Three term AVO waveform inversion

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Abstract

Density reflectivity is a desirable AVO attribute sought by the explorationist since it has the potential to say something about the fluid saturation. However accurate density reflectivity estimates are difficult to obtain due to the ill-conditioned nature of the inverse problem. A small amount of noise will lead to large errors in the estimates. To improve the stability of the inversion, large angles and offsets are required but these bring their own problems. One set of related problems is NMO stretch and offset dependent tuning. This paper develops and demonstrates an AVO waveform inversion that incorporates into its forward model these factors allowing for accurate estimates even in their presence. Well constraints and various regularization strategies are employed to further enhance the reliability of the solution.

Introduction

There is a great deal of interest in the exploration community in the use of long offset seismic data to perform three term AVO analysis to predict density reflectivity. This paper develops and demonstrates a three term AVO waveform inversion. By incorporating the waveform and the NMO operator, NMO stretch and offset dependent tuning can be modeled as part of the inverse problem leading to more accurate estimates of the reflectivity. The input seismic data can be either NMO corrected or left uncorrected.

Van Koughnet et al. (2003) published a series of examples from the Gulf Coast showing that density reflectivity can be practically solved for and used in an exploration environment. The technique requires data with good signal to noise, large angles and offsets. Unfortunately data recorded at these offsets often have amplitude and character distortions introduced from the wave propagation and processing. One such distortion is NMO stretch and the related effect offset dependant tuning. Dong (1999) quantified the affect of both these on AVO. Downton et al. (2003) showed these affects were particularly problematic for class III and IV AVO anomalies. Swan (1997) suggested a way of correcting for NMO stretch but the technique is only applicable for two term AVO inversion. Downton and Lines (2002, 2003) proposed a waveform inversion to correct for NMO stretch and offset dependent tuning demonstrating again a two-term avo model. This paper extends the approach to three-terms.

In the first part of this paper the relevant theory is developed using a Bayesian formalism. The likelihood function is developed assuming Gaussian statistics and extends the AVO waveform inversion of Downton and Lines (2003) to three terms. This original formulation assumes the seismic data has not been NMO corrected. A modification is introduced allowing NMO corrected seismic data to be used as input to the inversion. Because of the band limited nature of the seismic data the inverse problem is underdetermined necessitating the use of constraints. Constraints from well control are introduced which help establish the relationship between the different parameters solved for. Based on this *a priori* information, a change of variables is performed so that the parameters solved for are statistically independent. After the change of variables the problem is still undetermined so the problem needs to be regularized. Similar to Downton and Lines (2003) this can be done by choosing a weighting function that treats certain reflection coefficients as being more reliable than others. Choosing a long tailed *a priori* distribution leads to such a weighting function. This leads to a nonlinear inversion which is solved using conjugate gradient. The number of iterations used in solving the conjugate gradient algorithm also acts as a regularization parameter.

The algorithm is demonstrated on both synthetic and real seismic data. The synthetic example was constructed so as to include NMO stretch and offset dependent tuning that would bias the AVO estimates for a traditional AVO inversion. Good estimates of all the reflectivity, including the density, are obtained even when the density reflectivity is uncorrelated with either the P-wave or S-wave velocity reflectivity at noise levels typical in real seismic data. The algorithm accurately estimates the reflectivity even on events undergoing NMO stretch and differential tuning. The seismic data example demonstrates how the algorithm successfully differentiated a known density anomaly at two well locations.

Theory

The convolutional model is used as the basis for the likelihood model. This model assumes the earth is composed of a series of flat, homogenous, isotropic layers. Ray tracing is done to map the relationship between the angle of incidence and offset. Transmission losses, converted waves, and multiples are not incorporated in this model and so must be addressed through prior processing. In theory, gain corrections such as spherical divergence, absorption, directivity, and array corrections can be incorporated into this model, but are not considered for brevity and simplicity, so must be previously applied in the processing. Any linear approximation of the Zoeppritz equations may be used as the starting point for this derivation.

