# Migration with compensation in a viscoacoustic medium

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**Summary**

Theory and research have shown that the propagation of seismic waves in real media is in many respects different from propagation in an ideal solid. In data processing, the attenuation and dispersion of the seismic wave has often been neglected. Presented here is a method for accommodating absorption and dispersion effects in migration schemes. Extrapolation operators that compensate for absorption and dispersion have been designed. The algorithm is developed in the frequency-wavenumber domain, and it is characterized by simplicity, speed, less dependence on stratum obliquity, and good stabilization. To demonstrate the absorption and dispersion in the viscoacoustic medium, forward modelling was done. The results showed that the amplitude of the wave was decreased, the frequency became lower, and the phase was influenced, when it propagated in the viscoacoustic medium.To test the validity of the method, a viscoacoustic model was designed. Synthetic seismic data were generated based on the model. Viscoacoustic and conventional (lossless) prestack depth migration were performed on the synthetic data. The two results obtained are compared in this paper and show the influence of absorption and attenuation. Without consideration of the absorption and dispersion in the conventional prestack migration scheme, the geological model could not be imaged properly. For the viscoacoustic prestack depth migration scheme, extrapolation operators could compensate for absorption and dispersion, and a proper image obtained.

**Introduction**

No real materials are perfectly elastic. Wave energy is gradually converted into heat. The propagation of seismic waves in real medium is in many respects different from propagation in an ideal solid. A real medium will cause dissipation of seismic energy, thus decreasing the amplitude and modifying the frequency content of the propagating wavelet. This attenuation and dispersion of the seismic wave is strongly affected by the saturation state and physical condition of the rock (Jones, 1986). Therefore, these effects are important in exploration geophysics since they may allow us to extract more detailed information about the subsurface from seismic data or to construct images with better resolution, if the quality factor Q is satisfactorily approximated. There is, at the outset, no justification for neglecting the absorption and dispersion of seismic energy, and the effect has been incorporated into seismic modelling schemes (Emmerich and Korn, 1987; Carcione et al., 1988) and migration schemes (Mittet al., 1995). Attenuation of propagating waveforms is, in some cases, quite significant and could be a source of erroneous results in forward modelling, inversion, and imaging if neglected (e.g., Samec and Blangy, 1992). In recent years, the inclusion of second-order effects, such as absorption and anisotropy, into seismic processing schemes has become more important. Finite-difference modelling in a viscoacoustic medium has been developed (Carcione, 1993), and 3-D prestack migration in anisotropic media has been performed (Dong and McMechan, 1993).

The main purpose of this work is to compensate for the absorption of energy from the source location to the receiver location. This implies that both forward-propagated and backward-propagated waves should be compensated. Formally, this is equivalent to starting at the receiver location and repropagating the wave to the source location and at the same time adding the lost frequency components to the wavefield. The implementation performed in this paper is 2-D, but there is nothing in the formalism that prevents us from using the same method in the 3-D case.

**Viscoacoustic wave equation migration**

In viscoacoustic media, the viscoacoustic wave equation can be expressed as:

$$\nabla^2 (P(x,z,t) + \frac{1}{\omega_0}\frac{\partial P(x,z,t)}{\partial t}) = \frac{1}{c^2}\frac{\partial^2 P(x,z,t)}{\partial t^2}. \tag{1}$$

where *P(x,z,t)* is the pressure, ρ is the density, c is the velocity, and $\omega_0$ is the transition frequency, $\frac{1}{\omega_0} = \frac{\eta_1 + \frac{4}{3}\eta_2}{\rho c^2}$, where $\eta_1$ and $\eta_2$ are the viscoacoustic coefficients. We perform a Fourier transform in the x-direction and t-direction of equation (1) to obtain

$$A \frac{\partial \overline{P}(k_x, z, \omega)^2}{\partial z^2} = B \overline{P}(k_x, z, \omega), \quad (2)$$

where $k_x$ is the wave number responding to x, and $\omega$ is apparent frequency responding to t. $\overline{P}(k_x, z, \omega)$ is a 2-D Fourier transform of $P(x,z,t)$, and

$$A = \frac{i\omega}{\omega_0} + 1, \qquad B = k_z^2 \left(\frac{ik_x^2 \omega}{k_z^2 \omega_0} - 1\right),$$

where $k_z$ is the wave number corresponding to z.

