale constituent clay minerals is needed. Illite is usually the most abundant clay mineral in shales (Kaarsberg, 1959) and contributes significantly to the overall shales elasticity. In this development assumption is made that generally multiphase shales matrix is composed solely by the illite and substituted in calculations by the muscovite as an elastic equivalent of illite. Unfortunately, because of its small size crystal acceptable measurement of the full elastic properties of the clay minerals has not yet been done. Due to extremely high anisotropy of the single muscovite crystal results of the modelling under assumption of only muscovite composition set the upper limit on the possible intrinsic anisotropy of shales.

Averaging elastic properties of polycrystalline aggregate

Following Bunge (1982) elastic constants of the textured polycrystalline aggregate $\widetilde{C}_{ijkl}$ can be approximated by the mean value $\overline{C}_{ijkl}$ that may be expressed through the elastic constants of the constituent single crystal C_{ijkl} and the orientation distribution function $f(g)$ of the aggregate. Integration of the single crystal elastic constants weighted by the ODF over the all possible values of the orientation domain g yields elastic constants in the Voigt approximation:

$$\overline{C}^{V}{}_{ijkl} = \int C_{ijkl}\, f(g)\, dg \qquad (1)$$

where $g=\{\varphi_1, \Phi, \varphi_2\}$ is the orientation domain that consists of Euler angles (c.f. Morse and Feshbach, 1953). If the orientation space g is discretized *equation (1)* can me rewritten in the form:

$$\overline{C}^{V}{}_{ijkl} = \sum_{n=1}^{N} C_{ijkl}\, f(g_n)\Delta g_n \quad \text{with} \quad \sum_{n=1}^{N} f(g_n)\Delta g_n \equiv 1 \qquad (2)$$

Equation (2) explicitly defines elastic constants of polycrystalline aggregate in the Voigt approximation as arithmetic mean of the single crystal elastic constants weighted by the ODF. The Voigt-Reuss (VR) bounds generally accepted as limits of the possible elastic constants of the polycrystalline aggregate (Hill, 1952). Application of the geometric mean concept allows obtaining unique solution that is invariant to the averaging domain. Following Matthies and Humbert (1993) elastic constants of the aggregate averaged by the geometric mean method may be expressed:

$$\langle C_{ijkl} \rangle = \prod_{n=1}^{N} \left[C_{ijkl} \right]^{f(g_n)\Delta g_n} = \exp\left[\sum_{n=1}^{N} \ln C_{ijkl}\, f(g_n)\Delta g_n \right] \qquad (3)$$

where $\langle C_{ijkl} \rangle$ are geometric mean averaged elastic constants of the polycrystalline aggregate. $\langle C_{ijkl} \rangle$ are independent of the averaging domain and, therefore, represent unique solution of the ODF averaging procedure.

In the averaging procedure ODF quantitatively defines the information about textural properties of the rocks and can be obtained from the experimental pole figures of the distributions of crystallographic axis of constituent minerals (Bunge, 1982). To modell intrinsic anisotropy of shales clay platelet orientation distribution computed from a digitized SEM image of shale sample (Hornby et al., 1994) was used. Orientation distribution of clay platelet was reproduced from the Hornby et al. (1994) paper and approximated by the normal distribution function (*Fig.1*). The normal distribution approximation was than used to compose ODF under the assumption of the transversely isotropic (TI) intrinsic shale symmetry (no preferable lateral variations of the clay platelet normals). ODF $f(g)$ in equations (1-3) may be expanded into the series of symmetrical generalized spherical harmonics (Bunge, 1982):

$$f(g) = \sum_{l=0}^{4} \sum_{\mu=0(2)}^{l} \sum_{\nu=0(2)}^{l} C_l^{\mu\nu}\, \ddot{T}_l^{\mu\nu}(g) \qquad (4)$$

where $\ddot{T}_l^{\mu\nu}(g)$ are symmetrical generalized spherical harmonics (SGSH) constructed to fulfil both shales TI symmetry and constituent single crystal hexagonal symmetry conditions and $C_l^{\mu\nu}$ are coefficients of the SGSH that carry an information about the shales texture. Note that in *equation (4)* equality is not strictly fulfilled due to truncation of the infinite expansion series.

