# Mapping Rock Mechanical Properties with a Seismic Attribute-based Support Vector Machine (SVM) Technique

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**ABSTRACT**
Rock physical properties such as Young's modulus, Lame parameters and rock strength are essential for applications like borehole instabilities during drilling. These properties play also an important role at the time of adding interpretative value to seismic data during exploration and development stages.

In this presentation, we propose a method that propagates sparsely distributed rock physical properties derived from boreholes to a dense grid where 2D/3D seismic data exist.

The technique, Support Vector Machine (SVM), will be used to link borehole mechanical properties and seismic attributes near the borehole to propagate properties from the given boreholes to any proposed location within the 3D seismic data volume.

Synthetic and real data examples are used to illustrate the ability of the SVM to predict rock physical from seismic data.

## Introduction

Rock mechanical properties are an essential piece of information for applications like borehole instabilities analysis during drilling. It is estimated that borehole instability problems cost the oil industry worldwide about 1.0 billion dollars per year (Erling, et al., 1996).

Traditional methods of propagating mechanical properties from a given borehole to a proposed borehole are based on parameter shifting and spatial interpolation. The validity of these methods, in general, decreased when dealing with a sparse and distant collection of boreholes. The propagation of mechanical properties in these situations carries a significant amount of risk. Fortunately, seismic data possess high lateral resolution, and therefore, it can be used in conjunction with borehole data to propagate mechanical properties in an area of interests.

In fact, seismic multi-attribute interpretation has been widely used during oil/gas development and exploration (Hampson, et al., 2001; Fouad, et al., 2002). A variety of seismic attributes such as: instantaneous frequency, impedance, energy, etc., can be extracted for the stacked seismic data. In addition, if high quality pre-stack seismic data are available, Radon Transform can help us extract pre-stack attributes by transforming the seismic intercept-offset domain data to intercept-velocity space; in this new domain velocity clusters represent the energy of different seismic signals (primary and/or multiples).

In this paper, the Support Vector Machine (Tipping M.E., 2001) will be used to link post-stack seismic data and/or pre-stack seismic attributes to rock mechanical properties.

SVM has been gaining popularity in regression and classification due to its excellent performance at the time of dealing with sparse data and good empirical performance. Our implementation of the SVM technique requires borehole mechanical properties and seismic attributes near the borehole. During the training phase, the SVM will develop a functional mapping between an input vector (attributes) and a target output (mechanical properties at the borehole). The functional, is later on used to predict rock physical properties at any location within the seismic volume.

**Rock Mechanical Properties**

Most materials have an ability to resist and recover from deformations produced by forces. This ability is called elasticity. The simplest relationship between applied stresses and resulting strains is the linear relationship.

Considering a sample of length L and width D, the cross sectional area is $A = D^2$. When the force F is applied on its surfaces, the length of the sample is reduced to L' (L > L') and the width of the sample is increased to D' (D' > D). The applied stress is then $\sigma_x = F / A$, and the corresponding elongation is $\varepsilon_x = (L - L') / L$; the lateral elongation is $\varepsilon_y = \varepsilon_z = (D - D') / D$. If the sample behaves linearly, there is a linear relation between $\sigma_x$ *and* $\varepsilon_x$, where the relation can be written:

$$\varepsilon_x = \frac{1}{E}\sigma_x \qquad (1)$$

The ratio between lateral and vertical elongation is defined as:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \qquad (2)$$

*Equation 1* is known as Hooke's law, where the coefficient *E* is called Young's modulus. The ratio in *equation 2* is known as Poisson's ratio. It is a measure of lateral expansion relative to longitudinal contraction.

