Petrophysical Reservoir Characterization and Uncertainty Analysis

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ABSTRACT

A methodology of petrophysical reservoir characterization is presented. The goal of this methodology is to provide a complete solution for reservoir properties determination, including estimation and uncertainty analysis. The strengths of this reservoir characterization methodology are:

i) uncertainty analysis and

ii) integration of multiple geophysical data-sets, rock physics analysis and prior information in a straightforward way provided by the Bayesian framework.

The inference problem reported on in this paper is formulated to solve the problem of porosity estimation. The sources of information are pre-stack seismic data, well log data and core samples. The waveform elastic pre-stack inversion is incorporated in this methodology to access the porous medium physical property information from pre-stack seismic data. Geostatistical modeling is incorporated to access the porous medium spatial variability information from well-log data.

The methodology is implemented considering a reservoir composed of block cells. One posterior pdf is computed for each reservoir cell. Two cell volumes contain the final result. One is constructed with the modes of the posterior pdfs related to each cell and represents the estimated porosity model, and the other is constructed with the confidence interval of the posteriors pdfs and represents a measure of the related uncertainties.

This paper is going to describe the general theory of the methodology, the practical implementation and results of tests with a 3-D pre-stack seismic data set (1995 3-D Blackfoot data).

INTRODUCTION

Despite the important decisions that are made after the reservoir quantification, the current reservoir characterization practices often fail to account for the uncertainties associated with each piece of information used in this process. Some uncertainty examples are related to: disturbances in the seismic data; data processing and inversion performed to obtain seismic attributes, the discrete earth model and the rock physics models relating attributes to petrophysical properties.

Basically, the goal is to compute a marginal posterior distribution for the average porosity in each cell of the discrete reservoir model. In this case, we have divided the data information into two classes of data:

i -carrying information propagated from the well locations, which are the experimental variogram and

ii -one carrying information from the surface seismic data, which are the seismic attributes.

The waveform seismic inversion is incorporated into this Bayesian formulation via an elastic Bayesian inference work from pre-stack seismic data. This step follows the work of Gouveia and Scales (1998). The result of this seismic elastic inference is a joint normal posterior pdf for the elastic parameters of a 1-D layered medium given an input seismic gather. A likelihood function for the seismic attributes that represents the beliefs about porosity after the knowledge of the pre-stack seismic data is constructed with the parameters of these posterior pdfs (i.e. the maximum a posteriori and the covariance matrix) together with rock physics models and petrophysic observations.

To constructing a class of likelihood that carries information propagated from the well we define the experimental variogram as a data-set that carry the information regarding the spatial variability of porosity.

Finally, the posterior pdf for porosity is the product of these two likelihoods and the prior pdf. This posterior pdf represents the beliefs about the cell porosity given the knowledge of the pre-stack seismic data, which carry the porous medium elastic information, and the knowledge of well-log data, which carry the porous medium spatial variability information.

This text is going to describe the theoretical development and some tests.

BAYESIAN FORMULATION

Bayes Theorem

Consider the reservoir model composed by a set of *N* block. The problem consists of making inferences about median porosity for each cell: ϕ_i, i=1,…N, using a set of data **d** and prior information I, which is any information obtained independently from the data. Following the Bayesian approach of inference the solution is given by the posterior distribution $p(\phi \mid \mathbf{d}, I)$. This function is the normalized product of the prior pdf and likelihood function;

$$p\,(\,\phi\,|\,\mathbf{d}\,) \propto l(\mathbf{d}\,|\,\phi)\;\; r(\phi\,|\,I), \tag{1}$$

where $r(\phi\,|\,I)$ is the prior pdf, $l(\mathbf{d}\,|\,\phi)$ is the joint pdf for the data, also known as the likelihood function.

In order to consider the posterior pdf as the solution of an inverse problem, the likelihood must be defined (i.e. the relation between the data **d** and the parameter ϕ exists and is known); and there are compatibilities between the prior understanding of the model and the final results, i.e. $l(\mathbf{d}\,|\,\phi)$>0 for some ϕ where $r(\phi\,|\,I)$>0. Now it is necessary to define the likelihood function and the prior distribution to access the posterior distribution.

Likelihood Function

This work follows standard steps to construct the likelihood function, which is summarized by: *i*- selecting the datasets which carry information about ϕ; *ii*- finding mathematical expressions relating each type of porosity and *Iii*- defining statistical models (pdfs) for data distributions, based on data uncertainty.