Averaging of the elastic constants by the ODF for transversely isotropic shales has been discussed in details by Sayers (1994). It was shown that for the transversely isotropic medium composed of hexagonal crystals only five of the expansion coefficients C_0^{11}, C_2^{21}, C_4^{11}, C_4^{21}, C_4^{31} are independent. Furthermore, if the infinite symmetry axis of the TI medium is aligned with Z axis of the right handed XYZ sample coordinate system coefficients C_2^{21}, C_4^{21} and C_4^{31} vanishes during the averaging procedure and only three non-zero coefficients C_0^{11}, C_0^{11} and C_0^{11} contribute to the elasticity of the aggregate.

Results of the implementation of geometric mean concept in the ODF averaging procedure is shown in *Fig. (2).* Elastic constants calculated by the geometric mean using single crystal stiffnesses and compliances coincide and provide unique averaging solution within the VR bounds. Elastic constants calculated under the Voigt, Reuss and Hill assumptions also shown for comparison. Despite the fact that intrinsic elastic anisotropy is reduced with respect to the single crystal anisotropy (*Fig. 2*) the aggregate is still highly anisotropic ($A_p=36.7\%$) compare to the majority of sedimentary rocks (e.g. Thomsen, 1986).

Velocity calculation

Elastic constants of the polycrystalline aggregate obtained from the averaging procedure described above were used to calculate intrinsic phase velocities of shales by solving Christoffel equation (cf. Musgrave, 1970). Solution of the cubic Christoffel equation for any specific slowness direction yields three positive values of the squared phase velocity, which correspond to the *quasi-P*-wave and two *quasi-S*-waves in anisotropic medium. Prefix *'quasi'* usually applied due to complications with regards to the relationships between wave propagation and polarizations directions. In anisotropic medium pure *P* and *S* modes exist only in the so-called 'longitudinal directions' (Helbig, 1993), e.g. along the symmetry axes. Slowness surfaces resulted from the solution of Christoffel equation for all wavefront propagation directions within XZ symmetry plane are shown on *Fig. 3*. Slowness surfaces are normalized to be comparable with the normalized slowness surfaces of the Chattanooga Shale (Johnston and Christensen, 1995). Chattanooga Shale elastic constants were obtained from the laboratory ultrasonic measurements at confining pressure of 50 MPa and reflect mainly intrinsic properties of the shale composed by illite as predominant clay mineral (Johnston and Christensen, 1995). Experimentally observed *P*-wave anisotropy of Chattanooga Shale can be explained by the intrinsic anisotropy and attributed to the textural shale properties, which is in agreement with Johnston and Christensen's (1995) conclusions. Calculated and measured shear wave surfaces have very similar elastic behaviour with higher calculated intrinsic anisotropy of shear waves due to relatively low value of modelled $\langle C_{44} \rangle$ elastic constant.

Discussion and conclusions

Intrinsic elasticity of shales has been modelled by the ODF averaging of the elastic constants of shale constituent minerals. The VR bounds allow significant variation in the values of intrinsic elastic constants of shales (*Fig. 2*). Therefore, the geometric mean method has been applied with the ODF averaging procedure. Unique geometric mean solution fulfills the requirement of the stiffnesses to compliances invertibility and lies within the VR bounds relatively close to the Hill average (*Fig.2*).

Elastic constants obtained by geometric mean averaging were used to calculate phase velocities (*Fig. 3*). Normalized compressional and shear waves phase surfaces are in good agreement with the Chattanooga Shale normalized phase surfaces calculated from the elastic constants reported by Johnston and Christensen (1995). Elastic constants obtained from ultrasonic velocity measurements at confining pressure of 50MPa have been used in the phase velocity calculation because at this confining pressure effects of pores and microcracks are significantly reduced and velocity anisotropy reflects mainly intrinsic elastic properties of Chattanooga Shale with illite as predominant clay mineral in its composition.

Modelling of the intrinsic anisotropy of shales is based on several assumptions including simplified shales mineralogical composition, elasticity of the constituent minerals and orientations of shales platelets. Each of these assumptions may not hold in shales and, therefore, calculated and observed phase velocity surfaces are not expected to coincide completely. Furthermore, overall elasticity of shales depends not only on its intrinsic properties but also on the presence of oriented microcracks (e.g Vernik, 1993), the amount of the fluid-filled porosity (e.g. Hornby, 1998) and the in situ distribution of stresses (Sayers, 1999), i.e. factors that significantly influence seismic anisotropy and cannot be ignored. Models that incorporate these factors should be based on the assumption of initial intrinsic anisotropy of shales for the proper estimation of overall shale elasticity.