For isotropic materials, the general relations between stresses and strains are written as:

$$\begin{aligned}\sigma_x &= (\lambda + 2\mu)\varepsilon_x + \lambda\varepsilon_y + \lambda\varepsilon_z \\ \sigma_y &= \lambda\varepsilon_x + (\lambda + 2\mu)\varepsilon_y + \lambda\varepsilon_z \\ \sigma_z &= \lambda\varepsilon_x + \lambda\varepsilon_y + (\lambda + 2\mu)\varepsilon_z\end{aligned} \tag{3}$$

The coefficient $\lambda$ is known as the Lame parameter; $\mu$ is shear modulus.

Another important elastic modulus is the bulk modulus *K*. It is defined as the ratio of hydrostatic stress $\sigma_p$ relative to the volumetric strain $\varepsilon_v$. For hydrostatic stress state we have $\sigma_p = \sigma_x = \sigma_y = \sigma_z$ and $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$. The ratio can be defined:

$$K = \sigma_p / \varepsilon_v = \lambda + \frac{2}{3}\mu \tag{4}$$

If the stress is uniaxial ($\sigma_y = \sigma_z = 0$), the Young's modulus (*E*) and Poisson Ratio $(\nu)$ are defined as:

$$E = \frac{\sigma_x}{\varepsilon_x} = \mu\frac{3\lambda + 2\mu}{\lambda + \mu} \qquad \nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{\lambda}{2(\lambda + \mu)} \tag{5}$$

For most rocks, the Poisson's ratio is typically in the range 0.15 - 0.25. But, for weak, porous rocks, the Poisson's ratio will approach zero or even become negative.

**Rock Mechanical Properties from Well Logs**

The most important and direct method for the estimation of rock mechanical properties is acoustic logging. The compressional and shear velocity can be easily extracted from acoustic logging tools such as Schlumberger DSI. According to the acoustic wave propagation theory, the primary and secondary wave velocities can be expressed as:

$$v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad v_s = \sqrt{\frac{\mu}{\rho}} \tag{6}$$

Because the $v_p$, $v_s$ *and* density $\rho$ are available after acoustic logging, we can express the elastic coefficients discussed in above section in terms of the acoustic velocities:

$$\begin{aligned}\mu &= \rho v_s^2 & E &= \frac{\rho v_s^2(3v_p^2 - 4v_s^2)}{v_p^2 - v_s^2} \\ \lambda &= \rho v_p^2 - 2\rho v_s^2 & & \\ K &= \rho v_p^2 - \frac{4}{3}\rho v_s^2 & \nu &= \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)}\end{aligned} \tag{7}$$

**Rock Mechanical Properties from Seismic**

Seismic velocity estimation is one of the main objectives of seismic data processing. Velocity provides us valuable information about subsurface structures and the behavior of rocks.

Since the mid 90', AVO inversion methods have been gradually gained popularity, especially in the exploration and development of gas pools. AVO research has shown that both P and S reflectivity can be estimated from seismic data and, in addition, rock mechanical properties such as the Lame Parameters can be extracted from pre-stack seismic data to provide more interpretative value to the seismic probe (Goodway 2002; Gray, 2002).

In the pre-stack case, our attributes are pre-stack CMP gathers transformed to velocity space via the hyperbolic Radon Transform. We use a direct implementation of the adjoint Radon operator (Beylkin G 1987; Liu & Sacchi 2002) given by

$$\widetilde{m}(v,\tau) = \sum_{h} d(h, t = \sqrt{\tau^2 + (h/\upsilon)^2}) \tag{8}$$

Where $\mathrm{d(h,t)}$ indicates CMP gather in offset-time domain, $\widetilde{m}(v,\tau)$ the Radon panel in velocity-intercept time space, *t* is two-way travel time, $\tau$ represents the two-way zero offset time, $v$ is NMO velocity and *h* denotes the range of offset.

In the post-stack case, we directly use the time samples of the seismic traces in the vicinity of borehole as attributes.

