# AVO inversion of multicomponent data for P and S impedance

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**Summary**

In this paper we investigate a method of inverting amplitudes of both PP reflections and converted PS reflections to P and S impedance. A program has been written to perform a joint AVO inversion on PP and PS pre-stack synthetic data in MATLAB. The program performs an AVO inversion for P and S-wave Impedance reflectivity utilizing a model-based, weighted stacking approach, and the total impedances are calculated by integration including missing low-frequency components from well log information. The inverted impedance results from PP or PS only inversion and joint (PP and PS) inversion are compared in presence of noise.

**Introduction**

The Aki and Richards approximation for the P-wave reflection coefficient, $R_{pp}$, and the reflection coefficient for a P-wave converting to an S-wave, $R_{ps}$, can be reformulated as function of P and S-wave impedance ( *I,J* ) (Larsen 1999) as:

$$R_{PP}(\theta_P) = A(\theta_P)\frac{\Delta I}{I} + B(\theta_P)\frac{\Delta J}{J} \tag{1}$$

$$R_{PS}(\theta_S,\varphi_S) = C(\theta_S,\varphi_S)\frac{\Delta I}{I} + D(\theta_S,\varphi_S)\frac{\Delta J}{J} \tag{2}$$

The coefficients A, B, C, and D are functions of the angle $\theta_P$ (the average of P-wave incident, and transmission angle) for the PP section, and angle $\theta_S$ (the average of P-wave incident and transmission angle and the angle $\varphi_S$ (the average of the S-wave reflection and transmission angle) for the PS section and the velocity model (rather than of the data itself).

Larsen (1999) outlined a least-square, weighted-stacking procedure incorporating both PP and PS seismic data to extract P and S-wave impedance reflectivity ($\Delta I/I$, $\Delta J/J$). In this method, imaged (i.e. prestack migrated or NMO corrected and post-stack migrated) common midpoint (CMP) (PP case) and common conversion-point (CCP) (PS case) gathers are weighted and stacked. The weights are derived from a smoothed background velocity model. If N is the number of offsets contributing to each imaged CMP or CCP gather at a particular time sample under consideration, the $\Delta I/I$ and $\Delta J/J$ are given by:

$$\Delta I/I \;=\; \sum_{k=offset} wI_{PP}(\theta_{Pk},\theta_{Sk},\varphi_{Sk})R_{PP}(\theta_{Pk}) + \sum_{k=offset} wI_{PS}(\theta_{Pk},\theta_{Sk},\varphi_{Sk})R_{PS}(\theta_{Sk},\varphi_{Sk}) \tag{3}$$

$$\Delta J/J \;=\; \sum_{k=offset} wJ_{PP}(\theta_{Pk},\theta_{Sk},\varphi_{Sk})R_{PP}(\theta_{Pk}) + \sum_{k=offset} wJ_{PS}(\theta_{Pk},\theta_{Sk},\varphi_{Sk})R_{PS}(\theta_{Sk},\varphi_{Sk}) \tag{4}$$

where $wI_{PP}$ , $wI_{PS}$ , $wJ_{PP}$ and $wJ_{PS}$ are the stacking weights, the function of coefficient A, B, C and D. The equations (3) and (4) have been coded in MATLAB to invert the reflection amplitudes to P and S-wave impedance reflectivity ($\Delta I/I$, $\Delta J/J$). This method has been demonstrated on real data by Larsen (1999), Larsen et al. (1999), Margrave et al. (2001), and Zhang et al (2003).

**Implementation**

Our joint inversion program is designed to have a PP and a PS gathers, as well as a velocity-depth model as input. The PP and PS gathers used here were synthetics created in SYNTH (CREWES MATLAB library). Both seismograms were created initially as broad-band responses and then convolved with the appropriate wavelet. The combination of PP and PS data in an inversion requires that the two data types be correlated in time or depth. We followed a procedure to correlate the PP and PS synthetic sections in depth. The angles $\theta_P$ , $\theta_S$ and $\varphi_S$ required to calculate stacking weights, are determined by ray tracing on a smooth background velocity model. Then for each depth sample the impedance reflectivity results, $\Delta I/I$ and $\Delta J/J$ were estimated using equations 3 and 4 respectively. In next step the estimated $\Delta I/I$ and $\Delta J/J$ were integrated to *I* and *J* using BLIMP routine. Band limited impedance (BLIMP, from CREWES MATLAB library) routine is a simple algorithm for band-limited impedance inversion.

