Two new approximations for AVO inversion

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Abstract

Standard two-parameter inversion methods are analyzed and shown to be equivalent to each other. New two-parameter methods are derived which are modifications of the method of Fatti et al. and which yield different estimates of the shear impedance reflectivity. The first is linear and its results can also be obtained by appropriate combination of the results of Fatti et al. The other is non-linear, containing a term quadratic in the shear impedance reflectivity, but it can be solved non-iteratively. Inversions of synthetic data are carried which show that these methods can improve on the Fatti method for large density and shear impedance reflectivities.

Introduction

In AVO inversion one seeks to determine earth-property contrasts across an interface from the angle-dependence of seismic amplitudes. The starting point is $R_{PP}(\theta_i)$, where R_{PP} is the P-wave reflection coefficient determined from seismic amplitudes and θ_i is the angle of incidence at the interface. The final objective is a set of relative contrasts of the form $\Delta x/x$, which can also be expressed as reflectivities, R_i. We set out these definitions as follows:

$$\frac{\Delta x}{x} = \frac{x_2 - x_1}{(x_1 + x_2)/2} \qquad x = \alpha, \beta, \rho, I, J, \mu \qquad \gamma = \left(\frac{\beta_1 + \beta_2}{\alpha_1 + \alpha_2}\right)^2 \equiv \left(\frac{\beta}{\alpha}\right)^2$$

subscript 1 = earth layer above interface subscript 2 = earth layer below interface

$$\begin{aligned}
\alpha &= \text{P-wave velocity} & R_\alpha &= (1/2)(\Delta\alpha/\alpha) \\
\beta &= \text{S-wave velocity} & R_\beta &= (1/2)(\Delta\beta/\beta) \\
\rho &= \text{density} & R_\rho &= (1/2)(\Delta\rho/\rho) \\
I &= \text{P-wave impedance} = \rho\alpha & R_I &= R_\alpha + R_\rho \\
J &= \text{S-wave impedance} = \rho\beta & R_J &= R_\beta + R_\rho \\
\mu &= \text{shear modulus} = \rho\beta^2 & R_\mu &= 2R_\beta + R_\rho
\end{aligned}$$

The Aki-Richards approximation (Aki & Richards, 1980), a linearization of the Zoeppritz equations in R_α, R_β, and R_ρ, has been the starting point for most AVO inversion work. While the Zoeppritz equations give exact coefficients for idealized transmission, reflection, and conversion events, their complicated structure necessitates the use of non-linear inversion techniques. Inversion with the Aki-Richards approximation is a one-step process, involving the least-squares solution of a set of linear equations.

In reality of course one requires some “background” parameters as input to the inversion. One requires an estimate of R_α, for use in raytracing, and an estimate of γ. These are required to set up the coefficients in the Aki-Richards equation.

In practical inversions, the three-parameter Aki-Richards approximation is itself often set aside in favor of a two-parameter approximation. The best-known of these are the approximation of Smith & Gidlow (1987), in which a differential form of Gardner’s relation (Gardner et al., 1974) is used to replace R_ρ with R_α, and the approximation of Fatti et al. (1994), in which the contribution of R_ρ is assumed to be negligible in comparison to that of R_I and R_J. Reducing the number of variables controls large errors resulting from noise in the seismic data.

In this research we develop two new AVO approximations. The first is a linear theory similar to the method of Fatti et al., but which has smaller errors in some situations. The second is a two-parameter method that is superior in some cases to the Fatti method for estimation of R_J. Although it is non-linear, it is unique in that it can be solved in a one-step process, without recourse to iterative techniques.

Theory I: Linear two-parameter inversion with a simple two-data-point model, and a new approximation

