Anisotropic parameters of shales: effect of averaging techniques

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Abstract

Parameterization is widely used in industry to approximately describe the seismic wave behaviour in anisotropic rock formations. By modelling of the intrinsic elasticity of shales, i.e. properties of a non-porous and non-fractured solid shale matrix, limits of the seismic anisotropy and anisotropic parameters of the solid shale matrix have been estimated. In order to obtain anisotropic parameters elastic constants of solid shale matrix were first calculated as a function of texture on the basis of the theory of polycrystalline aggregates using the Voigt, the Reuss and the Hill volume averaging techniques. Wide separation of the Voigt and the Reuss (upper and lower) bounds of anisotropic shale aggregate enable implementation of the method of Geometric Mean Averaging (GMA) to further refine the elasticity. Constrained by the physical requirement of the invertibility of the elastic stiffnesses of textured shale aggregate into its elastic compliances, the GMA method yields a unique set of elastic constants independent of the Hill or the Reuss averaging assumption of uniform strain and stress, respectively. Strong dependence of absolute values of elastic constants (and, consequently, *P* and *S* velocities) on the averaging assumptions has, however, minor effect on the properties of elastic waves and their anisotropic parameters. In particular, the *P*-wave anellipticity is practically independent on the averaging procedures employed.

Introduction

The fourth rank elasticity tensor that also may be represented as a matrix of elastic constants, fully describes the elastic properties of anisotropic solids. The complex dependence of *P*- and *S*- velocities on these elastic constants motivated the simplier parameterization of anisotropy. In general, parameterizations are usefull for simplification of the anisotropic elastic behaviour via reduction of the number of parameters needed to descibe anisotropy. It should, however, be noted that parameterization cannot fully replace the more fundamental elastic constants but rather conveniently assist in the description of the elastic wave behaviour for specific cases when it is applicable. For a transversely isotropic medium *Thomsen* (1986) introduced three anisotropic parameters ε, δ and γ to describe the behaviour of *P*, *SV* and *SH* modes of elastic wave. The character of these parameters as a function of shale textural properties is presented in this contribution.

The strong textural features of elastically anisotropic shales have been previously reported in the literature (e.g. *Sayers*, 1994; *Johnston and Christensen*, 1995; *Hornby*, 1998, *Cholach and Schmitt*, 2003). SEM images of Colorado shale from the Cold Lake region of Alberta shown on the Fig.1 display strong clay minerals alignment within the bedding plane. The SEM images crystal shape orientation distribution can be correlated with the lattice preferred orientation (LPO) for phyllosilicate minerals. This allows the normals to the clay platelets to be considered as a crystallographic normal of basal plane in layer-lattice clay minerals in order to estimate the distributions (texture) of the clay platelets (*Hornby*, 1998). Knowing these textural properties and the elasticity of constituent minerals allows the anisotropic parameters of the aggregate to be calculated by volume averaging techniques.

Assumptions behind the volumetric averaging

In geophysics the Voigt (*Voigt*, 1928) or the Reuss (*Reuss*, 1929) volume averaging techniques of either uniform strain or stress within the isotropic aggregate, respectively, are employed. The Voigt-Reuss (VR) bounds were originally developed for an isotropic medium and generally accepted as the limits to the possible elastic constants to such a polycrystalline aggregate (*Hill*, 1952) with the true solution expected to be at an intermediate value $\overline{C}^{R}{}_{ijkl} \le \overline{C}_{ijkl} \le \overline{C}^{V}{}_{ijkl}$. The elastic constants in the Voigt approximation of uniform strain throughout the textured aggregate can be rewritten in the form:

$$\overline{C}^{V}{}_{ijkl} = \sum_{n=1}^{N} C_{ijkl}\, f(g_n)\,\Delta g_n \qquad (1)$$

where C_{ijkl} are elastic constants of constituent mineral, *f(g)* is orientation distribution function (ODF) that quantitatively describe aggregate texture (*Bunge, 1980*) and *g* is the orientation domain composed by the Euler angles. Eqn. 1 explicitly defines the elastic constants of polycrystalline aggregate according to the Voigt approximation: an arithmetic mean of the single crystal elastic

