High Resolution Radon Transform in the T-X Domain

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2004 CSEG National Convention

Summary

This paper describes a technique for computing a high resolution Radon transform of velocity gathers. First, it computes a 'preliminary' forward Radon transform in the t-x domain using the Gauss-Seidel method described in Yilmaz (1994). Next, it incorporates a priori information into the 'current' Radon tranform calculation by measuring the energy of the t-p traces from the 'preliminary' transform; then it prioritizes the p trace estimation sequence in the current transform according to the descending energy of the 'preliminary' p traces.

Introduction

The Radon transform has been one of the most widely used methods in attenuating multiples in seismic signal processing over the past two decades. An ongoing interest is to obtain a high resolution Radon transform to identify and discriminate multiples from the primary reflections. When the offset of a CMP t-x gather becomes smaller, the clustering of the energy in the Radon transformed tau-p domain becomes more smeared. Thorson et al. (1985) suggested a time domain hyperbolic least-squares method which can give a high resolution result at the expense of very large matrix computation. Hampson (1986) proposed a much more efficient frequency domain least-squares parabolic transform method which immediately became the industry standard, but it is rather smeared in the tau-p domain. Bradshaw et al. (1987, unpublished) and Yilmaz et al. (1994) used a fast time domain semblance weighted Gauss-Seidel method to approximate Thorson's solution, and the trajectories can be either parabolic or hyperbolic. But the drawback of the Gauss-Seidel is that the result can be biased towards the earlier sequential order of the p parameter. Otherwise, it can give very high resolution results. To solve this bias problem is the motivation of this paper. Sacchi et al. (1995) improved Hampson's frequency method by using the previous transform estimation as a priori information giving a high resolution result, and his technique has become quite popular. The downside is that it is not time varying by nature and can over emphasize the whole p trace (Trad, 2003); although it can give sparse results, the energy clusters are not necessarily at the optimal p locations (Cary, 1998). The proposed method here seems to yield higher resolution results than the high resolution frequency method, and it seems to focus the energy at more optimal p locations. It is computationally simple, accommodates both parabolic and hyperbolic trajectory implementations, and it is time varying due to its use of the time domain semblance weights.

Methods

The following describes one iteration of the time domain semblance weighted Gauss-Seidel forward Radon transform method (Bradshaw at el., 1987, Yilmaz et al., 1994, p.978):

1. Stack the t-x data gather along the p trajectory and weight (and/or threshold) that stack trace by the semblance associated with that p trajectory. This will give a p trace estimate.
2. Compute the inverse Radon tranform of this particular p trace estimate and subtract it from the t-x data gather which is now gradually degenerating to a t-x residual gather as iteration continues.
3. Accumulate the p trace estimate in the tau-p domain at the p trace location.
4. Increment to the next p value and repeat steps 1 to 4 until all p values are covered.

Usually, about three Gauss-Seidel iterations are sufficient to reduce the t-x residual gather energy below a certain level, and then the iterating can be stopped. After this process is completed, the accumulated p trace estimate in the tau-p domain is the forward Radon transform.

The method in this paper incorporates the Bayesian a priori notion proposed by Sacchi at el. (1995, equation 26) and Trad et al. (2003, equation 3). Sacchi improved Hampson's frequency method by adding a non-constant pre-whitening or damping to the diagonal of the covariance matrix [L^TL + diagonal]. The diagonal terms are the scaled reciprocal of the clustering energy of the p traces from the previous 'preliminary' transform. This in turn enhances the more powerful p traces and damps down the the weaker ones giving a high resolution Radon transform result. This is an application of the iteratively re-weighted least-squares (IRLS) algorithm.

In the time domain Gauss-Seidel method, a similar concept of utilizing the p trace energies from the previous transform as the Bayesian a priori information is used: the p sequence in step 4 is prioritized according to the descending order of the p trace energy. In summary: first, a forward Radon transform according to increasing sequential p parameters in the Gauss-Seidel method described above is computed; second, the energy of each p trace is measured; lastly, the forward Radon transform is re-computed, where this time the p parameters are computed in order of the descending power of the p trace from the previous transform. The way this last step is carried out ensures that the stronger energy will get transformed and removed from the t-x residual gather first.

Data Examples

The objective of the example is to show the parabolic Radon transform results of four different methods.