To begin we first take a new look at what is actually being calculated in a two-parameter inversion. Consider a simple case involving only two offsets, one of which is zero. In this case the equations for a general two-parameter linear inversion can be written as

$$R_{PP}(0) = A_0 X + B_0 Y \qquad \text{and} \qquad R_{PP}(\theta_i) = A_1 X + B_1 Y \ ,$$

where X and Y are each some combination of reflectivities, and A_i and B_i are initially undetermined coefficients. To linear order, $R_{PP}(0) = R_\alpha + R_\rho$ and $R_{PP}(\theta_i) = R_\alpha/\cos^2\theta - 4\gamma \sin^2\theta\, R_\mu + R_\rho$ (as per the Aki-Richards equations, where $\theta = [\theta_i + \theta_t] / 2$, and θ_t is the P-wave transmission angle). The general solutions for X and Y are linear combinations of $R_{PP}(0)$ and $R_{PP}(\theta_i)$. Thus they are also linear combinations of $R_{PP}(0)$ and $R_{PP}(\theta_i) \cos^2\theta - R_{PP}(0)$. This choice is convenient because $R_{PP}(0)$ is independent of R_μ and $R_{PP}(\theta_i) \cos^2\theta - R_{PP}(0)$ is independent of R_α. These solutions can be obtained by setting $B_0 = 0$ and $A_0/A_1 = \cos^2\theta$. The solutions then simplify to

$$X = (R_\alpha + R_\rho) / A_0 = R_I / A_0 \qquad \text{and} \qquad Y = -\left(4\gamma \sin^2\theta R_\mu + \tan^2\theta R_\rho\right) / B_1 \,.$$

Any two parameter inversion will yield these two quantities or linear combinations thereof. An analysis of the method of Fatti et al. shows that it corresponds to $A_0 = 1/2$ and $B_1 = -4\gamma \sin^2\theta$, yielding

$$X(\text{Fatti}) = 2R_I \tag{1}$$

$$Y(\text{Fatti}) = R_\mu + \frac{R_\rho}{4\gamma\cos^2\theta} = 2R_\beta + \left(1 + \frac{1}{4\gamma\cos^2\theta}\right) R_\rho = 2R_J - \left(1 - \frac{1}{4\gamma\cos^2\theta}\right) R_\rho \tag{2}$$

Thus Y(Fatti) is, to linear order, strictly equal to $2R_J$ only when $\gamma \cos^2\theta = 1/4$. Analysis of the Smith-Gidlow method shows that

$$X(\text{S-G}) = \frac{8}{5}(R_\alpha + R_\rho) = \frac{8}{5} R_I = \frac{4}{5} X(\text{Fatti})$$

$$Y(\text{S-G}) = 2R_\beta + \frac{4R_\rho - R_\alpha}{5}\left(1 + \frac{1}{4\gamma\cos^2\theta}\right) = Y(\text{Fatti}) - \frac{X(\text{Fatti})}{10}\left(1 + \frac{1}{4\gamma\cos^2\theta}\right)$$

From these results we can see that the Smith-Gidlow X and Y are, to linear order, equal to $2R_\alpha$ and $2R_\beta$ if $R_\rho = R_\alpha/4$ (as per the Gardner relation). Furthermore the results of either the Fatti or Smith-Gidlow methods can be combined to obtain the results of the other. These results have been obtained assuming two noise-free data points, but in Figure 1 the results obtained by least squares inversion on 31 noisy data points ($\theta = 0°, 1°, 2°, \ldots 30°$) show that the conclusions above are, with θ set to θ_{max}, still valid.

The result of any two-parameter inversion can thus, to linear order, be reduced to Eqs (1) and (2). Eq. (1) of course is the P-impedance, and Eq. (2) is closest to R_J, as there is a minus sign inside the third error term in Eq. (2), allowing for some cancellation. A better approach may be to approximate R_ρ by $R_I/5$ and then obtain $R_J \cong Y(\text{S-G}) + X(\text{S-G})/4$. This can equivalently be obtained as

$$Y(\text{Fatti}) + \frac{X(\text{Fatti})}{10}\left(1 - \frac{1}{4\gamma\cos^2\theta}\right) = 2R_J - \left(1 - \frac{1}{4\gamma\cos^2\theta}\right)\frac{4R_\rho - R_\alpha}{5} \,. \tag{3}$$

Comparing with Eq. (2) we see that this is the same as Y(Fatti) except that R_ρ in the error term has been replaced by $(4 R_\rho - R_\alpha)/5$. The latter quantity is generally smaller, at least for large R_ρ. This quantity can be obtained from Smith-Gidlow or Fatti results, or, because this is a linear theory, one can invert for it and R_I directly using the same value of B as in the Fatti method, but replacing A by

$$A = \frac{1}{2\cos^2\theta} + \frac{1}{10}(4\gamma\sin^2\theta - \tan^2\theta) \,. \tag{4}$$

Eq. (4) constitutes a new AVO approximation designed to give a more accurate estimation of R_J for large R_ρ.

