Migration in Transversely Isotropic Media Using an Anisotropic Screen Propagator

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ABSTRACT
Seismic anisotropy produced by shales and fine layering is a basic feature in sedimentary basin and occurs as slow propagation velocity in vertical direction and fast propagation velocity in horizontal direction. This kind of anisotropy may have significant influence on seismic modeling and imaging. In this work, the screen propagator in isotropic medium is extended to transversely isotropic medium with a vertically symmetric axis (VTI medium). An acoustic approximation for the quasi-P wave in VTI medium is assumed. However, the medium may be strong anisotropic. An explicit dispersion relation is derived. Generalized-screen approximation for lateral heterogeneities is adopted. An anisotropic screen propagator is developed and the influence of overburden anisotropy on migration is studied.

Introduction

A universal feature of depositional sequences is that they tend to be layering and have alternative impedance contrasts because of cyclic sea-level changes. The thickness of individual sedimentary layer within depositional sequences is usually much less than seismic wavelength. This kind of thinly layered structure is seismic anisotropy and can be described by transversely isotropic effective media (Backus, 1962). Furthermore, most shales are intrinsic anisotropies (Wang, 2002). The combination of the two kinds of anisotropic effects may produce strong seismic anisotropies. A seismic wave propagating through the media has slow propagation velocity in the vertical direction and fast propagation velocity in the horizontal direction. This is a basic seismic feature of the overburden in sedimentary basin and may have significant influence on seismic modeling and imaging.

Speaking theoretically, all of migration methods are only suitable for weak heterogeneity. The use of effective properties can incorporate heterogeneity in anisotropy. The depositional sequences with strong impedance contrasts (or strong heterogeneity) may be seen as weak anisotropic heterogeneity. Therefore, the use of effective properties facilitates the analysis of overall response of heterogeneity and widens the using region of migration methods.

Anisotropic screen propagator

Fourier-based migration methods (Gazdag, 1978) apply a correction to the vertical wavenumber and do downward continuation with the goal of accommodating lateral velocity variations. There are various approximation methods to expand the vertical wavenumber for isotropic medium. For example, the phase screen propagator (Liu and Wu, 1994), the pseudoscreen propagator (Jin et. al. 1998; Huang et. al. 1999), and the wide-angle screen propagator (Wu, 1994; 1996; Xie and Wu, 1999). For transversely isotropic medium, the scalar generalized screen (Le Rousseau and de Hoop, 2001a) and phase shift (Meadows and Abriel, 1994; Ferguson and Margrave, 2002) migrations are also developed. This work first derived an explicit dispersion relation for the transversely isotropic media. Then a generalized screen expansion for VTI medium with acoustic approximation is given. From this general formulation, the screen propagators for isotropic medium can be obtained when anisotropy disappears. Finally, the influence of the overburden anisotropy on migration is analysed. For simplicity, we will only discuss 2D case, but extend the result to 3D is straightforward.

Velocity model

A periodic layered medium can be replaced by a transversely isotropic effective medium when seismic wavelength is "long" enough compared to the spatial layering period. The corresponding 5 elastic constants of the replacement medium are a weighted averaging of the elastic constants of constituents (Backus, G. E., 1962). The larger the differences of elastic properties of the constituents are, the stronger the anisotropies are. Thomsen (1986) introduces 5 independent elastic constants to characterize the strength of the anisotropy. These five parameters are the vertical P-wave velocity α_0, the vertical SV-wave velocity β_0, and three dimensionless parameters ε, δ, and γ. The parameter ε, which describes the difference between the horizontal and vertical P-wave velocities, is called as "P-wave anisotropy". The parameter γ, which describes the difference between SV-wave velocity and SH-wave velocity, is called as "shear wave splitting parameter", and the parameter δ describes the difference between the P-wave and SV-wave anisotropies. Although the original design of Thomsen parameters is for weak anisotropy, these 5 parameters are also convenient for arbitrary anisotropic medium.

In sedimentary basin, shales (or clays) and fine layering are the main causes of seismic anisotropy. A seismic wave propagating through the medium has slow propagation velocity in the vertical direction ($\theta = 0^0$) and fast propagation velocity in the horizontal direction ($\theta = 90^0$). The changes of velocities are small for small incident angles ($\theta < \text{about}\ 30^0$) and large for large incident angles ($\theta > \text{about}\ 30^0$). This indicates that anisotropy produced by clays and fine layering can approximately be seen as isotropic medium for small incident angles or near offsets. The velocities on the surface are very low and the seismic reflections are usually large angle reflections for exploration geophysics. Therefore, the anisotropy will have important influence on seismic modeling and imaging for large incident angles or far offsets.