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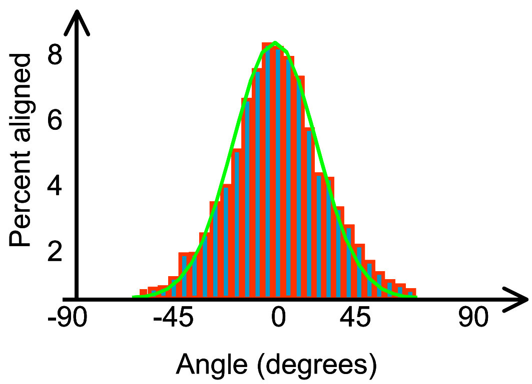

Fig. 1. Orientation distribution of clay platelets reproduced from the Hornby et al. (1994) and approximation by the normal distribution function.

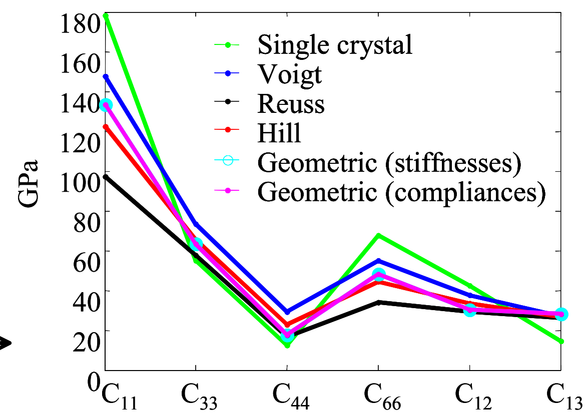

Fig. 2. Elastic constants of polycrystalline aggregate calculated with Voigt, Reuss, Hill and Geometric mean approximations. Single crystal elastic constants are shown for comparison.

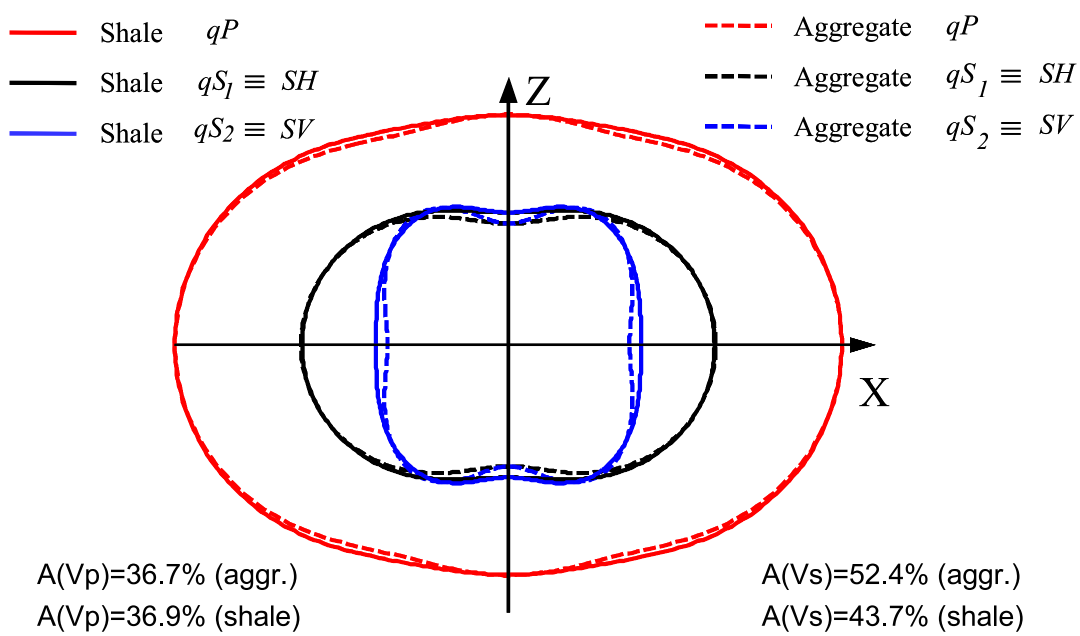

Fig. 3. Phase velocities of Chattanooga Shale from Johnston & Christensen (1995) and polycrystalline aggregate calculated from the geometric mean averaged elastic constants (Fig. 2).