Theory II: Importance of the R_J^2 term and the resulting AVO approximation

Eq. (2) suggests that deviations of Y(Fatti) from R_J are dominated by a linear trend in R_ρ. However it was found that deviations from expected behavior are strongly correlated with R_β^2 (or R_J^2). Such deviations obscure the linear R_ρ trend, except at very large values of R_ρ. Such behavior with R_J^2 has been noted in some of our earlier studies on other error quantities (Ursenbach 2003a,b). As a result it is reasonable to propose an AVO approximation of the form

$$R_{PP}(\theta) = A\frac{\Delta\alpha}{\alpha} + B_1\frac{\Delta\beta}{\beta} + B_2\left(\frac{\Delta\beta}{\beta}\right)^2 . \tag{5}$$

A and B_1 can be assigned the same values as in the Fatti method. B_2 could be obtained by extending the approach of Aki and Richards to second order in $\Delta\beta/\beta$. The correct result which removes $(\Delta\beta/\beta)^2$ error terms in the inversion result is

$$B_2 = B_1\frac{\beta}{\alpha}\frac{\gamma\sin^2\theta - \cos^2\theta}{\cos\theta\cos\varphi} = B_1\frac{\beta}{\alpha}\frac{\sin^2\theta - \cos^2\varphi}{\cos\theta\cos\varphi} \tag{6}$$

where φ is the average of converted-wave reflection and transmission angles at the interface. The quantity $\cos\varphi$ is well approximated by $\sqrt{(1 - \gamma \sin^2\theta)}$, and so requires no more input than that required by the linear theories. Eq. (5) is a non-linear theory, but it can be solved exactly without resorting to iterative techniques. Seeking a least-squares solution leads to a cubic polynomial in R_β. This has three solutions to choose from, and R_β is equal to the real root having the smallest magnitude. With R_β known, R_α can then be obtained as a quadratic function of R_β.

Eqs (5) and (6) constitute an augmented Fatti method. Solution of this two-parameter theory will again yield the quantities of Eqs (1) and (2) as its results, but without R_J^2 error.

Application

In this section we compare the abilities of different methods to accurately calculate R_J. We carry out the inversion for 125 interfaces, as decribed earlier (Ursenbach, 2003a,b). We first consider noise-free data, and then add random noise to the P-P amplitudes.

Figure 2 shows that Eq. (3) (or Eq. (4)) does not affect the R_J^2 error, but that most of the outlying points (where the linear R_ρ term is large) are improved. Figure 3 shows that Eq. (5) removes the R_J^2 errors from the R_J estimates, but that the outlying points are often worse. However Figure 4 shows that when Eq. (5) is used with Eq. (3) or Eq. (4) that the outlying points are now as good or better than in the Fatti result. Figure 5 replicates the results of Figure 3, but with random noise added to the P-P amplitudes. One can still see similar trends, and the noise causes a similar degree of scatter in both two-parameter methods. Figure 6 is instructive. We have found that the range of inversion errors induced by a given level of noise is inversely proportional to γ and $\sin^2\theta_{max}$. Note that outlying points in the previous figures typically possess small γ. The non-linear theories are also proportional to $1+2R_\beta$, and the slope in both cases varies roughly as $\sqrt{(\theta_{i,max}/n)}$, where n is the number of data points (n=31 in Figures 1-6).

Conclusions

We have demonstrated that the methods of Smith-Gidlow and Fatti et al. (and all other linear, two-parameter methods) are equivalent. We have shown that a new linear, two-parameter method generally gives improved estimates of R_J when R_ρ is large. We have also developed an augmented version of the Fatti method which includes a non-linear term. This method is more complicated to calculate, but is still non-iterative, and removes errors quadratic in R_J. It should always be used in combination with the first method, as that adds no extra difficulty and appears to give better results when R_ρ is large.