Anisotropy screen propagators

VTI medium The Christoffel equation for VTI medium (e.g., Auld, 1973) expressed by the Thomsen parameters can be derived as

$$(1+2\varepsilon)k_x^4 + 2[1+\delta+\frac{\alpha_0^2}{\beta_0^2}(\varepsilon-\delta)]k_x^2k_z^2 - \frac{\omega^2}{\beta_0^2}(1+2\varepsilon+\frac{\beta_0^2}{\alpha_0^2})k_x^2$$
$$-\frac{\omega^2}{\beta_0^2}(1+\frac{\beta_0^2}{\alpha_0^2})k_z^2 + k_z^4 + \frac{\omega^4}{\alpha_0^2\beta_0^2} = 0 \qquad (1)$$

Substituting $k_z^2 = \omega^2/\alpha^2(\theta) - k_x^2$ into equation (1) and rearranging terms we have

$$k_z^2 = [\frac{\alpha_0^2}{\beta_0^2}(\delta-\varepsilon)-\delta-1]k_x^2 + \frac{1}{2}\frac{\omega^2}{\beta_0^2}(1+\frac{\beta_0^2}{\alpha_0^2}) \mp \frac{1}{2}\sqrt{\Omega} \qquad (2a)$$

where

$$\Omega = \frac{\omega^4}{\beta_0^4}f^2 + 4\frac{\omega^2}{\beta_0^2}[\varepsilon - \frac{\beta_0^2}{\alpha_0^2}\delta + \frac{\alpha_0^2}{\beta_0^2}(\delta-\varepsilon)]k_x^2$$
$$+8\frac{\alpha_0^2}{\beta_0^2}\{(\varepsilon-\delta)[\delta+1+\frac{1}{2}\frac{\alpha_0^2}{\beta_0^2}(\varepsilon-\delta)-\frac{\beta_0^2}{\alpha_0^2}]+\frac{1}{2}\frac{\beta_0^2}{\alpha_0^2}\delta^2\}k_x^4 \qquad (2b)$$

Equation (2a) is the exact qP-wave and qSV-wave vertical wavenumbers expressed by horizontal wavenumbers for arbitrary transversely isotropic media. The minus sign in front of the square root is corresponding to qP-wave and plus sign qSV-wave. *Equation (2a)* is similar to *equations (25)* and *(28)* in ver der Baan and Kendall (2002) but in a different form.

Acoustic approximation For acoustic case, $\beta_0 = 0$, *equation (1)* becomes

$$(1+2\varepsilon)k_x^2 - \frac{\alpha_0^2}{\omega^2}(\varepsilon-\delta)k_x^2k_z^2 + k_z^2 - \frac{\omega^2}{\alpha_0^2} = 0 \qquad (3)$$

Substituting $k_z^2 = \omega^2/\alpha^2(\theta) - k_x^2$ into *equation (3)* and rearranging terms we obtain the dispersion relation for acoustic approximation

$$k_z^2 = \frac{\frac{\omega^2}{\alpha_0^2} - (1+2\varepsilon)k_x^2}{1 - \frac{\alpha_0^2}{\omega^2}(\varepsilon-\delta)k_x^2} \qquad (4)$$

Equation (4) depends on three Thomsen parameters, α_0, ε, and δ (Tsvankin, 1996; Alkhalifah, 1998). Isotropy case can be obtained by taking Thomsen parameters ε and δ as zeros.