Following these three steps, let us define the data set. This methodology defines two types of independent data set: *i*- a data-set of density, p-wave and s-wave velocity seismic attributes, represented by **S** $\in R^{3N}$ associated with the N cells of the reservoir model and *ii*- a dataset carrying information about the spatial variability of reservoir porosity, which is represented by **v** $\in R^{L}$. **v** is a set of experimental porosity-porosity variogram values computed from well log data.

Considering **v** and **S** as statistically independent datasets, the likelihood function should be expressed as the product of two independent distributions:

$$l(\mathbf{d}\,|\,\phi) = l(\mathbf{v},\mathbf{S}\,|\,\phi) = l_1(\mathbf{v}\,|\,\phi)\,l_2(\mathbf{S}\,|\,\phi)\,; \tag{2}$$

Data v distribution $l_1(\mathbf{v}\,|\,\phi)$: Assuming additive errors in the data **v**, it should be written as

$$\mathbf{v} = \mathbf{f}_1(\phi) + \mathbf{e}_1, \tag{3}$$

where $\mathbf{e}_i$ is a random variable representing a set of additive and independent errors. The error $\mathbf{e}_i$ incorporates the uncertainties, which are associated with the porosity estimates in the well, the discrete earth model and the mathematical functions f_1 that relates these data with porosity. The modeling operator f_1 is the variogram function from geostatistics.

Next step is to establish the criteria to select the probability density models for $l_1(\mathbf{v}\,|\,\phi)$ data pdf. We chose to use the principle of maximum entropy to assign probabilities and assume that the first and second order moment information is sufficient to describe the errors. According to these choices, the $l_1(\mathbf{v}\,|\,\phi)$ data pdf is normal and should be expressed as

$$l_1(\mathbf{v}\,|\,\phi,\sigma_1^2) = \frac{1}{2\pi\sigma_1^{\mathrm{L}}} exp\left\{-\frac{1}{2\sigma_1^2}\left[\mathbf{v}-\mathrm{f}_1(\phi)\right]^{\mathrm{T}}\left[\mathbf{v}-\mathrm{f}_1(\phi)\right]\right\}, \qquad (4)$$

and σ_1^2 considered the unknown data error variance. This choice criterion to construct a likelihood pdf will be used as a standard criterion elsewhere in this work.

Seismic attributes data distribution $l_2(\mathbf{s}\,|\,\phi)$: Let $\mathbf{s}_{\text{rho}}$, $\mathbf{s}_{\text{p}}$ and $\mathbf{s}_{\text{s}}$ be a set of vectors representing density, p-wave and s-wave velocity seismic attributes respectively. These data vectors should be represented as the sum of a function of porosity, which are deterministic variables, and an error component, which is a probabilistic variable

$$\begin{aligned} \mathbf{s}_{\text{rho}} &= \mathbf{f}_2(\phi) + \mathbf{e}_2 \\ \mathbf{s}_{\text{p}} &= \mathbf{f}_3(\phi) + \mathbf{e}_3, \\ \mathbf{s}_{\text{s}} &= \mathbf{f}_4(\phi) + \mathbf{e}_4, \end{aligned} \qquad (6)$$

The error $\mathbf{e}_i$, i=2,3,4 incorporate the uncertainties, which are associated with the process of data acquisition, data processing, the elastic inversion, the discrete earth model, and associated with the mathematical functions f_i , i=1,2,3 that relate these data with porosity.

The next step is to define the relationships between data vectors and porosity, represented by the functions f_1,f_2 and f_3. For density, we use

$$\mathrm{f}_1 = \rho_{\mathrm{m}} + \phi(\rho_{\mathrm{f}} - \rho_{\mathrm{m}}); \qquad (7)$$

where ρ_m is the matrix density and ρ_f is the pore fluid density and are considered unknown parameters. For f_2 and f_3 the choice is the rock physics models studied by Han et al (1986).

$$\mathrm{f}_2(\phi) = \mathrm{a}_2 + \mathrm{b}_2\phi + \mathrm{c}_2\gamma; \qquad (8)$$

$$\mathrm{f}_3(\phi) = \mathrm{a}_3 + \mathrm{b}_3\phi + \mathrm{c}_3\gamma; \qquad (9)$$

where γ is the unknown clay content and a_i, b_i and c_i for i=1,2 are the unknown regression coefficients.