References

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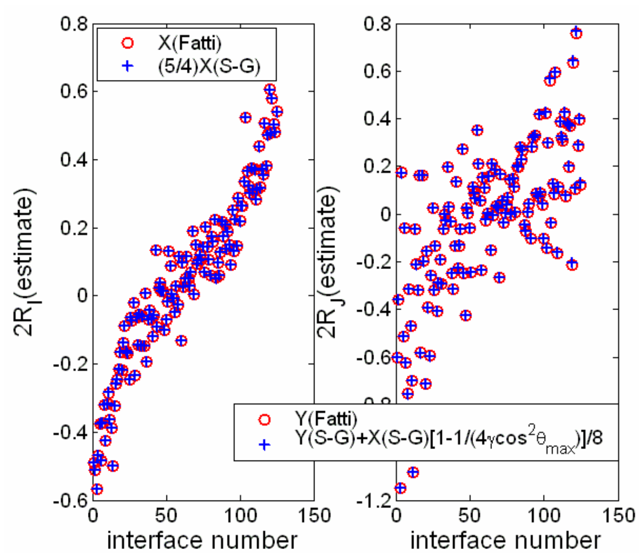


Figure 1. A comparison showing that R_I and R_J estimates as calculated by the Fatti and Smith-Gidlow methods are equivalent. AVO inversion has been carried out by both methods on 125 interfaces, using data of Castagna and Smith (1994) as described in Ursenbach (2003a,b).

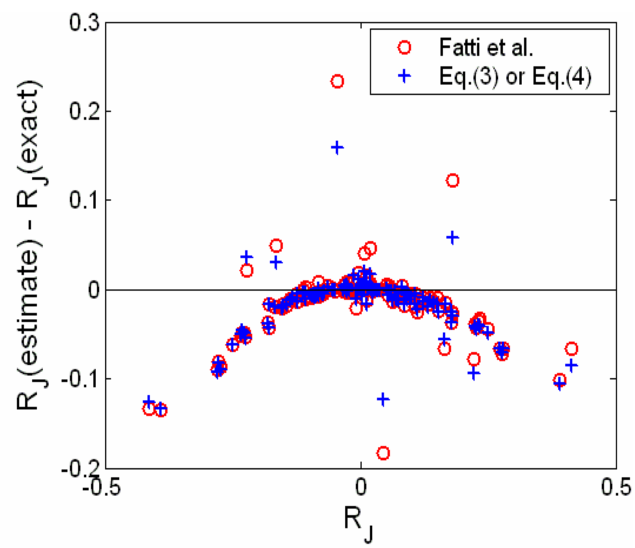
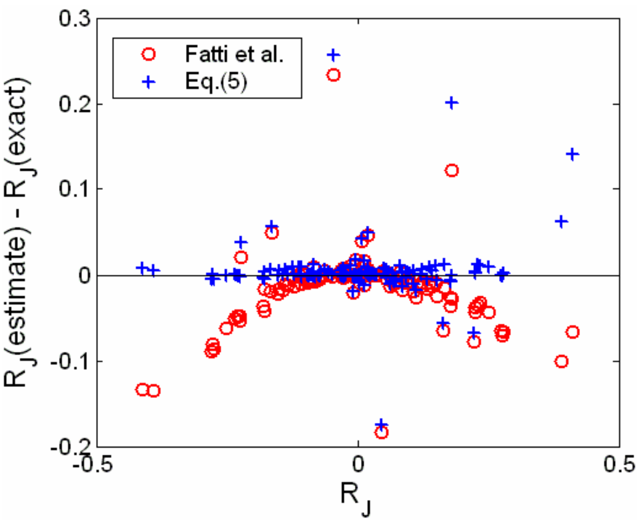


Figure 2. A comparison of Fatti inversion with inversion based on Eq. (4). Points away from the quadratic trend are associated with large linear R_ρ error terms. When these errors are large, Eq. (4) reduces them by replacing R_ρ by $0.8R_\rho - 0.2R_\alpha$ in the error term.



