Can we retain simple yet efficient: eikonal equation solver revisited

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For the exploitation of great speed and efficiency of Kirchhoff migration, there have been numerous methods of travel time computation in the literature. One of the very popular ones is the solution of Eikonal equation. Reshef and Kosloff (1986) combined a Runga-Kutta method in depth and a finite difference method in horizon to solve the eikonal equation; Vidale (1988) proposed to use a finite difference scheme in both depth and horizon for solving the eikonal equation, Van Trier and Symes (1991) adopted a finite difference approach in cylindrical coordinates; and there have also been a number of modifications to these ideas (Podvin and Lecomte 1991; Qin et al 1992; Gray and May 1994).

A close look to above schemes shows that they are either less accurate due to the lower order of finite difference method, or computationally complicated, mainly due to the fact that wave motion in our seismic experiments have a cylindrical (2D) or spherical (3D) symmetry, whereas the way of data collection has a rectangular or cubic symmetry. For retaining simplicity in solving the eikonal equation while still achieving reasonable accuracy and efficiency, we suggest to adopt a 4th order Runga-Kutta method and a 2nd order finite difference method in cylindrical (2D) or spherical (3D) coordinates. This is a simple but more natural approach, and therefore will produce more reliable result.

For a 2D experiment, the eikonal equation governing travel time propagation in cylindrical coordinates is expressed as

$$(\frac{\partial T}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial T}{\partial \theta})^2 = \frac{1}{V^2}\,, \tag{1}$$

where T is the travel time, r is the radius, θ is the angle from horizontal surface, and velocity V is a function of r and θ, which is obtained by interpolation from velocity in rectangular coordinates. To utilize Runga-Kutta method (4th order), equation (1) can be re-arranged as

$$\frac{\partial T}{\partial r} = f(r, \theta, V)\,, \tag{2}$$

where

$$f = [\frac{1}{V^2} - \frac{1}{r^2}(\frac{\partial T}{\partial \theta})^2]^{1/2}\,. \tag{3}$$

At each radius step, $\frac{\partial T}{\partial \theta}$ is approximated with a 2nd order central finite difference representation

$$\frac{\partial T}{\partial \theta} = \frac{T_{j+1} - T_{j-1}}{2\Delta\theta}\,, \tag{4}$$

except at the surface, where either backward or forward difference formula are used. Figure 1 illustrates the grids arrangement for travel time computation. Step sizes of radius r and angle θ are determined according to complexity of the velocity model, and can be adjusted during expansion process. For example, at areas closer to the source, $d\theta$ can be a few degrees coarse, and as propagation moving outward to areas far away from the source, $d\theta$ can be adjusted to smaller values. After obtaining travel time map in this cylindrical coordinates, a bi-linear interpolation scheme can be employed to scatter travel time to rectangular (2D)/cubic (3D) grids.

This scheme should work well in shot domain Kirchhoff depth migration, where travel time maps for each station with reasonable aperture size can be calculated and stored on hard disks in advance. For migrating each shot, only relevant ravel time maps to this shot need to be loaded up to the memory, therefore overhead computer time for disk access can be limited.

Figure 2 is a practical velocity model with complicated structures, evidenced by higher velocity layers on the right side of the panel, and lower velocity layers on the left side of the panel. Figure 3 is the one-way travel time map from a source point for this model. It is clear that the travel time map models the given structure well. With travel times to depth points from source and from receivers, a depth migrated shot will thus be obtained.

We will show some shot domain depth migration results using this approach.