To approximate the impedance of the subsurface using seismic data, it is necessary to account the band-limited nature of seismic data, especially for low-frequencies (Ferguson and Margrave, 1996). Their BLIMP algorithm uses impedance estimated from well logs to provide the missing low-frequency components of input seismic data. In this way we integrated $\Delta I/I$ and $\Delta J/J$ to *I* and *J* including the low-frequency component which significantly improves the inversion results.

**Synthetics examples**

Several models were used to test the inversion program. We present the inversion results of one sample model here. The examined model is shown in Figure (1). Both PP and PS synthetics have the same initial zero-phase (100 HZ Ricker) wavelet and the same offset range from 0 to 1000 m.

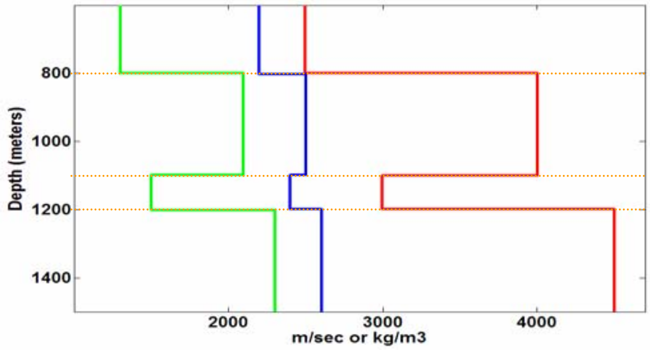


*Figure 1: A simple log model in depth.*

Figure (2) shows the estimated *I of* the simple logs model (Figure 1) by PP inversion only, PS inversion only and joint inversion. This figure clearly shows that adding low-frequency trend improves inversion results significantly.

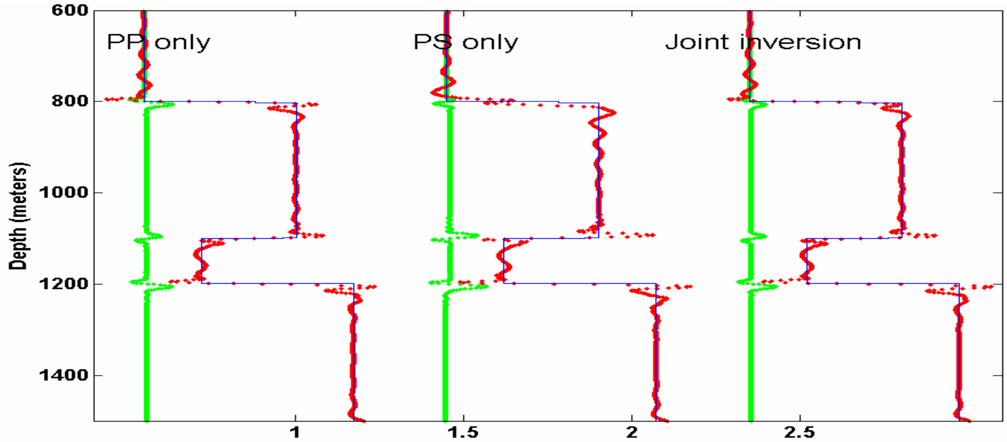


*Figure 2: P-wave Impedance (I) as estimated from PP inversion only (left), PS inversion only (middle) and joint inversion (right), for the log model in Figure 10a. In each plot, the red curve is estimated Impedance after including low-frequency content, green curve is estimated impedance before including low-frequency content, and the blue curve is the true impedance.*

In real seismic sections, the PP and PS data have different frequency content. Typically PS data have less frequency content compared to PP data. To simulate this, PP and PS synthetics of the real log model in Figure (3), were generated with different input wavelets. The zero-phase wavelet 5-10-80-100 (butterworth) is used for PP synthetics and zero-phase wavelet 5-10-30-40 is used for PS synthetics. The example comes from Blackfoot Field, owned and operated by Encana, in southeastern Alberta, Canada.