Fig. 1 is the synthetic input velocity-corrected gather with 75 offsets labeled on the top. The three events are of equal amplitude: the top event at 400 ms is a flat event; the middle event at 1500 ms is a hyperbolic event, and the bottom event at 1900 ms is a parabolic event.

Fig. 2 shows the Radon transform of the frequency domain least-squares Hampson's method. Note the smeared character of the result.

Fig. 3 is the Radon transform of the high resolution frequency domain least-squares Sacchi method. It shows an improvement in focusing the p traces over the Hampson's method.

Fig. 4 shows the Radon transform of the time domain semblance-weighted Gauss-Seidel Bradshaw/Yilmaz method. It demonstrates the smearing is biased towards those p traces which were computed first in the estimation sequence.

Fig 5 shows the Radon transform of the high resolution time domain semblance-weighted Gauss-Seidel method. It gives the most focused results. The slight smearing of the event at 1500 ms is due to the fact that a parabolic transform is being used to model a hyperbolic event.

Discussion

The smearing effect present in any Radon transform arises because of the limited range of offsets present in the input t-x gather. When the maximum offset becomes sufficiently large (assuming uniform offset sampling), all of the discussed techniques will produce focused results due to the enhanced statistics.

Those who are well-versed in the Gauss-Seidel technique will appreciate that the Bradshaw/Yilmaz method corresponds to "the early bird gets the worm" philosophy, whereas the new approach suggests that the "largest worm should always be eaten first" (assuming one wants to eat worms).

Acknowledgements

We would like to acknowledge Geo-X Systems Ltd. for allowing us to publish this work.

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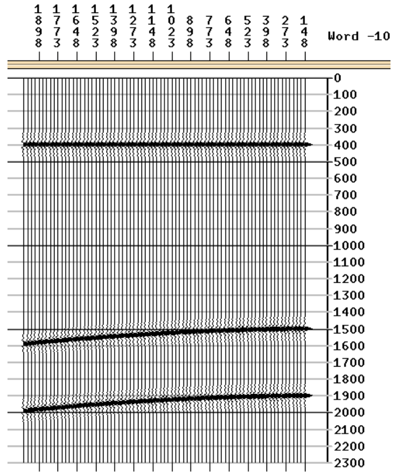

Fig. 1: Input CMP gather in t-x domain

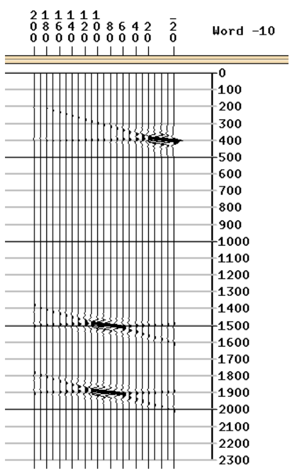

Fig. 2: RT by freq domain LS

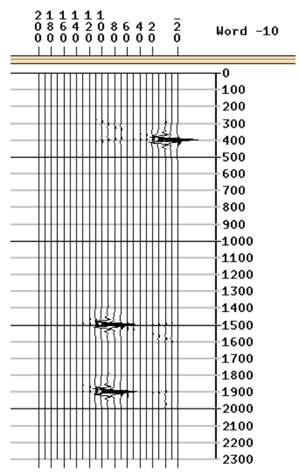

Fig. 3: RT by high-res freq domain LS

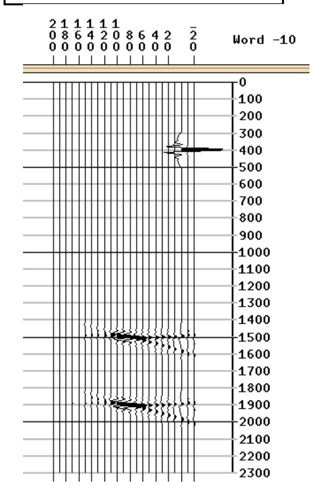

Fig. 4: RT by time domain Gauss-Seidel

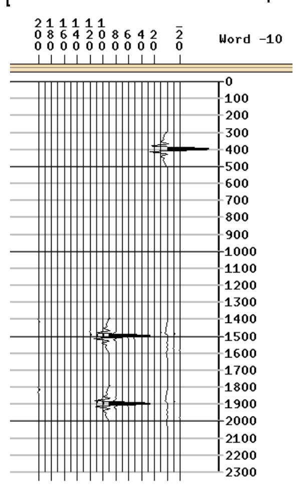

Fig. 5: RT by time domain high-res Gauss Seidel