NUTS AND BOLTS OF BEAM MIGRATION

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Introduction

Gaussian beam migration (Hill, 1990, 2001) is an elegant, accurate, and efficient depth migration method. It has the ability to image complicated geologic structures with fidelity exceeding that of single-arrival Kirchhoff migration and approaching that of wave-equation migration. In fact, its accuracy can exceed that of most wave-equation migrations in imaging very steep dips, especially in three dimensions and especially in the presence of anisotropy.

The success of Gaussian beam migration has sparked interest in beam migrations, some based on Gaussian beams, others not. Some variations of beam migration have as their sole purpose the speedup of Kirchhoff migration (Sun et al., 2000); others use wavefield extrapolation to build migration Green's functions (Brandsberg-Dahl, 2003); still others use wavelet decomposition of seismic wavefields to emulate the simultaneous space/wavenumber localization property of Gaussian beam migration (Wu et al., 2002). Here, I present an overview of beam migration with emphasis on Gaussian beam migration. My objectives are to explain Gaussian beam migration as an improved version of the more familiar Kirchhoff migration, to compare Hill's common-offset migration with common-shot migration (which may sometimes be preferable to common-offset migration), and to express the recent wave-equation-based beam migrations as generalizations of Gaussian beam migration.

Anatomy of Gaussian beam migration

Gaussian beam migration is a generalization of Kirchhoff migration that operates in the frequency-wavenumber, as well as the time-space, domains, and allows for multipathing in a natural way. The key to this generalization is the use of Gaussian beams for the Green's functions. Beams are obtained from the solution of dynamic ray equations; they produce the wavefield at points some distance away from the raypath. Propagating the recorded energy along a particular beam, and applying an imaging condition, gives part of the total migrated image; the total image is the sum of the contributions from all the beams. Each beam can be specified to be planar at the Earth's surface, and the downward continuation can be performed on local slant stacks of the input traces. Migrating a limited number of local slant stacks over a limited number of initial directions makes Gaussian beam migration efficient, and assuring that the number of directions is adequate to sample the entire wavefield makes it accurate. The migration of a single slant stack in a single direction is shown in Figure 1 (Hale, 1992), and the full migrated image is accumulated from all the partial migrations.

The above description is oversimplified: the details of Gaussian beams complicate the complete picture considerably. Gaussian beams have traveltimes and amplitudes that are both complex-valued. In fact, it is the imaginary part of the traveltime within the beam, not the taper applied to the local slant stack of the input traces, that causes the decay of the migrated image away from the central ray in Figure 1. The complex-valued amplitudes and traveltimes complicate the full description of Gaussian beam migration, but they also provide a regular migration Green's function. In contrast, the Green's functions used in standard Kirchhoff migration become singular at caustics, requiring *ad hoc* fixes. A wide latitude of choices exists for initial conditions for Gaussian beams; Hill's formulation of Gaussian beam migration uses Green's functions that are planar at their take-off points on the Earth's surface.

In addition to the complex-valued ray quantities, the traveltime tables used in Gaussian beam migration are considerably different from those used in standard Kirchhoff migration. Standard Kirchhoff migration uses tables of traveltimes from point sources. Each table contains the traveltimes from a station location to all image locations. In a constant-velocity Earth, contours of equal traveltime sweep out spherical shells in three dimensions. In an Earth with considerable velocity variation, single-arrival traveltime tables cannot accommodate energy that travels to an image location along more than one path. In Gaussian beam migration, the point-source response is built as a superposition of plane-wave responses, and the plane-wave traveltime tables are collimated, as the partial image in Figure 1 is. If several beams from the same source or receiver location strike the same image location, Gaussian beam migration accumulates several arrivals into that location in a natural fashion.

Gaussian beam migration is thus a generalization of Kirchhoff migration, and it improves Kirchhoff migration in its natural handling of both wavefield caustics and multi-valued traveltimes.

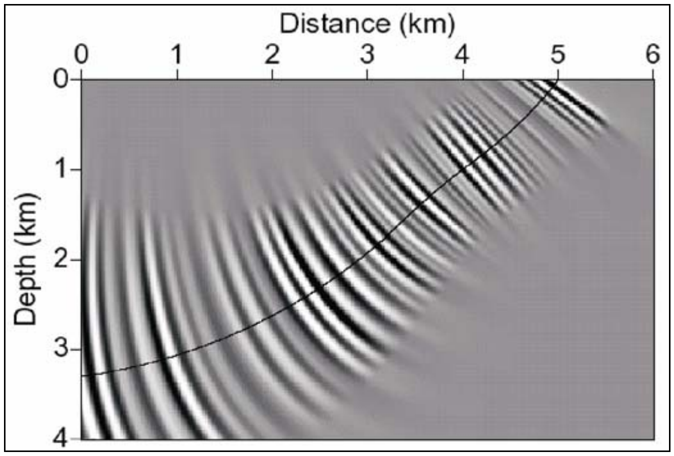


Figure 1. Poststack Gaussian beam migration of a single slant-stacked trace in a single direction. From Hale (1992).