The set of equations (6), after the substituting equations (7), (8) and (9), should be treated as multi-regression model with auto-correlated errors (Zelner, 1996). In rock physics and elsewhere we encounter sets of regression equations and it is often the case that the disturbances are correlated. It is important that non-independence of disturbances terms be taken into account in making inferences. If this is not done, inferences may be greatly affected.

The l_2 pdf is defined as a normal distribution and the regression coefficients of the rock physics models and the matrix and fluid density are unknown. We made the choice to eliminate these unknown parameters of the Bayesian formulation by **a marginalization process** and to incorporate the information from laboratory petrophysical experiments following the **Bayesian technique to predict a pdf for a new observation given an old observation**.

Following this Bayesian technique, the predictive pdf for a vector of future observation **s**, which is assumed to be generated by the multiple regression process specified by the set of equations (6), is derived after the knowledge of an old set of observations $\mathbf{S}^*$, which is assumed to have the same statistical properties of **s**.

For now, consider an available set of seismic attributes related to the reservoir cells and the associated covariance matrix $\mathbf{C}_s$. Consider the existence of high resolution estimates of porosity and clay volume $\mathbf{X}^*=[\phi^* \gamma^*]$ in some position where core samples are available. Let's consider $\mathbf{S}^* = [\mathbf{s}_{rho}{}^* \mathbf{s}_p{}^* \mathbf{s}_s{}^*]$ the seismic attributes estimated for the cells at these core sampled position. A multiple regression process with the set of equations (6) can generate the vector $\mathbf{S}^*$, where the regression coefficients and the matrix and fluid densities are unknown variables but the petrophysical properties porosity ϕ and clay content γ, which represent the control variables are known. The pdf for these unknown parameters (regression coefficients and matrix and fluid density), given the seismic attributes $\mathbf{S}^*$ and the associated petrophyscial estimates $\mathbf{X}^*$ should be represented by $l_2{}^+(a_i, b_i, c_i, \rho_m, \rho_f | \mathbf{C}_s, \mathbf{S}^*, \mathbf{X}^*)$, for i=1,2.

Now, let's consider a cell at some position of the reservoir space with the unknown porosity and clay volume $\mathbf{x} = [\phi\ \gamma]$. The joint pdf for the sesmic attributes $\mathbf{s} = [s_{rho}\ s_p\ s_s]$ and the unknown a_i, b_i, c_i ρ_m, and ρ_f (for i=1,2) associated with this cell given the knowledge of seismic attributes $\mathbf{S}^*$ and the associates core samples petrophysical estimates $\mathbf{X}^*$ is represented by $l_2(\mathbf{s}, a_i, b_i, c_i, \rho_m, \rho_f \mid \mathbf{C}_s, \mathbf{S}^*, \mathbf{X}^*, \mathbf{x})$. This pdf should be decomposed as the product of two independent pdfs:

$$\begin{aligned} & l_2(\mathbf{s}, a_i, b_i, c_i, \rho_m, \rho_f \mid \mathbf{C}_S, \mathbf{S}^*, \mathbf{X}^*, \mathbf{x}) = \\ & l_2{}^*(\mathbf{s}, \mathbf{C}_S, | a_i, b_i, c_i, \rho_m, \rho_f, \mathbf{x}) l_2{}^+(a_i, b_i, c_i, \rho_m, \rho_f \mid \mathbf{C}_S, \mathbf{S}^*, \mathbf{X}^*) \end{aligned} \quad i=1, 2. \quad (10)$$

One way of deriving the predictive pdf is to write down the joint pdf $l_2(\mathbf{s}, a_i, b_i, c_i, \rho_m, \rho_f | \mathbf{S}^*, \mathbf{X}, \mathbf{x})$, i=1,2 and marginalize (i.e. integrate) with respect to a_i, b_i, c_i, ρ_m and ρ_f (for i=1,2) to obtain the marginal pdf for **s**, which is the predictive pdf.

Prior distribution

This work considers the definition of a non-data base prior distribution (NDBP) described by Jeffreys (1936) to access the prior distribution. A NDBP is derived from theoretical knowledge of the physical medium and from the investigator's background experiences. Following the Bayesians' most conservative practice to define the prior pdf, consider that with all previous available information, the only veritable statement is that porosity should fall between 0 and 1 interval. The use of a boxcar function is consistent with expressing this prior information.