The wave equation (2) has two independent solutions, corresponding to extrapolation of upgoing waves and downgoing waves, respectively.

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{\pm ir z}, \quad (3)$$

where $\overline{P}(k_x, z, \omega)$ is 2-D Fourier transform of $P(x,z,t)$.

Thus, for a downgoing wave, we have

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{ir z}, \quad (4)$$

where $r = \sqrt{|B|/|A|}\left[\cos((\beta - \alpha)/2 + \pi) + i\sin((\beta - \alpha)/2 + \pi)\right]$, $\alpha, \beta$ is the phase of *A* and *B*. The extrapolator $e^{ir z}$ will estimate the energy-loss that the wave has experienced during propagation from source to the reflecting interface (see Figure 1).

The upgoing wave has been propagated upwards, losing amplitude on its way. The extrapolation will give

$$\overline{P}(k_x, z, \omega) = \overline{P}(k_x, 0, \omega) e^{-ir z}. \quad (5)$$

The extrapolator $e^{-irz}$ will boost the amplitude, thereby compensating for the lost amplitude on the way up from the reflection point (see Figure 1).

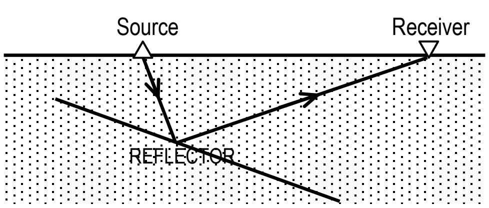


FIG. 1. Raypath of a reflected signal. The wave is compensated for absorption on the way from the source down to the reflector, and up to the receiver.

We perform an inverse Fourier transform in the $k_x$-direction and $k_z$-direction of equation (4) to obtain

$$P(x,z,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{P}(k_x, 0, \omega) e^{\pm ir z} e^{i(k_x x + \omega t)} dk_x d\omega \quad (6)$$

According to the exploding reflector theory, the reflected point is located in the position when *t=0*. So, let *t=0*,

$$P(x,z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{P}(k_x, \omega) e^{\pm ir z} e^{ik_x x} dk_x d\omega \quad (7)$$

then equation (7) is the basic migration formula in wave number domain.

These algorithms are limited to homogeneous media with constant-velocity. In order to overcome this kind of limitation, Gazdag and Squazerro (1984) developed an approximate extension of phase shift to laterally variant media (PSPI). We have similarly extended our methods.

**Numerical Example**

In order to demonstrate the property of attenuation and dispersion of seismic wave in viscoacoustic media, forward modelling synthetic seismic data were generated by using viscoacoustic forward modelling scheme (Cui, 2001). Figure 2 is the geological model. It also contains the relative parameters of the modelling. In this model, there is two exploding points located at 250m and 750m depth under the No. 64 geophone, respectively. At zero time, these two exploding points generate a minimum phase wavelet, respectively. The streamer shown in Figure 2 consists of 128 geophones with a separation of 30m. The velocity and transition frequency $\omega_0$ of model are constant here.