Downton and Lines (2002) use the two-term Fatti approximation to develop a waveform inversion. This formulation extended to three terms is

$$\begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_N \end{bmatrix} = \begin{bmatrix} \mathbf{WN}_1\mathbf{F}_1 & \mathbf{WN}_1\mathbf{G}_1 & \mathbf{WN}_1\mathbf{H}_1 \\ \vdots & \vdots & \vdots \\ \mathbf{WN}_N\mathbf{F}_N & \mathbf{WN}_N\mathbf{G}_N & \mathbf{WN}_N\mathbf{H}_N \end{bmatrix} \begin{bmatrix} \mathbf{r}_P \\ \mathbf{r}_S \\ \mathbf{r}_d \end{bmatrix}, \quad (1)$$

where $\mathbf{r}_p$,$\mathbf{r}_s$,$\mathbf{r}_d$ are the P- and S-velocity, and density reflectivity respectively. These are all vectors whose elements correspond to different time samples. Likewise the elements of the data vector $\mathbf{d}_n$ represent the processed seismic data for the n[th] offset for the corresponding time samples. The block matrices describe the physics of the problem. The matrices **F**, **G**, and **H** are diagonal matrices that contain weights that describe how the amplitude changes as a function of offset. These weights follow from the three-term Aki and Richards equation (1980, equation 5.44). Following Claerbout (1992), the block matrix $\mathbf{N}_n$ performs NMO. This operator can be constructed using whatever offset traveltime relationship one desires. In order to invert data at large angles of incidence, it is important to correctly position the event without introducing residual NMO. In this case, a higher order correction is used following Castle (1994). This has the advantage of introducing high-order terms without introducing the theoretical complications of intrinsic anisotropy. Implicit in this derivation is that the velocity is known *a priori* and that static corrections are applied. Lastly, **W** is a convolution matrix which contains the source wavelet. Applying these three operators in series, the block matrices $\mathbf{F}_n$,$\mathbf{G}_n$,$\mathbf{H}_n$ model the offset dependent reflectivity from the zero offset reflectivity, $\mathbf{N}_n$ applies NMO and **W** convolves the offset dependent reflectivity with the source wavelet modeling the band limited seismic data with NMO. The inversion of equation (1) can be thought of as three separate inversion problems, deconvolution, inverse NMO and AVO inversion.

Data that has been previously NMO corrected may be input to this algorithm with a slight modification. In processing, for stability reasons, the inverse of NMO is practically never done, instead its conjugate is applied (Claerbout, 1992). NMO processing may be simulated by applying the conjugate NMO operator to both the left and right hand sides of equation (1) resulting in

$$\begin{bmatrix} \mathbf{d'}_1 \\ \vdots \\ \mathbf{d'}_N \end{bmatrix} = \begin{bmatrix} \mathbf{N}_1^T\mathbf{d}_1 \\ \vdots \\ \mathbf{N}_N^T\mathbf{d}_N \end{bmatrix} = \begin{bmatrix} \mathbf{N}_1^T\mathbf{WN}_1\mathbf{F}_1 & \mathbf{N}_1^T\mathbf{WN}_1\mathbf{G}_1 & \mathbf{N}_1^T\mathbf{WN}_1\mathbf{H}_1 \\ \vdots & \vdots & \vdots \\ \mathbf{N}_N^T\mathbf{WN}_N\mathbf{F}_N & \mathbf{N}_N^T\mathbf{WN}_N\mathbf{G}_N & \mathbf{N}_N^T\mathbf{WN}_N\mathbf{H}_N \end{bmatrix} \begin{bmatrix} \mathbf{r}_P \\ \mathbf{r}_S \\ \mathbf{r}_d \end{bmatrix}, \quad (2)$$

where $\mathbf{d}_n$' is the NMO corrected data for the n[th] offset. Note that $\mathbf{N}_n^T\mathbf{WN}_n$ is not an identity matrix. This operator is responsible for NMO stretch and offset dependent tuning. By applying the inverse of this operator these artefacts can be removed. Depending on the processing performed to the seismic either equation (1) or (2), may be inverted with similar results. For future reference and simplicity, the linear model (Equation 1 or 2 as appropriate) is written as

$$\mathbf{d} = \mathbf{Lm}, \quad (3)$$

where **L** is the linear operator, **m** is the unknown reflectivity vector¸ and **d** is the seismic data before or after NMO correction as appropriate.