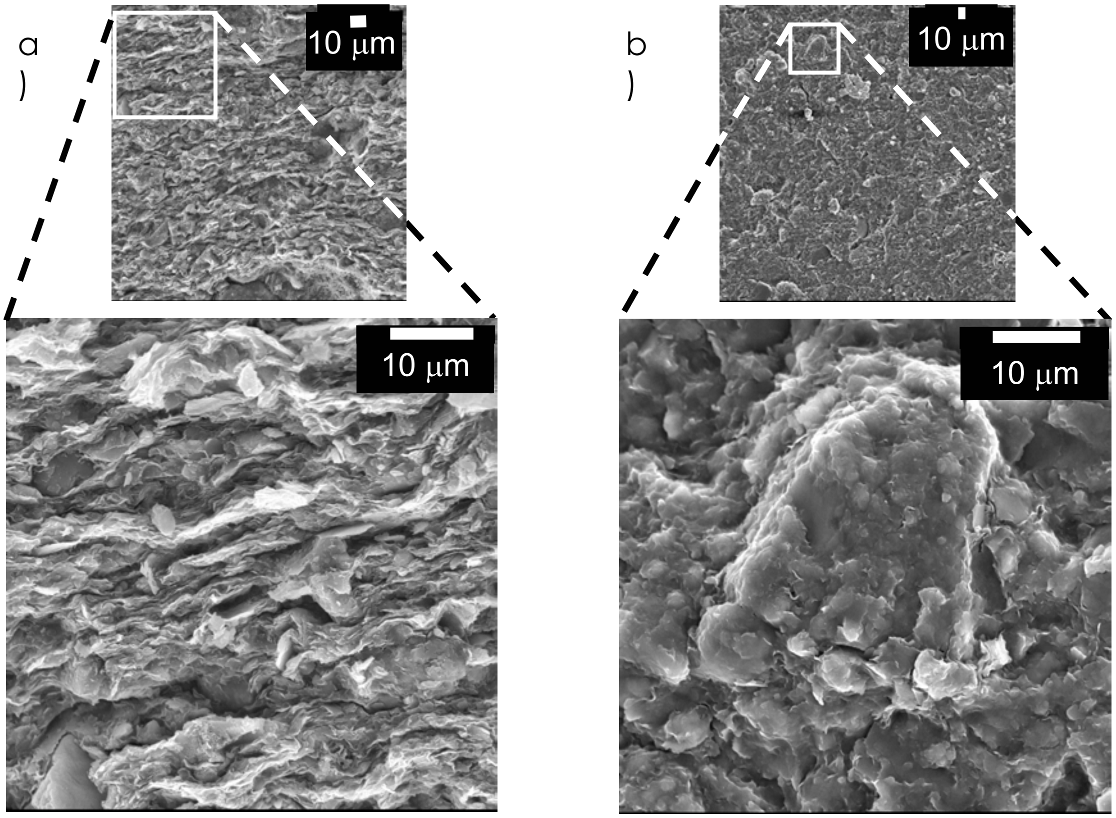

Fig. 1. SEM image SEM images of Colorado Shale from the Cold Lake area of Alberta, Canada: a) section perpendicular to the bedding plane; b) section parallel to the bedding plane. Average size of clay platelets is several microns and almost order of magnitude smaller then embedded randomly distributed silt (quartz, calcite etc.) particles.

constants weighted by the ODF *f(g)*. The Reuss approximation of constant stress throughout the textured aggregate can be implemented through the procedure similar to Eqn. 1 by averaging the elastic compliances S_{ijkl}:

$$\overline{C}^R{}_{ijkl} = \left[\sum_{n=1}^{N} S_{ijkl}\, f(g_n)\Delta g_n \right]^{-1} \tag{2}$$

Note that the averaging procedure in Eqn. 2 yields elastic compliances $\overline{S}^R{}_{ijkl}$ that should be inverted into the elastic stiffnesses $\overline{C}^R{}_{ijkl}$. If aggregate is textured and composed of highly anisotropic crystals the VR bounds are widely separated. In such an aggregate both the Voigt and Reuss assumptions of uniform strains and stress, respectively, are violated. Consequently, the elastic properties of textured shales should be unbiased by the VR bounds. In the Geometric Mean Averaging method a unique solution is obtained independent of whether the averaging is carried out using the stiffnesses or compliances (Eqn. 1 and 2, respectively). The GMA solution can implicitly be written as:

$$\left\langle C_{ijkl} \right\rangle = \prod_{n=1}^{N} \left[C_{ijkl} \right]^{f(g_n)\Delta g_n} = \left[\prod_{n=1}^{N} \left[S_{ijkl} \right]^{f(g_n)\Delta g_n} \right] \tag{3}$$

The resulting matrix of elastic constants $\left\langle C_{ijkl} \right\rangle$ is independent of the domain of averaging (i.e. either stiffnesses or compliances) and yields a unique solution of the ODF averaging procedure.