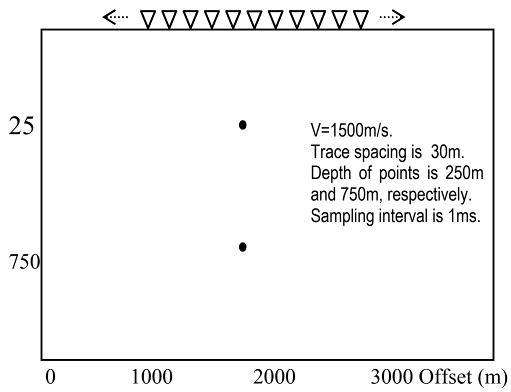


Fig 2. Geological model for forward modeling

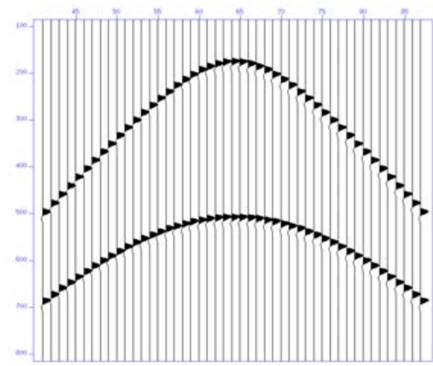

FIG.3. Acoustic forward modelling result from the same geological model of Fig.2 but the attenuation is zero

For the lossless model, the attenuation is zero. So the frequency, amplitude and phase of the wave are same at different traces, as well as different depths (see the Fig. 3). In Figure 4 we show the results of using the viscoacoustic wave equation forward modelling scheme. The transition frequency $\omega_0$ equals 2000. It is very clear that the energy of the seismic wave is absorbed when it is propagating in the viscoacoustic media. The longer propagation, the absorption the more serious. In Figure 5 we partially enlarge the figure 4, in order that the absorption can be observed easily. Note with exploding point being far away from the receiver, amplitude of wave is decreased, and frequency become lower, and the phase is also influenced.

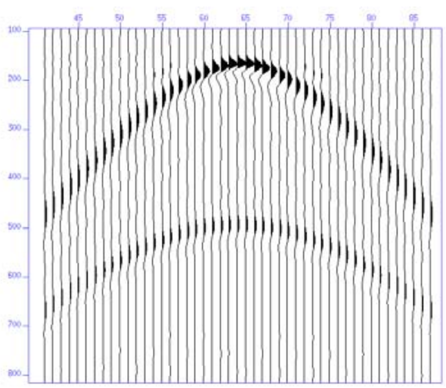

FIG.4.Viscoacoustic forward modeling result of Fig. 2. The attenuation is 2000.

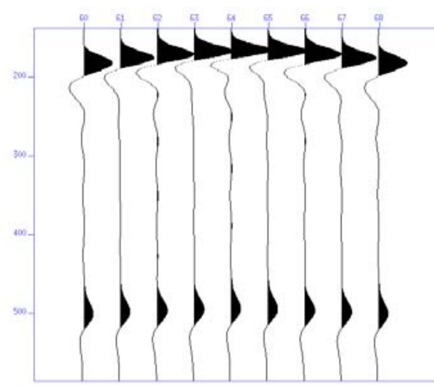

FIG.5. Partially enlarged of fig.4

In order to check the formalism developed in the previous section and to demonstrate the compensation for absorption, synthetic seismic data were generated. A geological model was designed (see Figure 6), including an inclined layer, two faults, and a flat seam. The velocity of each layer is labelled in the figure. The transition frequency $\omega_0$ of this model is 20000. Figure 6 contains the geometry of the model used, together with the shot and receiver configuration. The dashed line defines the area to be imaged in this test. The test contains 30 shots. The streamer shown in the figure consists of 128 geophones with an interval of 20m; the offset is from 20m to 1280m, and shot in the middle. Data were recorded for 2,048ms with a simple rate of 1ms. The shot gather is shown in figure 7. The source signature is a Ricker wavelet. First, viscoacoustic modeling with absorption was performed. The result of the modeling is shown in Figure 7. The synthetic data were generated without surface multiples. For field data, one should preprocess with surface multiple removal schemes, e.g., using methods such as those proposed in Fokkema and Vanden Berg (1990) and Wapenaar et al. (1990). The data were processed with the prestack depth migration scheme outline earlier. In Figure 8, we show the result using conventional prestack depth migration. Note the diffuse image of the reflectors due to the dispersion of the wavelet as it propagates down into the subsurface and up again. Also the locations of some of the reflectors are incorrect. This is a consequence of the fact that both an amplitudes and arrival times are changed in a viscoacoustic medium with absorption (Carcione et al., 1988). In Figure 9, we show the result of our proposed viscoacoustic prestack depth migration scheme. Note the improved quality of the image; at the points of the faults and reflectors are imaged clearly.