**Support Vector Machine**

Given a set of Radon based attributes vectors $\{x_n, n = 1,..., N\}$ along with the corresponding rock mechanical property targets $\{t_n, n = 1,..., N\}$ from several boreholes. The SVM makes predictions based on a function of the form:

$$t(x) = \sum_{n=1}^{N} \omega_n K(\widetilde{x}, \widetilde{x}_n) + \omega_0 \tag{9}$$

where $\{\omega_n\}$ are the model weights and *K(.,.)* is a kernel function.
$\widetilde{x}$ is a set of samples of input vectors. *t(x)* is the vector of target values. Hence, training involves estimation of the appropriate weighting parameters $\{\omega_n\}$. In *Table 1* we list some choices of the kernel function.

Given the dataset of input-target pairs *(x,t)*, we follow the standard formulation and assume *p(t|x),* the conditional probability of the target vector given the input vectors, is well represented by a Gaussian distribution.

The likelihood of the dataset can then be written as:

$$p(t \mid w, \sigma^2) = (2\pi\sigma^2)^{-N/2} \exp\{-\frac{1}{2\sigma^2}\|t - \phi w\|^2\} \tag{10}$$

Where $t = \{t_1, t_2, ..., t_n\}$ , $\{\omega_n\}$ is the *NX(N+1)* *'design'* matrix with $\phi = \{\phi(x_1), \phi(x_2), ..., \phi(x_N)\}$ transition, wherein

$$\phi(x_n) = [1, k(x_1, x_n), k(x_2, x_n), ..., k(x_N, x_n)]' \tag{11}$$

is a kernel function. Once the priors are defined, the posterior over the weights is then obtained from Bayes' rule:

$$\begin{aligned} p(w \mid t, \alpha, \sigma^2) &= \frac{p(t \mid \omega, \sigma^2) p(\omega \mid \sigma)}{p(t \mid \alpha, \sigma^2)} \\ &= (2\pi)^{-(N+1)/2} |\Sigma|^{-1/2} \exp\{-\frac{1}{2}(w-\mu)^T \Sigma^{-1}(w-\mu)\} \end{aligned} \tag{12}$$

Where the posterior covariance and mean are $\Sigma$ and $\mu$ respectively with

$$\begin{aligned} \Sigma &= (\phi^T B \phi + A)^{-1} \qquad & A &= diag(\alpha_{0,} \alpha_1, ..., \alpha_N) \\ \mu &= \Sigma \phi^T B t \qquad & B &= \sigma^{-2} I_N \end{aligned} \tag{13}$$

By integrating out the weights, the maximization of $p(t \mid \alpha, \sigma^2)$ yields:

$$\begin{aligned} p(t \mid \alpha, \sigma^2) &= \int p(t \mid \omega, \sigma^2) p(\omega \mid \alpha) d\omega \\ &= (2\pi)^{=N/2} \mid \sigma^2 I + \phi A^{-1} \phi \mid^{-1/2} \exp\{-\frac{1}{2} t^T (\sigma^2 I + \phi A^{-1} \phi)^{-1} t\} \end{aligned} \tag{14}$$

To obtain $\alpha$ , we differentiate *equation 14* and after a few mathematical manipulations we arrive to the following expression:

$$\alpha_i^{new} = \gamma_i / \mu_i^2 \tag{15}$$

Where we have defined the quantities: $\gamma_i = 1 - \alpha_i \sum_{ii}$ and $\mu_i$ being the $i^{th}$ posterior mean weight, and $\sum_{ii}$ is the $i^{th}$ diagonal element of the posterior weight covariance computed with the current $\alpha$ and $\sigma^2$ values.

For the noise variance, the differentiation leads to the following estimator,

$$(\sigma^2)^{new} = \frac{\| t - \phi \mu \|^2}{(N - \sum_{ii} \gamma_i)} \tag{16}$$

In practice, during iterative re-estimation, many of $\alpha_i$ tend to infinity. Hence, the associated $p(\omega_i \mid t, \alpha, \sigma^2)$ becomes highly peaked at zero. The corresponding basis functions are thus pruned. This, in general, leads to a sparse representation of the weights.