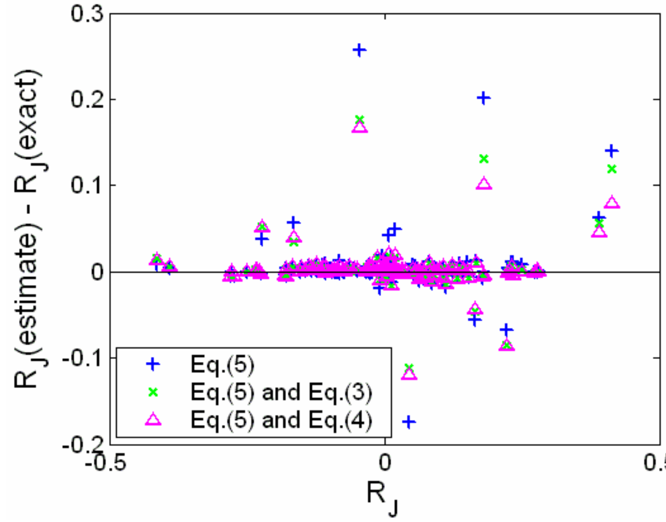
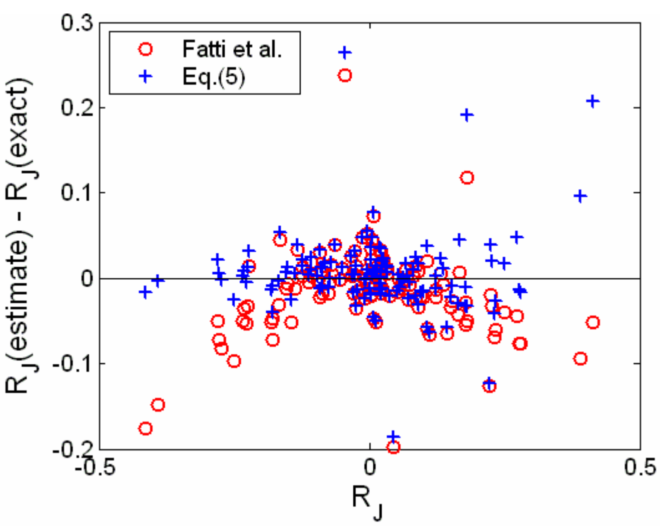


Figure 3. A comparison of Fatti inversion with inversion based on Eq. (5). The R_J^2 error is clearly absent in the latter result. However errors associated with the linear R_ρ term, represented by outlying points, appear slightly worse in the new method.

Figure 4. A comparison of the method of Eq. (5) with two other methods derived from it. In one method the results of Eq. (5) inversion are combined in the manner of Eq. (3). In the other method the *A* parameter in Eq. (5) is defined by Eq. (4). For a linear theory these procedures would give identical results. Eq. (5) is non-linear, however, so they differ, but both improve the estimate of outlying points. Comparison with Figure 3 indicates that they compare favorably with the Fatti method as well.



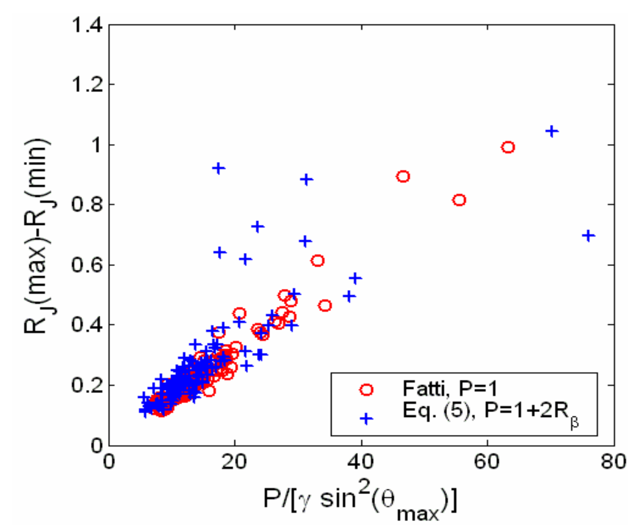


Figure 5. This figure is identical to Figure 3 except that random noise has been added to the P-P amplitudes prior to inversion. The same trends can be discerned, however, and one can see that the noise induces a similar degree of scatter in both two-parameter methods.

Figure 6. To obtain this data, 100 inversions were carried out (for each of the 125 interfaces) using different random errors each time. The value of R_J predicted for each interface varied with the error used, and the difference of maximum and minimum predictions for each interface was obtained and plotted above. The range of predictions for each interface was found to correlate with γ and $\sin^2\theta_{max}$, with a slope that depends on the interval between data points, and, for fixed $\theta_{i,max}$, varies as $1/\sqrt{n}$, where n is the number of data points. For quadratic methods, such as Eq. (5), the results also correlate with $(1+ 2\ R_\beta)$. Results from noisy data are distributed on both sides of the noise-free results, so the latter are a good indication of the average behavior of noisy data.