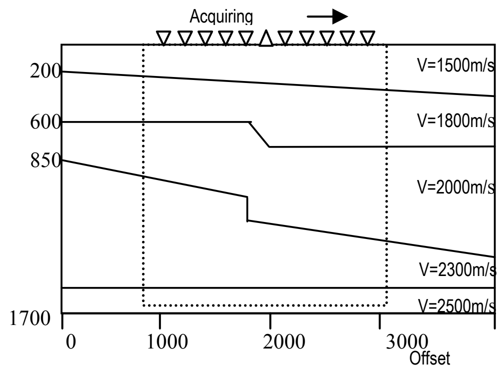


FIG.6. Subsurface model. The dashed line defines the image window. The velocity for each layer is labelled in the model. The transition frequency $\omega_0$ of this model is 20000.

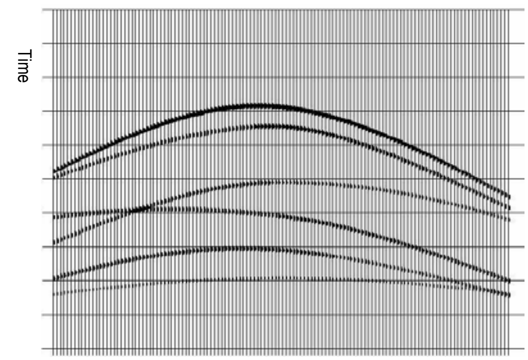


FIG. 7. Modelled shot gather data

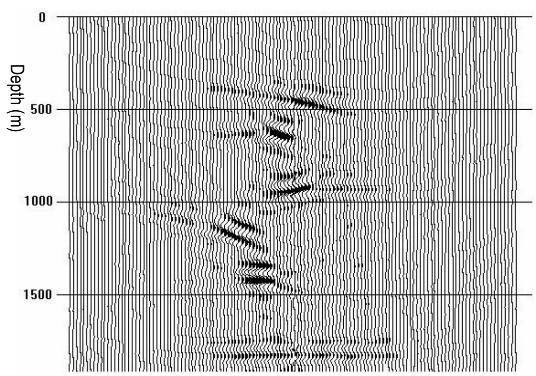


FIG. 8. The images resulting from acoustic wave equation prestack depth migration

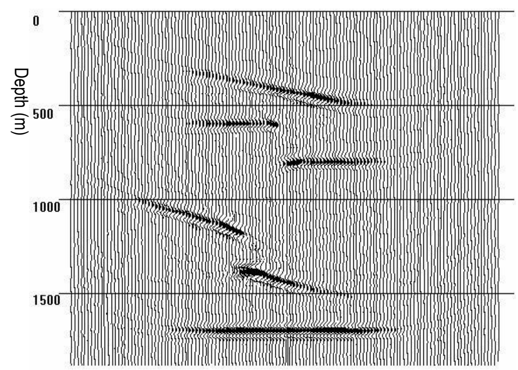


FIG. 9. The images resulting from viscoacoustic wave equation prestack depth migration.

## Conclusion

We have shown that the effect of absorption and dispersion of seismic energy in a viscoacoustic medium can be compensated for in migration schemes. New extrapolator coefficients that account for the attenuation and dispersion of wavefield have been designed in this paper. From prior information about the variable velocity and the absorption coefficient of the medium, the correct extrapolator coefficient for a given point in space can be accessed and used in the depth extrapolation. The synthetic data used in the numerical example were obtained from a designed viscoacoustic model. The extrapolation operator, however, can handle the absorption. The results of the numerical test show a significant improvement of the images when migrating with compensation for absorption, as compared to images using conventional prestack migration. The images are less diffuse, and the locations of the reflectors are improved.

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