Results

The effect of the different averaging procedures on the intrinsic anisotropic parameters of shale was investigated through their dependence on degree of texture. If the texture peak of the clay platelet normals in an aggregate of the TI symmetry is aligned with the bedding normal of shale, then the elasticity of such an aggregate will depend only on the elasticity of the single crystal. The textural peak of the distribution of the clay platelet normals in real shales is not fully aligned but rather distributed around the bedding normal (*Hornby et al.*, 1994; *Ho et al.*, 1999). This distribution can be represented by the normal (Gaussian) distribution function characterized by its standard deviation σ. The Gaussian broadens as the standard deviation of the normal distribution increases and, eventually, distribution of the clay platelet normals can be treated as *quasi* uniform within the aggregate with no specific preferential orientation. This resembles a random distribution of crystals (Fig. 2) (i.e. equal probability in all directions) of the constituent anisotropic minerals resulting in an elastically isotropic medium. Perfectly aligned constituent minerals, on the other hand, produce a medium that does not differ from the single anisotropic crystal.

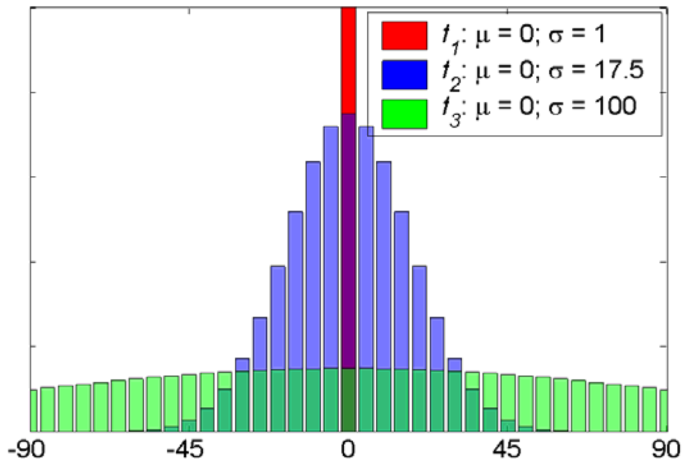

Fig. 2. Schematic representation of shale texture by Gaussian distributions with different standard distributions (not to scale).

Anisotropic parameters are calculated from the elastic constants obtained by different averaging techniques and plotted as a function of the strength of texture on Fig. 3. Both, ε and γ behave similarly and decay from the maximum single crystal value to zero for the isotropic aggregate as expected. It is interesting to note that the different averaging approximations insignificantly affect these anisotropic parameters, except for a certain range of values of ε (corresponding to highly textured aggregate). The parameter δ has rather vague physical meaning but it reflects the *P*-wave phase velocity dependence on the direction in the vicinity of the vertical incidence angle. It has a more complex behaviour (Fig. 3) being negative for the single crystal, but increasing to at least the same positive value before eventually decaying towards zero for the *quasi* isotropic aggregate. It is interesting to note that for a certain range of textural strengths, δ is sensitive to the averaging method with the Voigt values several times those for the Reuss. Anellipticity characterizes the deviation of the *P*-wave slowness surface from elliptical. The resulting curves of anellipticity η (defined here as $\eta=\delta-\varepsilon$) for the Voigt, Reuss, Hill, and GMA averages all yield nearly the same value as shown on Fig. 3. Despite the fact that the various averaging procedures give different absolute values of elastic constants (see *Cholach and Schmitt*, 2003), the overall behaviour of the elastic waves (especially anellipticity) predicted by these approximations is similar. Finally, solving the Christoffel's equation (e.g. *Musgrave*, 1970) for set of elastic constants obtained from the GMA procedure elastic wave slowness surfaces has been calculated and plotted on Fig. 4. The initial highly anisotropic slownesses for the textured aggregates approach circular slowness surfaces for the *quasi*-isotropic aggregate. The *P* slownesses are generally anelliptic while those of *SH* mode are always elliptical in TI medium. The *P* and *S* vertical velocities for intrinsically anisotropic shales are always less than those for the corresponding isotropic aggregate.

Conclusions

The anisotropic parameters of shales have been calculated from the elastic constants modelled by the Orientation Distribution Function (ODF) averaging with incorporated Geometric Mean Averaging (GMA). Absolute values of elastic constants of textured shales depend on the averaging assumptions (viz. the Voigt, the Reuss, the Hill and the GMA). Anisotropic parameters, however, are less influenced by the choice of the averaging technique (Fig. 3), i.e. GMA solution is close to the VR results. Consequently, while the choice of the averaging technique is essential for proper estimation of the absolute values of the elastic constants (and, eventually, wave velocities), it is less significant when describing the behaviour of the elastic velocities.