Posterior distribution

With the Bayes Theorem applied, the posterior distribution should be

$$p(\phi, \gamma, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{v}, \mathbf{s}, \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, I) \propto \\ l_1(\mathbf{v} \mid \phi)\, l_2(\mathbf{s}, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, \mathbf{x})\, r(\phi \mid I) . \quad i = 1, 2 \quad (11)$$

The regression coefficients and the matrix and fluid density are not the target of investigation. Petrophysical property inference and associated uncertainty is all that matters in this problem. These uninteresting parameters are eliminated by integration of the joint posterior. This procedure, which is known in statistics as marginalization of the joint distribution, is applied and the marginal posterior pdf is obtained:

$$p(\phi, \gamma \mid \mathbf{v}, \mathbf{s}, \mathbf{C}_\mathrm{d}, \mathbf{S}^*, \mathbf{X}^*, I) \propto \\ \int p(\phi, \gamma, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{v}, \mathbf{s}, \mathbf{C}_\mathrm{d}, \mathbf{S}^*, \mathbf{X}^*, I)\, \mathrm{da}_i\, \mathrm{db}_i\, \mathrm{dc}_i\, \mathbf{d}\rho_\mathrm{m}\, \mathbf{d}\rho_\mathrm{f} \; ; i=1, 2. \quad (12)$$

All inference questions can be addressed to the posterior. For example, one can use the mean, median or mode as estimates for the interval porosity and the standard deviation or confidence intervals as measure of uncertainty.

REAL DATA EXAMPLE

The Blackfoot area was chosen for tests. The Blackfoot area is located 15 km southeast of Strathmore, Alberta, Canada. The target rocks are incised valley-fill sediments, which consists of the lower Cretaceous sandstone of Glauconitic Formation at the Blackfoot Field. In the Blackfoot area the Glauconitic sands thickness varies from 0 to over 35 m, at the depth of approximately 1,580 meters (between 1.0 s and 1.1). It is subdivided into three phases of valley incision. The lower and upper members are made of quartz sandstone, 0.18 porosity average, while the middle member consists of low porosity tight lithic sandstone.

The seismic elastic inference: a full waveform inversion: The first step in this methodology is a seismic elastic inference, which provides an elastic model of the medium **S** and the associated covariance matrix $\mathbf{C}_\mathrm{s}$. This work follows the methodology presented by Gouveia and Scales (1998), which developed a Bayesian formulation for the pre-stack seismic inverse problem to estimate elastic velocities and density or elastic impedances. In their work, all uncertainties are described by normal distributions, but careful consideration is taken to construct the covariance matrices.

This elastic inversion methodology considers a 1D reservoir with n-layers. The layers' thicknesses do not vary during the inversion process. Elastic velocities (P and S-wave velocities) and densities from layers of a target interval are inverted from a seismic gather.

According to the Bayes' Theorem the general formulation of this seismic inverse problem, can be written as

$$p(\mathbf{S} \mid \mathbf{d}, I) \propto l(\mathbf{d} \mid \mathbf{S})\, r(\mathbf{S} \mid I)\,, \qquad (13)$$

where $p(\mathbf{S} \mid \mathbf{d}, I)$ is the resulting posterior pdf for the seismic attributes and $l(\mathbf{d} \mid \mathbf{S})$ and $r(\mathbf{S} \mid I)$ are, respectively, the likelihood and the prior pdf. **S** represents the elastic attributes and **d** the pre-stack seismic data.

In defining the probability models, one problem arises due to the non-linearity of the forward seismic problem **d**=g(**s**), where the g is the seismic modeling operator defined by the elastic reflectivity method (Muller, 1985). Even when the prior pdf and likelihood are Gaussian, the posterior pdf $p(\mathbf{S} \mid \mathbf{d})$ cannot be obtained in closed form.