**Rock Mechanical Properties Propagation**

As stated above, incorporation of seismic information to the property propagating process confers an important advantage because it enables the incorporation of geological constraints and lateral variations to the propagating solution. However, defining empirical relations between borehole property and seismic attributes may require a correct depth-to-time conversion, and more importantly, require a bridge to connect the seismic attributes and borehole properties.

In this paper we have presented a method that propagates borehole mechanical properties using seismic attributes extracted from post and/or pre-stack seismic data based on the SVM. The SVM-based property propagation can be summarized as:

1. Select a corridor of interest in the section and borehole derived properties at location within the seismic volume.
2. Extract the pre-stack seismic attributes from CMP gathers near selected boreholes via the Radon Transform (in the pres-stack case). Alternatively, use stacked traces as attributes.
3. Define rock mechanical properties and seismic attribute pairs to train the SVM.
4. Use the Bayesian algorithm outlined above (Mackay, 1992; Tipping, 2001) to train the SVM.
5. Propagate the desired properties to new spatial locations in the seismic volume.

**Applications**

We illustrate the performance of the proposed method with two examples. First, we analyze a synthetic example. Then, we examine a field data example.

We generate a 100-CMP gather synthetic dataset in a six-layer model (*Table 2*), with variable dipping reflectors, each having different velocity and density. Each synthetic CMP gather consists of 91 traces of 542 samples per trace.

We first assume that there exist 3 boreholes, Well 1,2, and 3 located at CMP positions 1001, 1049 and 1099, respectively. The six-layer model and three boreholes are shown at *Fig. 1. Fig. 2* portrays the three CMP gathers (left) in the vicinity of the three boreholes; we also display the Radon gathers (right) that were used as input attributes.

We test our algorithm by training the SVM with 1, 2 and 3 boreholes, respectively. In all these scenarios, we have attempted to reconstruct the mechanical parameter $\lambda$ on the entire volume. The results are portrayed in *Figs 3, 4 and 5*. Note the excellent agreement between the true model and the predicted model obtained with the SVM when the three boreholes are used in the training process (see *Fig. 5*).

The second example is a field data example. The SVM was trained using post-stack seismic attributes instead of pre-stack seismic attributes. We encountered difficulties at the time of extracting pre-stack attributes from this particular data set. We believe that the low SNR of this particular survey might have hampered our efforts. *Fig. 6* portrays the stack section corridor of interest. There are three boreholes, at CDP positions 148, 1523 and 1911. In *Fig. 7* we portray the synthetic seismic traces obtained from sonic derived reflectivity

During the training phase, post-stack seismic data will be the SVM input and log-derived velocity will be the SVM output. Well 2, for example, is selected to train the SVM. The log-derived velocity will be propagated from CDP 100 to CDP 2480 through the seismic corridor. Figure 8 shows a velocity section predicted from SVM. The three blank curves are log-derived velocity. It clearly demonstrates that the predicted velocities are in agreement with log-derived velocities at Well 3; the agreement is not as good at Well 1. However, if the three Wells are selected to train the SVM, the propagated velocity section (*Fig. 9*) seems reasonable, especially at the position of Well 1.

## Conclusions

A new method to integrate borehole and seismic attributes was proposed. The method is based on the application of an unsupervised learning method, Support Vector Machine, to guide borehole-derived properties through a seismic volume. The SVM can be either trained with pre-stack or post-stack seismic attributes.

## Acknowledgements

We would like to acknowledge the following companies and agencies: Geo-X Ltd., Pan Canadian Energy Ltd., Veritas Geo-services, NSERC, and AERI (Alberta Energy Research Institute).