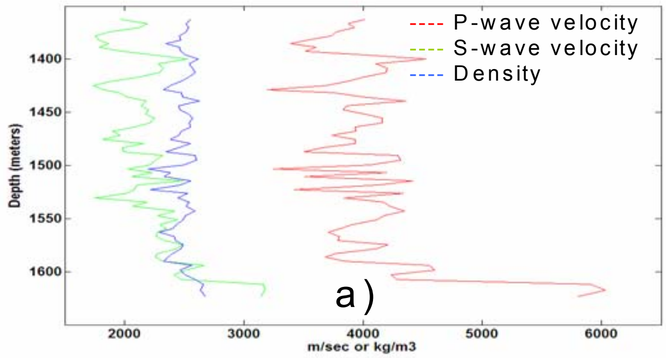


*Figure 3: Real well logs at Blackfoot Field.*

With synthetics gathers from the log model in Figure (3), the impedance estimation by joint inversion is examined (Figure 4). Impedance estimation has significantly improved by low-frequency restoration also a very good match between true impedances and estimated impedances are clear (Figure 4).

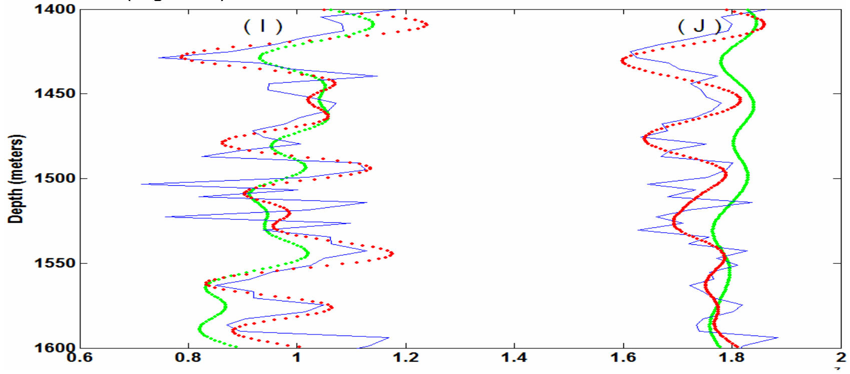


*Figure 4: P-wave Impedance (I) and S-wave impedance (J) as estimated from joint inversion. In each plot the red curve is estimated Impedance after including low-frequency content, the green curve is estimated impedance without including low-frequency content, and the blue curve is the true impedance.*

A careful look at Figure (4), reveals that the *J* estimation has less frequency content compared to the *I* estimation. To understand this, the stacking weights for joint inversion were examined. Figure 5 shows the stacking weights for estimation of Δ*J*/*J* . It can be seen that the PP weights are very small compared to weights applied to the PS section. Thus in joint version, PP data have a smaller influence on Δ*J*/*J* estimation than do PS data, which results in less frequency content in Δ*J*/*J* estimation. Also for Δ*I*/*I* estimation in joint inversion, the PS weights are small compared to the PP weights, thus PP data dominates in the Δ*I*/*I* estimation.

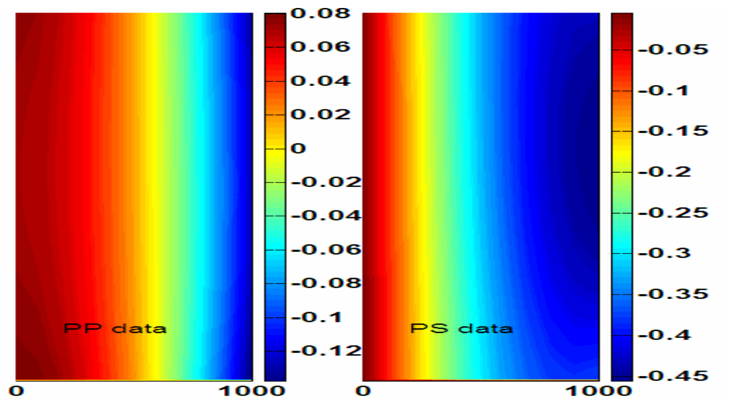


*Figure 5: The stacking weights required to estimate J from joint inversion method, for the log model in Figure 3.*

**Inversion results in presence of noise**

Any given seismic recording will contain some amount of noise in relation to the desired signal. To examine the effect of noise upon inversion accuracy, random noise was added to the synthetic data. In Figures (6)-(8), the *I* and *J* estimation of the log model in Figure (3) from noisy synthetics, are shown. In all these Figures, the red curve is estimated Δ*I*/*I* and Δ*J*/*J* from noise-free data, the green curve is estimated Δ*I*/*I* and Δ*J*/*J* from noisy data and the blue curve is the true Δ*I*/*I* and Δ*J*/*J*.