Generalized screen expansion For inhomogeneous anisotropic medium, Thomsen parameters α_0, ε, and δ are dependent on positions. For convenience we let α denotes vertical inhomogeneous velocity and α_0 vertical background velocity. Assuming background medium with three Thomsen parameters α_0, ε_0, and δ_0, which are constants in the slab but may change from one slab to another. The Thomsen parameter perturbations will affect the vertical wavenumber. Thin-slab propagator can handle lateral heterogeneity. In sedimentary basin, the vertical velocity increases with depth and so the influence of vertical velocity perturbation on vertical wavenumber is large. However, the influence of ε and δ perturbations will be small because their changes are small within a thin-slab. This work will only consider the influence of vertical velocity perturbation and ignore the influence from Thomsen parameter ε and δ perturbations. We introduce small vertical velocity perturbation and let

$$u_\alpha(x_i)=[\alpha(x_i)]^2-[\alpha_0(x_3)]^2 \tag{5}$$

Substituting Eq. (5) into (4) and doing Taylor expansion we get generalized anisotropy screen expansion for the vertical velocity perturbation.

$$\begin{aligned}
k_z &= k_{0z} + k_{u\alpha z} \\
&= k_{0z} + \frac{1}{2}k_{0z}b_1u_\alpha + \frac{1}{2}k_{0z}\left[\frac{b_2}{a_1} - \frac{1}{4}b_1^2\right]u_\alpha^2 \\
&+ \frac{1}{2}k_{0z}\left[-\frac{a_5}{a_4}b_3 - \frac{b_1b_2}{2a_1} + \frac{1}{8}b_1^3\right]u_\alpha^3 \\
&+ \frac{1}{2}k_{0z}\left[\frac{a_5^2}{a_4^2}b_3 + \frac{a_5}{2a_4}b_1b_3 - \frac{b_2^2}{4a_1^2} + \frac{3}{8}\frac{b_1^2b_2}{a_1} - \frac{5}{64}b_1^4\right]u_\alpha^4 \\
&+ o[u_\alpha]^4
\end{aligned} \tag{6}$$

where

$$b_1 = \frac{a_2}{a_1} - \frac{a_5}{a_4}$$

$$b_2 = a_3 - \left(\frac{a_2}{a_4} - k_{0z}^2\frac{a_5}{a_4}\right)a_5$$

$$b_3 = \frac{a_3}{a_1} - \frac{a_5}{a_4}b_1$$

$$a_1 = \frac{\omega^2}{\alpha_0^2}\left[(1+2\varepsilon_0)k_x^2 - \frac{\omega^2}{\alpha_0^2}\right]$$

$$a_2 = (1+2\varepsilon_0)k_x^2\omega^2 - 2\frac{\omega^4}{\alpha_0^2}$$

$$a_3 = -\omega^4$$

$$a_4 = (\varepsilon_0 - \delta_0)k_x^2 - \frac{\omega^2}{\alpha_0^2}$$

$$a_5 = -\omega^2$$

$$k_{0z}^2 = \frac{\dfrac{\omega^2}{\alpha_0^2} - (1+2\varepsilon_0)k_x^2}{1 - \dfrac{\alpha_0^2}{\omega^2}(\varepsilon_0 - \delta_0)k_x^2} = \frac{a_1}{a_4} \qquad (7)$$

For *equation (6)*, the zero order approximation is corresponding to phase-shift approximation, the first order approximation is corresponding to phase screen propagator, the high order approximations are corresponding to generalized screen. The corresponding isotropy may be obtained by taking $\varepsilon_0 = \delta_0 = 0$.

$$\begin{aligned} k_z &= k_{0z} + \frac{1}{2}\frac{\omega^2}{k_{0z}}u_\alpha - \frac{1}{8}\frac{\omega^4}{k_{0z}^3}u_\alpha^2 \\ &+ \frac{1}{16}\frac{\omega^6}{k_{0z}^5}u_\alpha^3 - \frac{5}{128}\frac{\omega^8}{k_{0z}^7}u_\alpha^4 + o[u_\alpha]^4 \end{aligned} \qquad (8)$$

Equation (8) is similar to *equation (18)* in Le Rousseau and de Hoop (2001b).

Generalized screen propagator For a small vertical step Δz and a smooth medium, the scalar generalized screen propagator becomes

$$\begin{aligned} U(x,z;x',z') &\approx \int \frac{\omega}{2\pi} dk_x \exp(ik_x x)\exp(ik_z \Delta z) \\ &= \int \frac{\omega}{2\pi} dk_x \exp(ik_x x)\exp[i(k_{0z} + k_{u\alpha z})\Delta z] \\ &= U^0(x,z;x',z') + U^1(x,z;x',z') \end{aligned} \qquad (9)$$