The regularization introduced in the next section requires that the parameters being solved for are statistically independent. Based on empirical rock physical relationships this is clearly not the case for the above parameterization. Castagna et al. (1984) showed that for clastics the S-wave velocity of the rock is highly correlated with its P-wave velocity. Likewise the Gardner et al. (1974) relationship makes use of the fact the density and P-wave velocity are highly correlated. Assuming Gaussian statistics, Downton and Lines (2001) showed that these correlations can be described by a 3×3 covariance matrix. Other probability distributions can be mimiced using a suitable weighting matrix in calculation of the covariance from the well log statistics. If it is assumed, as is typically done in deconvolution, that the reflectivity time samples are ergodic and independent then this covariance matrix may trivially be extended to N time samples resulting in a 3N×3N sparse covariance matrix $\mathbf{C}_m$. This matrix describes the correlations between the different variables. A transform matrix **m=Vm**' which does a change of variables to independent variables **m'** is calculated by doing an eigenvector analysis of the covariance matrix $\mathbf{C}_m = \mathbf{V\Lambda V}^T$. The eigenvectors form the transform matrix while the eigenvalues describe the expected variance of the tranformed variables. Since stationarity has been assumed, there are 3 distinct eigenvalues σ_1^2,σ_2^2, and σ_3^2 corresponding to the variance of the three transformed variables. Under the change of variables equation (3) becomes

$$\mathbf{d} = \mathbf{L'm'}, \quad (4)$$

where **L'=LV**.

In equation (1) or (3) both the matrices **W** and **N** are typically underdetermined or ill-conditioned. This is due to the fact the data is band-limited and the differential tuning introduces null spaces into the NMO operator **N**. Because of this, the problem needs to be regularized. Similar to Downton and Lines (2003) this is done by choosing a weighting function that treats certain reflection coefficients as being more reliable than others. Choosing a long tailed *a priori* distribution leads to such a weighting function.

A long tailed distribution or sparse reflectivity may be argued for based on physical arguments. The P-wave impedance reflectivity may be modeled as a long tailed distribution, such as the L1 distribution (Levy and Fullagar, 1981; Shapiro and Hubral, 1999). Under the change of variables the second variable is similar to the fluid factor. The fluid factor reflectivity is sparse by its nature since it only responds to anomalous fluids or large changes in lithology. The third variable is similar to a difference between the scaled density and the velocity reacting to places where the density is uncorelated to the velocity. The reflectivity of this will be sparse as well. After the change of variables in the proceeding section, the variables are independent so the resulting parameter covariance matrix is diagonal. Thus the three reflectivity series can be modeled by a variety of distributions including the Huber, Cauchy or L_p norm. The Cauchy distribution leads to a weighting a diagonal matrix **Q** whose elements are defined by

$$Q_{kk} = \frac{1}{\sigma_1^2}\begin{cases} \dfrac{1}{\left(\dfrac{\mathrm{m'}_n^2}{2\sigma_1^2}+1\right)} m'_n & n \le \dfrac{N}{3} \\ \dfrac{\sigma_1^2}{\sigma_2^2}\dfrac{1}{\left(\dfrac{\mathrm{m'}_n^2}{2\sigma_2^2}+1\right)} m'_n & \dfrac{N}{3} < n \le \dfrac{2N}{3}. \\ \dfrac{\sigma_1^2}{\sigma_3^2}\dfrac{1}{\left(\dfrac{\mathrm{m'}_n^2}{2\sigma_3^2}+1\right)} m'_n & \dfrac{2N}{3} < n \le N \end{cases} \tag{5}$$

These weights rely on parameter estimates themselves and so must be calculated in a bootstrap fashion. The methodology is similar to that of Sacchi and Ulrych (1995).