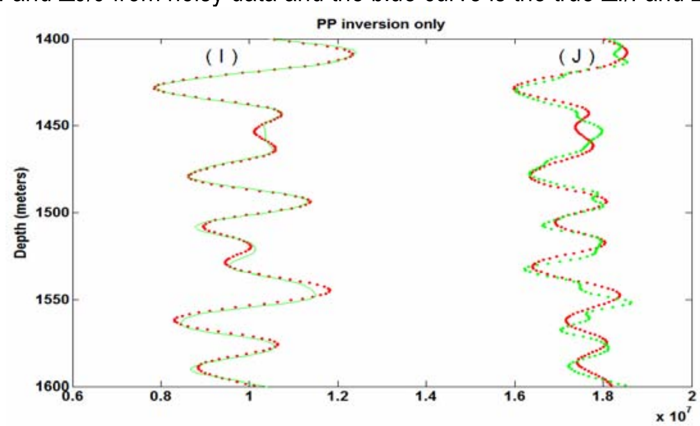


*Figure 6: I and J as estimated from PP inversion only for the log model in Figure (3) with signal-to-noise ratio of 2.*

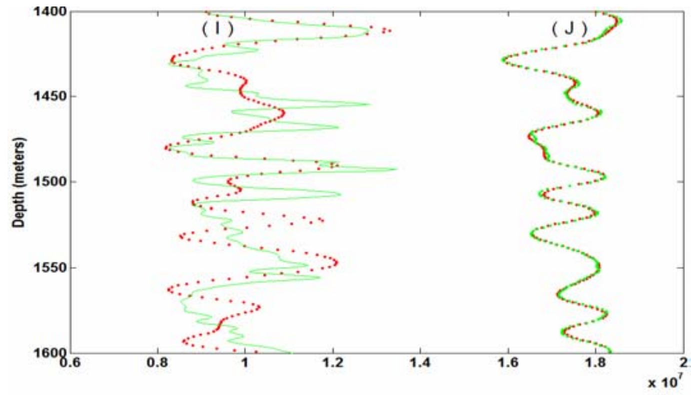


*Figure 7: I and J as estimated from PS inversion only for the log model in Figure (3) with signal-to-noise ratio of 2.*

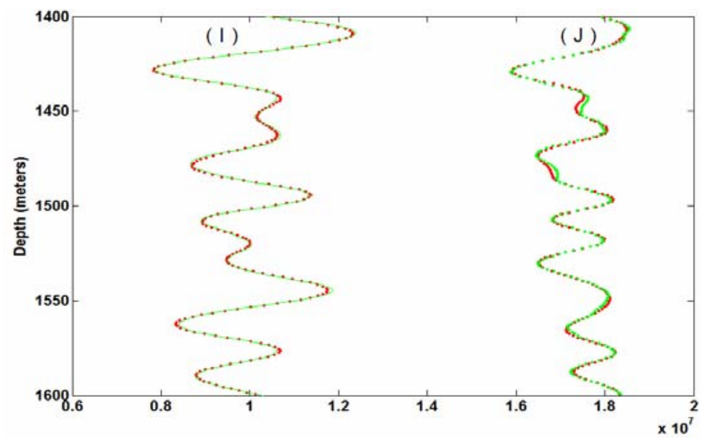


*Figure 8: I and J as estimated from joint inversion for the log model in Figure (3) with signal-to-noise ratio of 2.*

We observed in the presence of noise the PS inversion only is poor method for *I* estimation also the PP inversion only, is poor method for *J* estimation while joint inversion method has very good results for both *I* and *J* estimation in presence of noise. This is because the joint inversion has effectively double the fold of the other methods.

**Conclusion**

The joint inversion program described by Larson (1999) has been cast as a MATLAB algorithm. This algorithm includes some modifications to Larsen's approach: (1) adding low-frequency trend to band-limited data; and (2) deriving impedance estimation from P and S reflectivity. We showed that the including the low-frequency trend can provide the impedance estimation significantly better than that without including the low-frequency trend. We also showed that in presence of random noise, the joint inversion was significantly more accurate than other methods. This effect was the result of doubling the data fold input into estimation of P and S reflectivity.


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