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*Table 1: Some possible kernel functions*

| Type of Regression | Kernel Function |
|---|---|
| Gaussian RBF | $k(x_i, x_j) = \exp(-\frac{\| x_i - x_j \|^2}{2\sigma^2})$ |
| Polynomial of degree d | $k(x_i, x_j) = ((x_i^T x_j) + 1)^d$ |
| Spline | $k(x_i, x_j) = 1 + x_i^T x_j + x_i^T x_j \min(x_i, x_j) - \frac{(x_i + x_j)}{2}(\min(x_i, x_j))^2 + \frac{1}{3}(\min((x_i, x_j))^3$ |

*Table 2 Parameters for six-layer*

| Layer No. | Compressional Velocity (*m/s*) | Density $(kg / cm^3)$ | Poisson Ratio |
|---|---|---|---|
| Layer 1 | 2100 | 1.8 | 0.35 |
| Layer 2 | 2400 | 2.2 | 0.15 |
| Layer 3 | 3400 | 2.4 | 0.32 |
| Layer 4 | 2600 | 2.5 | 0.12 |
| Layer 5 | 4400 | 2.4 | 0.30 |
| Layer 6 | 5400 | 1.8 | 0.40 |

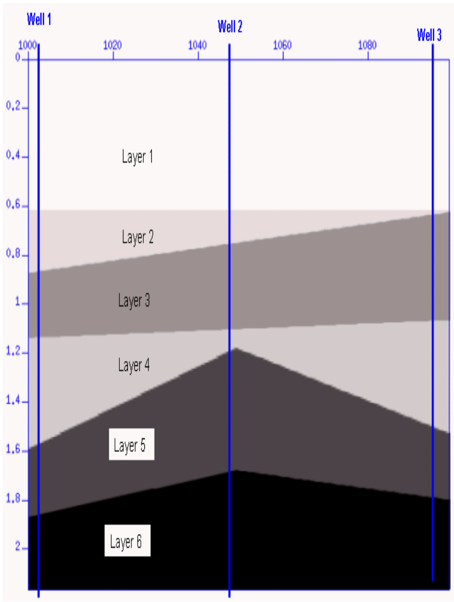


*Fig. 1: Model Space with three Wells*

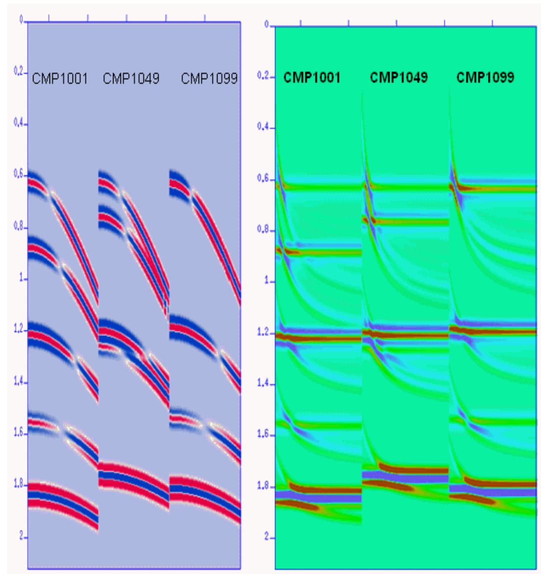


*Fig. 2: CMP gathers (left) and Radon attributes (right) in the vicinity of the three wells*

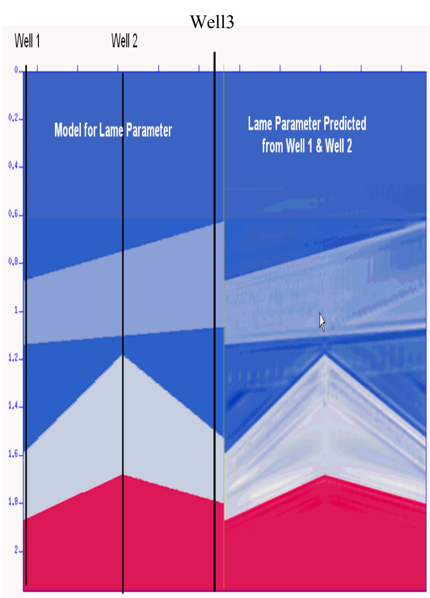


*Fig. 3: Lame Parameter Propagation from Well 1 and Well 2. Left is the true model, and right is the SVM predicted model.*