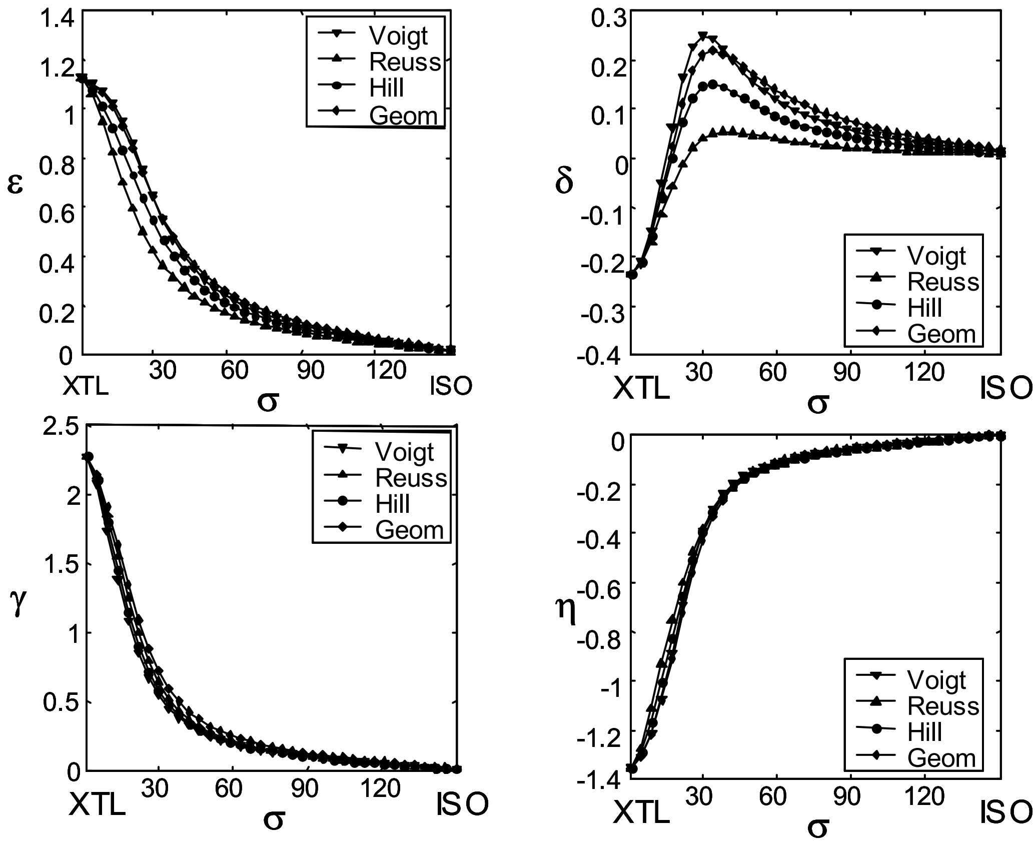

Fig. 3. Anisotropic parameters for TI medium as a function of texture. Right end of the ordinate axis for each plot represents values of fully aligned polycrystalline with the properties of a single crystal (XTL) All parameters approach zero for an isotropic aggregate. Notice dependence of ε and, especially, δ on the averaging procedure. Anellipticity (η) values are very close for all approximations.

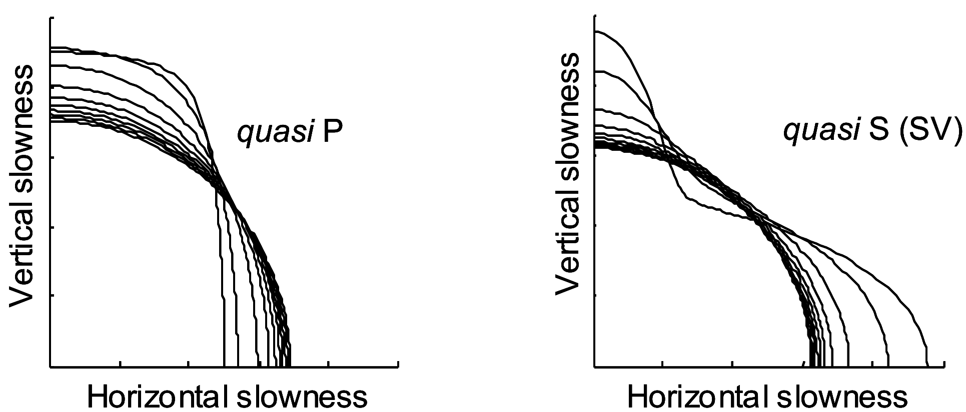

Fig. 4. Slowness surfaces for *qP* and *qS* (*SV*) waves for intrinsically anisotropic shale of TI symmetry calculated from the elastic constants obtained by the GMA procedure. Slowness surfaces vary from highly anisotropic for a single crystal to almost circular for *quasi* isotropic aggregate.

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