We expand the perturbation term in terms of the vertical propagation, thus we have

$$U^0(x,z;x',z') = \exp\left\{i\omega\left[\alpha^{-1}(x,z) - \alpha_0^{-1}(z)\right]\Delta z\right\}$$
$$\times \int \frac{\omega}{2\pi} dk_x \exp(ik_x x)\exp(ik_{0z}\Delta z) \tag{10}$$

$$U^1(x,z;x',z') = \exp\left\{i\omega\left[\alpha^{-1}(x,z) - \alpha_0^{-1}(z)\right]\Delta z\right\}$$
$$\times \int \frac{\omega}{2\pi} dk_x \exp(ik_x x)\exp(ik_{0z}\Delta z) \tag{11}$$
$$\times \Delta z i\left[k_{u\alpha z}(x,z;k_x) - k_{u\alpha z}(x,z;0)\right]$$

Equations (6), (8), and (9) construct the basic formula for screen propagators. The phase-shift propagator, the scalar phase-screen propagator, and the scalar generalized screen propagator can be obtained by remaining different Taylor orders.

Numerical tests

In order to study the influence of overburden anisotropy, we consider an isotropic fault model overlain by a VTI medium with ε =0.3 and δ =0.2 (*Fig. 1a*). *Fig. 2b* is the impulse response calculated by scalar anisotropic generalized-screen propagator. For comparison, the case for isotropic overburden is also shown in Figure 1c. It can be seen that overburden anisotropy causes flattened wavefront for large incident angles. This is because the propagation velocities are faster in oblique incidences than normal incidence. Usually, the thickness of overburden in sedimentary basin in thick and so overburden anisotropy may results in large position error in both horizontal and vertical directions.

Conclusions and discussions

This work extends the screen propagators in isotropic medium to transversely isotropic medium with a vertical symmetry axis (VTI medium). Scalar generalized screen expansion for vertical velocity in VTI medium is given to the fourth order. When two Thomsen anisotropic parameters become zeros the generalized screen expansion for VTI medium degrades to the case of isotropic medium. Numerical result shows that overburden anisotropy may results in large position error if the anisotropy is not considered

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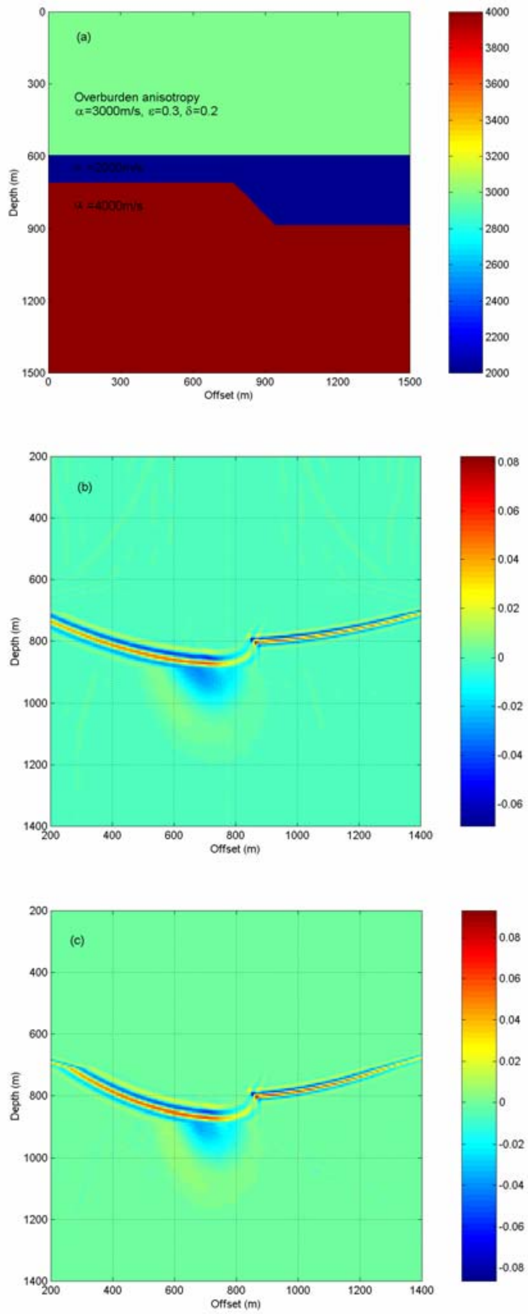

Fig. 1: (a) A fault model with anisotropic overburden; (b) Impulse response for model (a); (c) Impulse response when the overburden in model (a) is isotropic.