The optimal solution is found with the aid of Bayes' theorem. Assuming uniform uncorrelated Gaussian noise the likelihood function is Gaussian with the linear model is defined by Equation (4). This is combined with the weights coming from equation (5) leading to the nonlinear constrained least squares solution

$$\left[\mathbf{L'^T\, L'} + \mu\mathbf{Q}\right]\mathbf{m'} = \mathbf{L'}^T\,\mathbf{d}, \tag{6}$$

where μ is the noise-to-signal ratio which may be estimated from the data itself. There are two sources of nonlinearity in Equation (6), the estimate of the regularization parameter μ and the calculation of **Q**. The matrix **Q**, equation (5) requires previous estimates of **m**' to construct the diagonal weighting terms. This must be done in a bootstrap fashion. Since the actual inverse problem being solved is large, it is most efficiently solved using iterative techniques such as conjugate gradient (Skewchuk, 1994). Solving the inverse problem requires two nested loops. In the inner loop the conjugate gradient algorithm is used to solve Equation (6) using the previously calculated values of **m** and **Q**. The maximum number of conjugate gradient iterations is used as a parameter to help stabilize the solution (Hansen, 1998). After solving for the reflectivity the estimate of **m** and the covariance matrix **Q** is updated. Iteratively updating these parameters and re-estimating the reflectivity parameters constitute the outer loop. Generally a satisfactory sparse solution is obtained after 3 to 5 outer loops. For the first loop the inversion is run as an unconstrained inversion by setting **m=0**. Care must be taken in the first outer loop not to put too much detail in the solution or the problem will not converge. This can be controlled by carefully setting the maximum number of conjugate gradient iterations parameter to a value that limits resolution.

Examples

Based on the results of Downton et al. (2003) a synthetic model was constructed (Figure 1) that included a class III and IV AVO anomaly undergoing NMO stretch at 1.5 and 1.7 seconds, a class III and IV AVO anomaly undergoing offset dependent tuning at 1.6 and 1.8 seconds and density anomaly at 1.9 seconds. The rest of the reflectors followed the mudrock and Gardner relationships. Noise was added to give a signal-to-noise ratio of 10:1. The NMO corrected gathers were taken through the inversion scheme, inverting angles up to 55 degress resulting in Figure 2. Note that the events undergoing stretch and offset dependent tuning are well estimated. The density anomaly at 1.9 seconds is acurately estimated even though it is completely uncorrelated with the velocity.

The real data example (Figure 3) is a line shot over two Halfway anomalies. The Halfway sand shows up as bright spots at around 0.72 seconds. This line was inverted using the AVO waveform inversion outlined in this paper. The resulting density reflectivity differentiates between the producing pool (wells C and E) and the uneconomic low gas saturated well A.

Conclusions

This paper developed and demonstrated a three-term AVO waveform inversion that works on data with NMO stretch and offset dependent tuning. Constraints are used to regularize the problem addressing the underdeterminedness and helping to stabilize the inversion in the presence of noise. The algorithm is successfully demonstrated on synthetic data that exhibit NMO stretch and offset dependent tuning that would bias a traditional three term AVO inversion. Density reflectivity is successfully estimated even when it is uncorrelated with the velocity reflectivity for signal-to-noise ratios typical for real seismic data.

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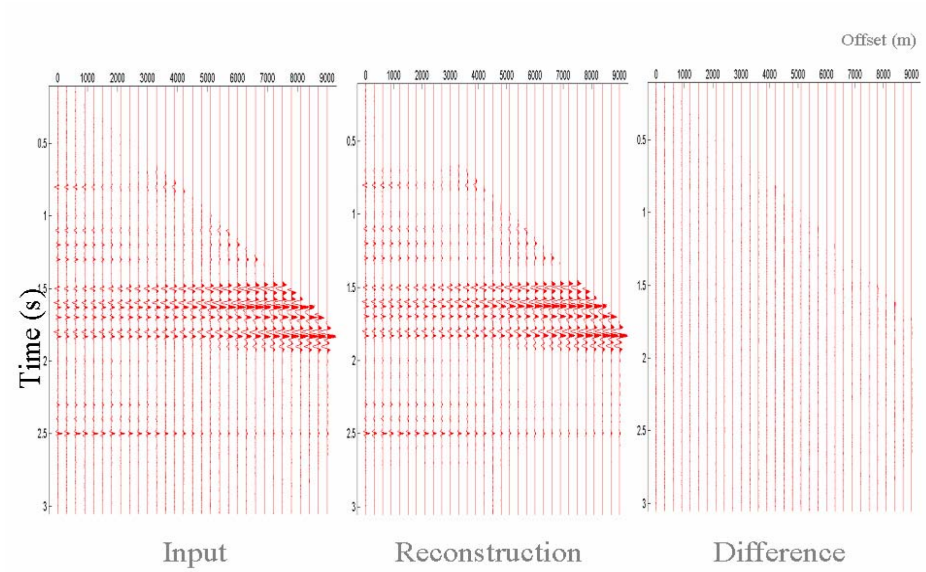