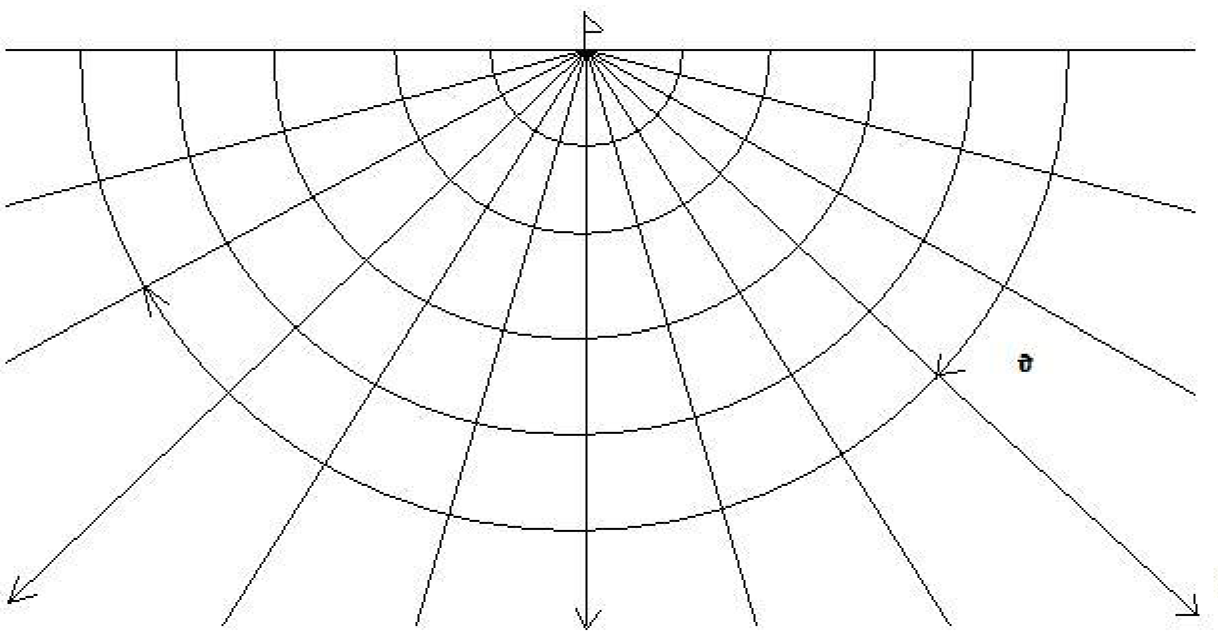

Fig. 1. A 2D grids in cylindrical coordinates

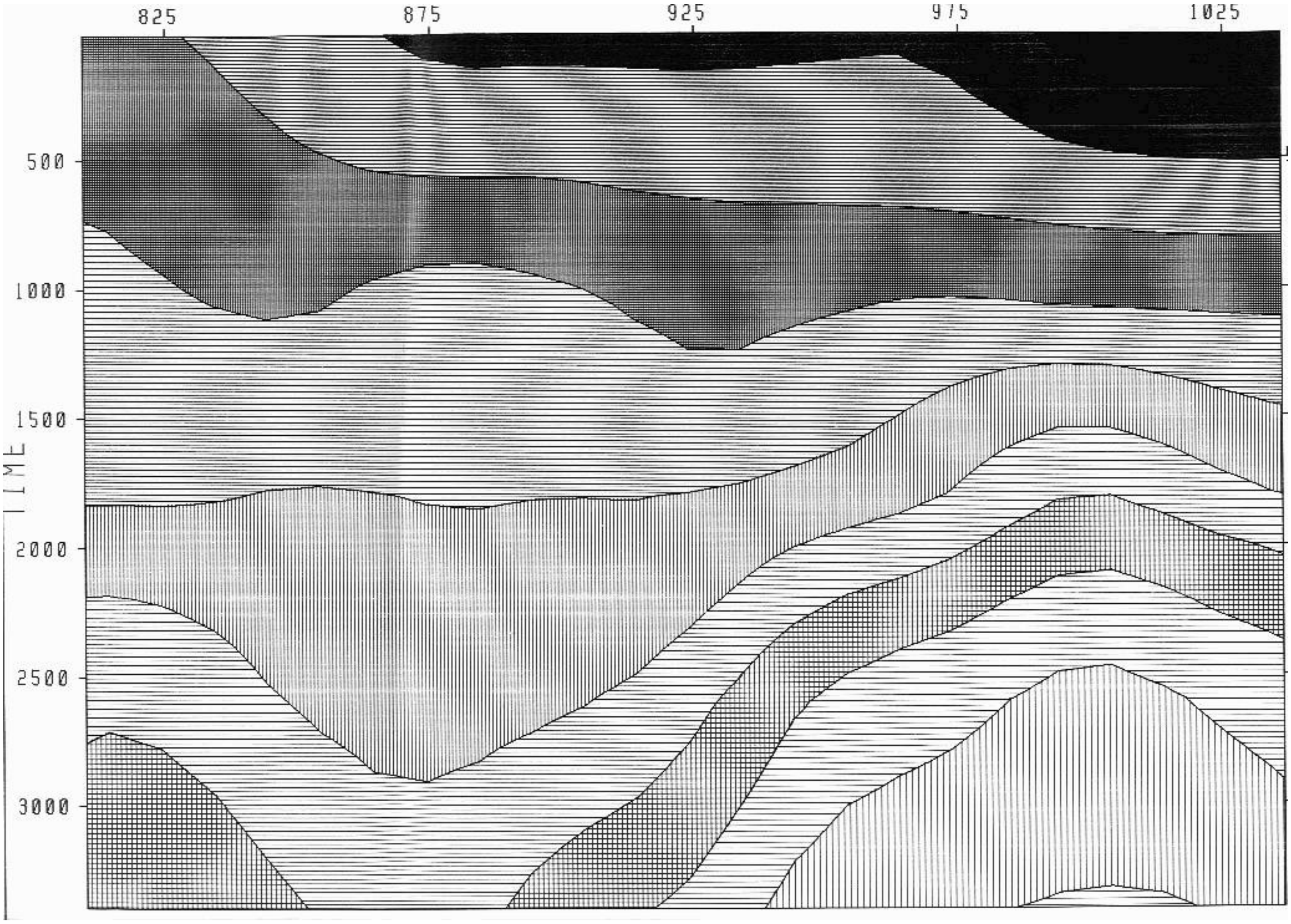

Fig. 2. A practical velocity model with higher velocity layers on the lower