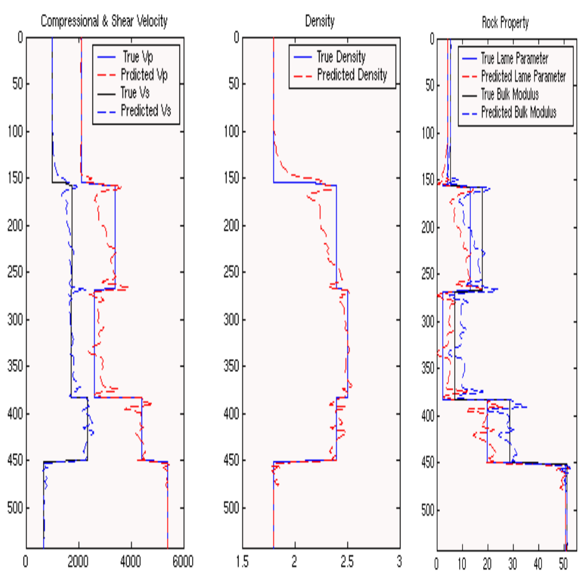


*Fig. 4: True properties (solid) and predicted properties (dash) of Well 3 propagated from Well 1 and Well 2.*

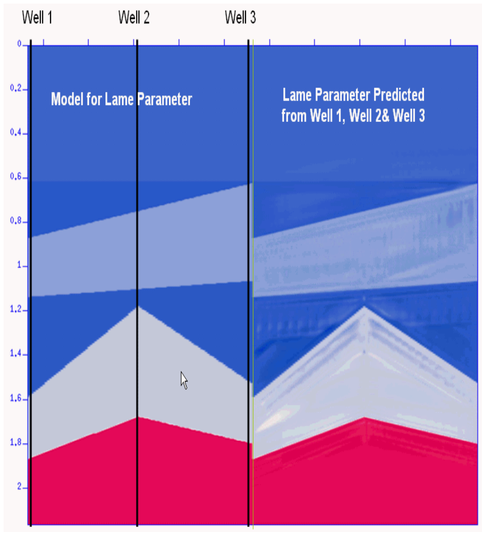


*Fig. 5: Lame Parameter Propagation from Well 1, Well 2 and Well 3. Left is the True Model, and right is the SVM predicted result*

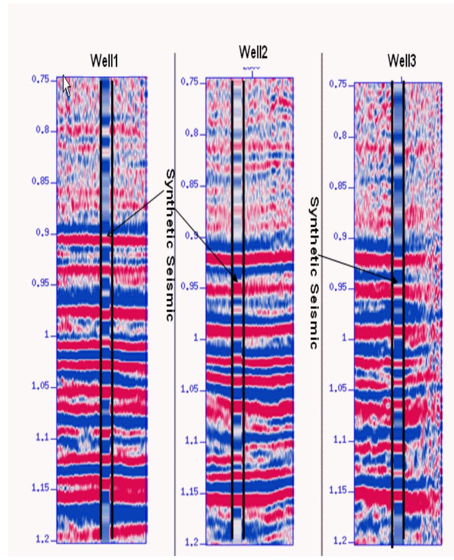


*Fig. 7: Synthetic seismic traces for the three wells plus traces in the vicinity of the wells.*