Figure 1: Input with NMO stretch and offset dependent tuning, model from estimated parmaters and difference

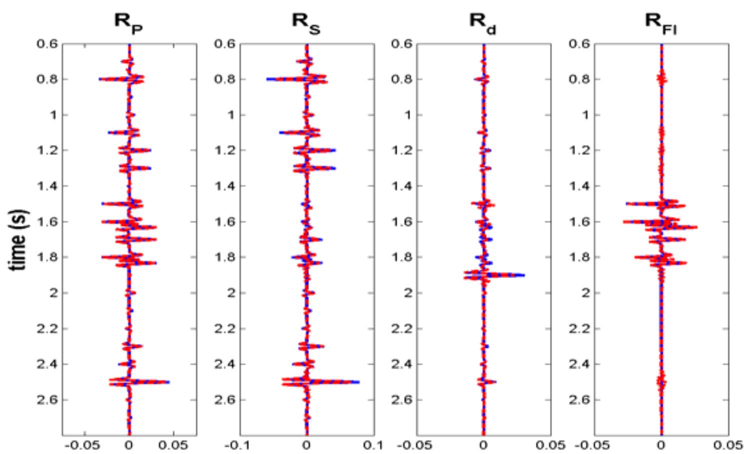

Fig. 2. The estimate of reflectivity (red) versus the ideal (blue). R_p, R_s, R_d and R_{fl} are the P-and S-impedance, density and fluid stack reflectivity respectively

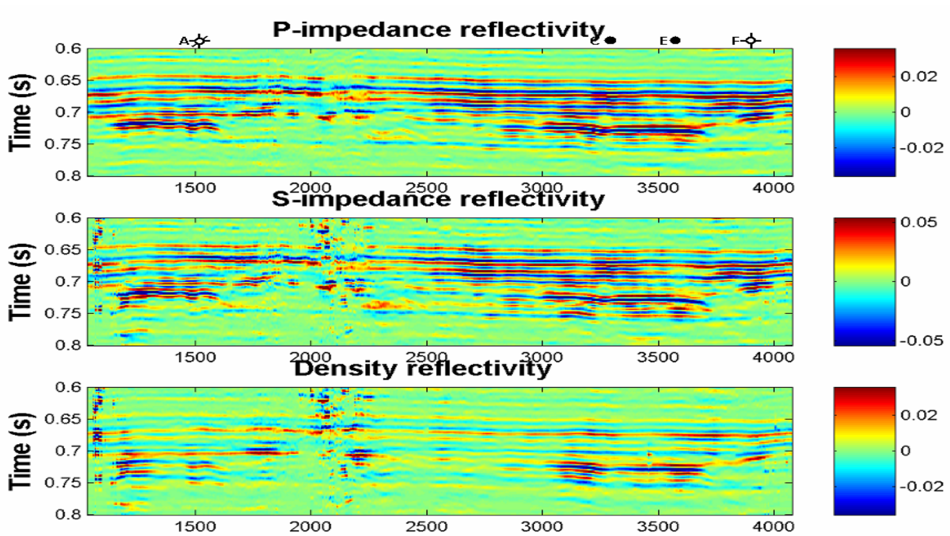

Fig. 3. Results of AVO waveform inversion.

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