right side and lower velocity layers on the lower left side.

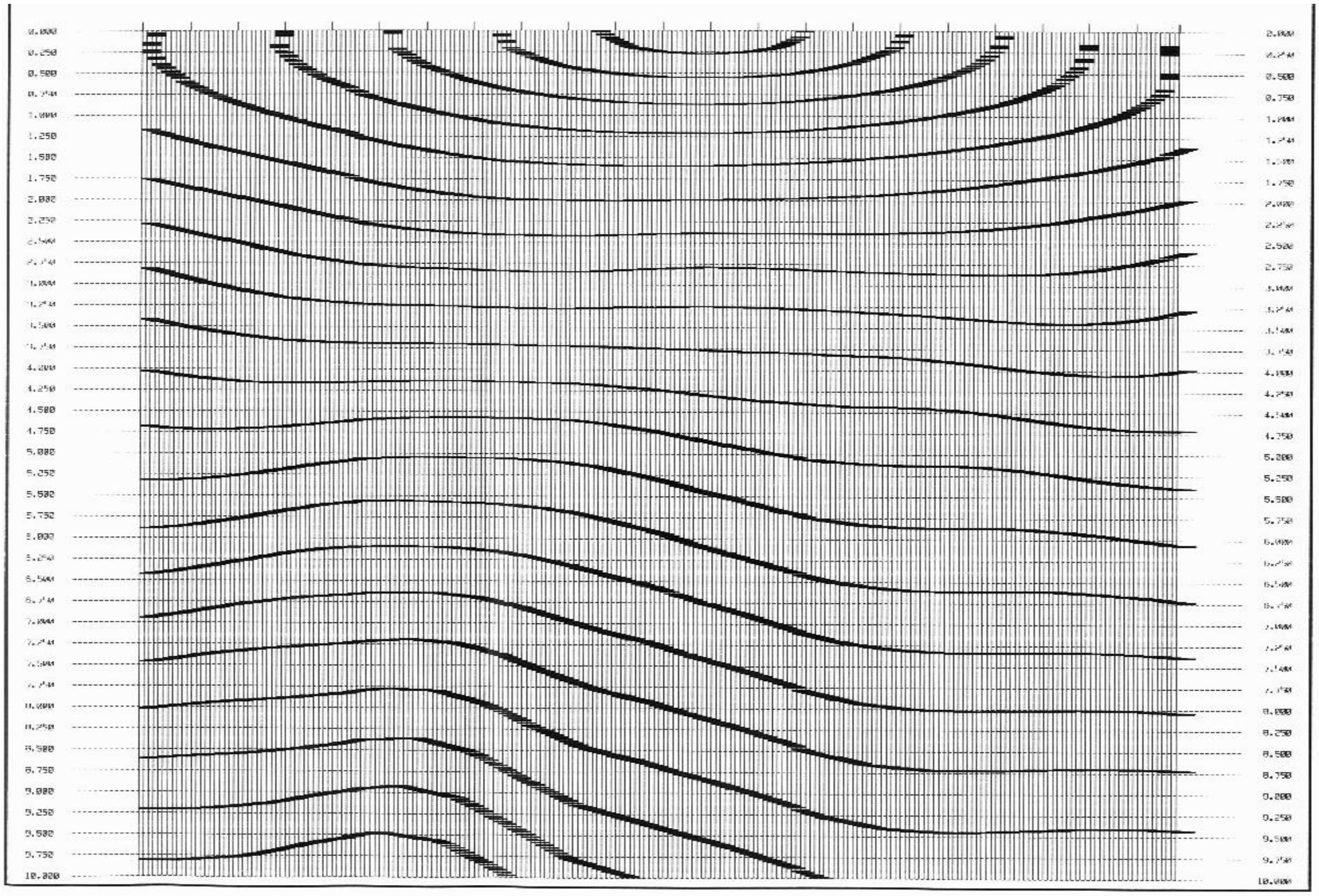

Fig. 3. One-way travel time map from a source in the middle on the surface. Time interval is 0.1 s.