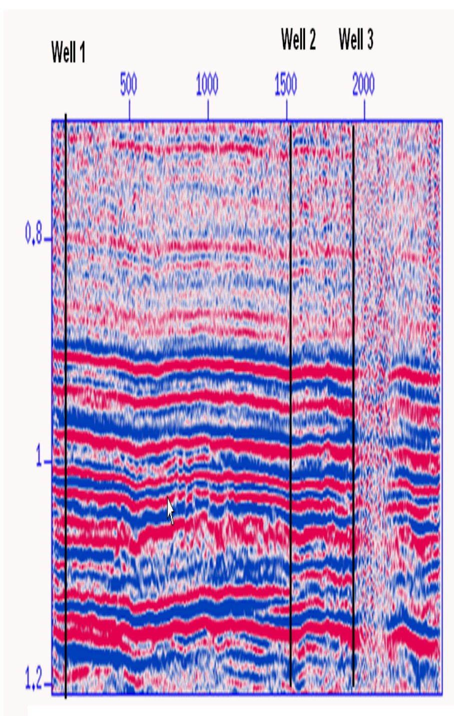


*Fig. 6: The seismic corridor of interest with three wells (Well 1, Well 2 and Well 3)*

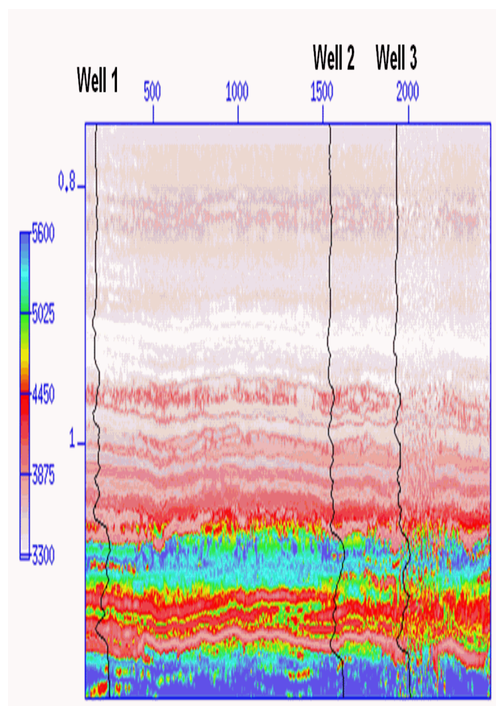


*Fig. 8: Predicted velocity section from Well 2. The three black curves are the true velocity profiles calculated from sonics.*

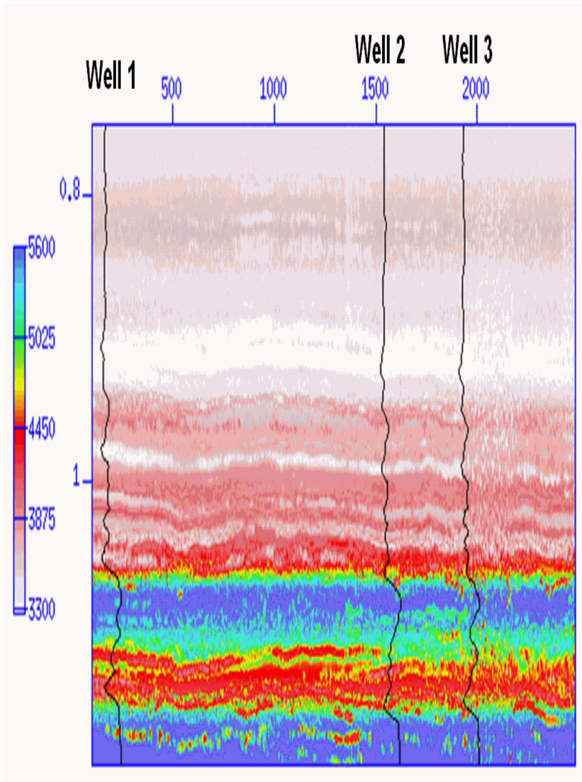


*Fig. 9: Predicted velocity section from Well 1, Well 2 and Well 3. The three black curves are true velocity calculated from sonic data.*