The standard solution is to first to obtain optimum estimates $\hat{\mathbf{S}}$, then a Gaussian approximation for $p(\mathbf{S} \mid \mathbf{d})$ is constructed on the basis of the linearization of the forward problem around point $\hat{\mathbf{S}}$. When $l(\mathbf{d} \mid \mathbf{S})$ and $r(\mathbf{S} \mid I)$ are both Gaussian, maximizing probability density is equivalent to minimizing the exponential argument of the Gaussian, leading to standard non-linear least square problem. The process of optimization by conjugate gradient used in Gouveia and Scales (1998), and adapted by the purpose of this work, can be summarized by the following expression:

$$\mathbf{S}_{n+1} = \mathbf{S}_{n} + \eta\delta_{n}\,, \qquad (14)$$

where δ_n is the direction of the *n*th iteration step and η is the step length. The gradient of the objective function is given by

$$\nabla\mathbf{\Theta} = \mathbf{G(s)C_d^{-1}(d - g(S))}\,, \qquad (15)$$

where **G**(**S**) is the matrix of Frechét derivatives of the forward term g(**s**) and $\mathbf{C}_d$ is the data variance matrix. After completing the optimization process, the resulting Gaussian approximation for the seismic attributes around $\hat{S}$ can be expressed by

$$p(\mathbf{S} \mid \mathbf{d}) \propto \mathbf{exp}\left\{-\frac{\mathbf{1}}{\mathbf{2}}\left[\mathbf{S}-\hat{\mathbf{S}}\right]^{\mathrm{T}} \mathbf{C}_{s}^{-1}\left[\mathbf{S}-\hat{\mathbf{S}}\right]\right\}, \qquad (16)$$

where the covariance matrix is $\mathbf{C}_s=[\mathbf{G}^T\mathbf{C}_d^{-1}\mathbf{G}]^{-1}$ which the Frechet derivatives evaluated at $\hat{S}$.

The vertical component of the 1995 Blackfoot 3D-3C data-set was selected for this example. A previous processing has followed with a pre-stack Kirchhoff Time Migration. The frequency range of the vertical component is from 20 hz to 115 hz and the bin interval is 30 x 30 m. The physical model consists of layers of 5 meters thickness inside of the target interval.

The inversion methodology was applied to each migrated image gather of a sub volume of this data-set. The Figure 1 shows a section of the elastic model obtained for the target interval. This section consists of result of the inversion performed on all image-gathers of cross-line 130. Figure 2 shows the position of this cross-line (red line) and well positions.

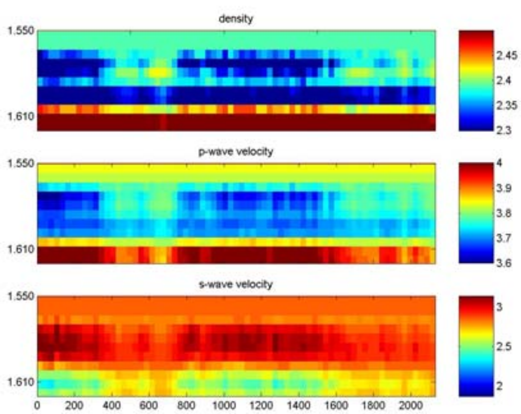

Figure 1: Seismic attributes density, p-wave velocity and s-wave velocity, respectively the images from top, meddle and bottom.

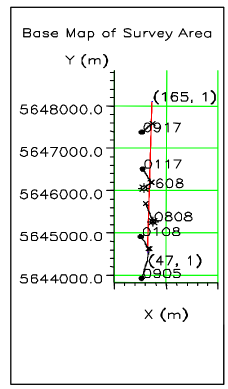

Figure 2: The base map of the area with the position of some well logs and the cross-line 130 (red line).

The petrophysical inference: The porosity inference is applied on the target interval along the cross-line represented by Figure 1.

To compute l_2 the clay content γ is considered an a priori known parameter. These parameters are estimated for the cells at inter well space using the clay content estimation from the gamma ray log and a variographic modeling. Then the posterior distribution for porosity, expressed by Equation (12), can be rewritten as $p(\phi, \gamma = \gamma^* \mid \mathbf{v}, \mathbf{s}, \mathbf{C}_\mathrm{d}, \mathbf{S}^*, \mathbf{X}^*)$.

The core samples data analysis from the wells 1608 and 0808 were used to construct the matrix $\mathbf{X}^*$ and the elastic model obtained from the image gather adjacent of these wells to construct the matrix $\mathbf{S}^*$.

Using a horizontal moving window, running across the reservoir section, a distribution l_2 (Equation 10) is calculated for a cell in the centre position of each window (the data vector **s** is the seismic attributes from cells falling inside the window). The Fresnel Zone is considered for defining the dimension of the window. In the same way, l_1 (Equation 4) is also computed for each cell position.

Finally, both distributions are combined by the application of Equation (11) to yield one posterior pdf for each cell of the reservoir.