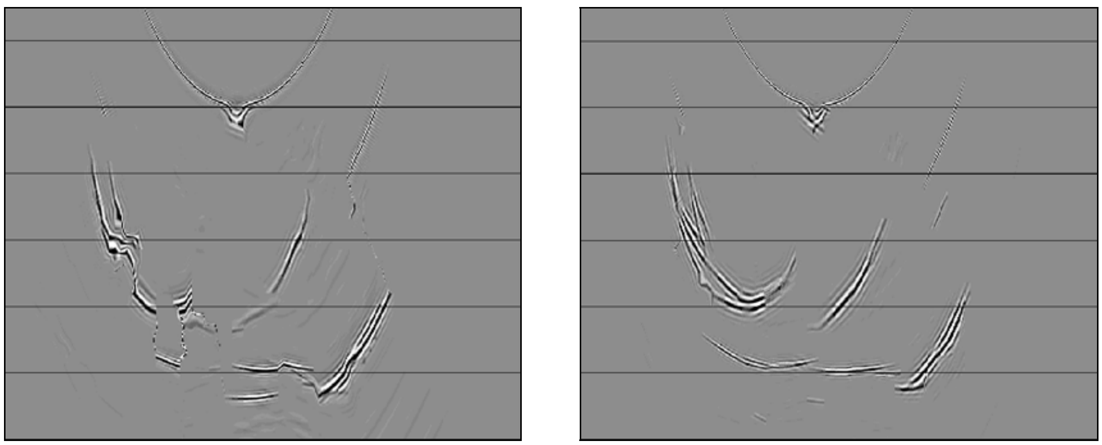
Common shot or common offset?

Hill (2001) describes common-offset, common-azimuth Gaussian beam migration. This continues the generalization of Kirchhoff migration, and it offers some advantages over common-shot migration (tomographic velocity analysis, post-migration AVO, etc.). However, it is not quite as natural to formulate common-offset migration as it is to formulate common-shot migration, and some instances arise where common-shot (or common-receiver) migration is desirable, for example where shot spacing is much greater (or much less) than receiver spacing or where velocities and/or elevations vary rapidly along the recording surface. These cases can occur for marine data with ocean bottom sensors, or for land data. In the first case, unaliased slant stacks of shot records are more easily obtained than unaliased slant stacks of offset records; in the second case, accurate slant stacks of shot records are more easily obtained than accurate slant stacks of offset records. Can Hill's formulation be successfully adapted to common-shot migration?

In poststack migration, as in Figure 1, the image is built up along individual beams. The input trace (local slant-stacked data) is distributed into the image space along the beam, using real and imaginary parts of traveltime and amplitude. In common-shot migration, we have a local slant stack of traces centered at a particular (receiver) beam location. We also have a source location. Downward continuing energy from both source and receiver beam locations implies sending beams from both locations, and imaging in the intersection of these beams. Performing this operation for all source beam directions and all receiver beam directions is an onerous task, and it will severely compromise the efficiency (which is one of the major appeals) of Gaussian beam migration. In performing common-offset migration, Hill replaces this double loop operation with a scan inside a single loop: To migrate a slant-stacked trace at a particular midpoint beam center in a particular *midpoint* beam direction (sum of source and receiver ray parameters) at a particular image location, Hill searches for the *offset* beam direction (difference between source and receiver ray parameters) with the minimum value of imaginary time, i.e., the least decay away from the raypath. Further, the scan is performed over a coarse-grid subset of image locations. Although the scan is the innermost loop of the migration program, it is performed on far fewer points than the actual migration summation. As Hill points out, this reduces the amount of multipathing that the migration can accommodate, although it is designed to accommodate the dominant multipathing.

Analogously, for common-shot migration, we can migrate a slant-stacked trace at a particular receiver beam center in a particular *receiver* beam direction at a particular image location by searching for the *source* beam with the minimum value of imaginary time. Imaging for the given receiver beam direction is performed once for each location within the receiver beam, and not once for every source beam that strikes an image location.