Two volumes of the discretized reservoir represent the final results. One shows the mode of the posterior pdfs, representing the final estimated porosity model, and another shows the length of 0.95 posterior probability centered at the mode, representing the associated uncertainty model.

Three different tests are performed using different amounts of information:

i – using only the variogram data **v**:

$$p(\phi \mid \mathbf{v},, I) \propto l_1(\phi, \gamma = \gamma^* \mid \mathbf{v})\, r(\phi \mid I) \;;$$

ii – using only the seismic attribute data (**s**):

$$p(\phi, \gamma = \gamma^* \mid \mathbf{s}, \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, I) \propto l_2(\mathbf{s}, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, \mathbf{x})\, r(\phi \mid I)\;;$$

iii - using both data types (**v** and **s**), i.e.

$$p(\phi, \gamma, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{v}, \mathbf{s}, \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, I) \propto$$
$$l_1(\mathbf{v} \mid \phi)\, l_2(\mathbf{s}, \mathrm{a}_i, \mathrm{b}_i, \mathrm{c}_i, \rho_\mathrm{m}, \rho_\mathrm{f} \mid \mathbf{C}_\mathrm{S}, \mathbf{S}^*, \mathbf{X}^*, \mathbf{x})\, r(\phi \mid I)$$

The Figure 3 shows the model estimated by the mode of these three tests respectively Figure 3a, b and c. On Figure 3a we can see that the model estimated by the variogram data has relatively high porosity values and shows us a smooth horizontal variation. On Figure 3b shows the channels with porosity varying between 0.10 and 0.2. The model showed on Figure 2c is similar that on Figure 2b.

The Figure 4 shows the length of the centered interval having 0.95 probabilities, corresponding to each one of the estimates in Figure 3. This gives a measure of the spread of the posterior pdf' and the resolution for porosity of each cell of the reservoir. Figure 4a shows that the data v is more informative for cells on the right portion of the section (south). This portion has more wells than the left portion of the section (north, check the well positions on Figure 2). Figure 4b shows that the information about porosity contained in the seismic attributes is homogeneously distributed across the section. From Figure 4c we conclude that the main source of information comes from the seismic data.

CONCLUSION

The tests show reasonable porosity models obtained from the mode of the posterior pdfs. The associated uncertainty, represented by the length of 0.95 probability intervals, consistently varies depending on the amount of information available. The variogram fitting procedure allowed describing the information about the porosity from the wells at inter-well locations. For the inversion of seismic attributes alone the level of uncertainty varies homogeneously across the model. When combining variogram and attribute data, we observe that the main source of information is the seismic data.

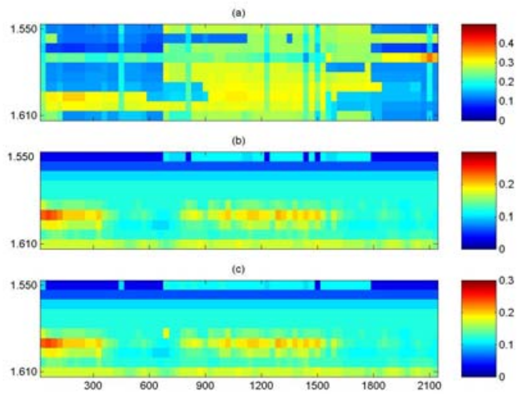

Figure 3: FIG 12: Images of the reservoir section representing the modes of the posterior distributions for porosity by the use of variogram data (a), seismic attributes (b) and both datasets (c).

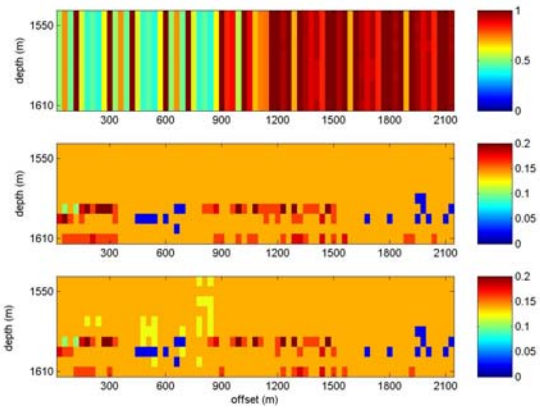

Figure 4: FIG 13: Images of the reservoir section representing the length of the 0.95 probability interval of the posterior distributions obtained from the inversion of variogran values (a), seismic attributes data (b) and both datasets (c).

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