For common-shot migration, however, there is an added negative effect. Moving from one image location to another within a given receiver beam will result in frequent jumps in selected source beam, sometimes with associated jumps in the real part of source traveltime. Since the receiver beam direction is fixed, the total (source plus receiver) traveltime can jump, potentially leading to the branch jumping that we often see with single-arrival Kirchhoff migration. For common-offset migration, both source and receiver beam directions change as we move over image locations, and changes in the real part of source traveltime are more likely to be compensated by changes in the real part of receiver traveltime. Figure 2 shows an example of this difference between common-shot and common-offset Gaussian beam migration of a pair of impulses on a single input trace. On the other hand, the final stacks, compositing these effects over all beams, are remarkably similar (Figure 3), suggesting that the technical compromise of common-shot migration is worthwhile.



a b

Figure 2. Gaussian beam impulse response migrations: (a) common-shot migration; (b) common-offset migration. The common-shot image shows branch jumping, and the common-offset migration does not.

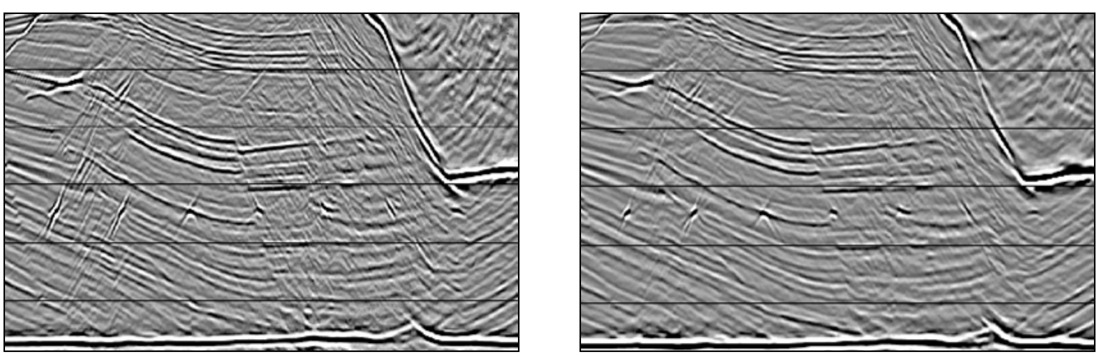


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Figure 3. Gaussian beam migrated images from the Sigbee2a salt model data set: (a) common-shot migration; (b) common-offset migration.

Obsolete, or ahead of the curve?

Gaussian beam migration generalizes Kirchhoff migration, but it still appears to have a fatal weakness compared with migration by downward continuation, namely its dependence on rays. In particular, using information from along a raypath to image away from the raypath poses a problem. How is the image affected, for example, if a raypath travels within high-velocity salt near low-velocity sediments? Using saltrelated traveltimes to image within sediments appears problematic. To overcome this problem, Brandsberg-Dahl (2003) uses rays in a beam migration, but only as a guide for a wave-equation migration. In his scheme, the tangent and normal directions to the raypath provide a coordinate system for the wavefield extrapolator. The extrapolator uses the actual velocity at all points in the subsurface, not the velocity along the ray. An additional benefit of this scheme is the potential to image dips well beyond the range of the chosen (low-dip) extrapolator.

Aside from this ray-based extension of Gaussian beam migration, Wu et al. (2002) have proposed a generalization that does not depend on rays at all. Beam sources and beam gathers are wavefields that are simultaneously localized in both space and direction. These are formed from localized plane-wave data in a way similar to Gaussian beam migration's local slant stacks, but they are propagated using plane-wave wavefield extrapolators, not Gaussian beams.

Conclusions

Gaussian beam migration provides a complete and correct formulation for multi-arrival Kirchhoff migration, extending our notion of Kirchhoff migration. It can also be extended and generalized to accommodate wavefield extrapolation during the downward continuation. When this is done, the term "generalized beam" should replace "Gaussian beam." I have described the component parts of Gaussian beam migration, indicating the relationship of each part to both Kirchhoff migration and wave-equation migration. Hill (1990, 2001) has presented the component parts in much greater detail; my goal has been to place the parts within the context of the more familiar migration methods.

References

Brandsberg-Dahl, S., 2003, Beam-wave imaging: 73rd Ann. Internat. Mtg., Soc. Expl. Geophys., 977-980.

Hale, D., 1992, Migration by the Kirchhoff, slant stack, and Gaussian beam methods: Colorado School of Mines Center for Wave Phenomena Report 121.

Hill, N. R., Gaussian beam migration: Geophysics, 55, 1416-1428.

Hill, N. R., Prestack Gaussian-beam depth migration: Geophysics, 66, 1240-1250.

Sun, Y., Qin, F., Checkles, S., and Leveille, J. P., 2000, 3-D prestack Kirchhoff beam migration for depth imaging: Geophysics, 65, 1592-1603.

Wu, R-S., Chen, L., and Wang, Y., 2002, Synthetic beam-sources and plane-sources for prestack beamlet migration: 72nd Ann. Internat. Mtg., Soc. Expl. Geophys., 1336-1339.