

# Bad Stacking Velocities in the Presence of Anomaly and their explanation.

Emil Blías, Revolution Geoservices Inc., Calgary, Canada

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## Abstract

A depth velocity model determination is one of the most important problems in seismic processing and interpretation. A velocity model, beyond its initial purpose to obtain a seismic stack, is used for depth migration, AVO analysis and inversion, pore pressure prediction and so on. Before a well is drilled, seismic data provides the only information for the velocity. From seismic data we can directly determine only stacking velocities, which give us the best image in time domain. Dix's formula gives us interval velocities in 1-D medium. In many cases the 1D assumption does not work, especially when we have the overburden local velocity anomalies. Not only do they reduce post-stack image quality, but also create big difference in stacking velocity behaviour for deep seismic reflectors with small dips. In this paper, we will see how we can analytically describe the connection between interval velocities, shallow anomaly and stacking velocities. This connection will provide us with clearer understanding of so called "anomalous" behavior of stacking velocities.

## Introduction

In the presence of a shallow velocity anomaly, we can often see an "anomalous" lateral oscillation of stacking velocities, increasing with reflector depth. In the 1D case, the stacking velocity is close to RMS velocity, which can be considered as a kind of average velocity. When we think about average velocity, we usually mean that this velocity is bounded by the minimum and maximum interval velocity above. The "anomalous" lateral behavior of the stacking velocity means that its value is beyond the minimum and maximum interval velocities. We expect the stacking velocity (many processors in everyday conversation even call it RMS velocity) to behave as kind of average velocity. That is, if we have a locally slow velocity in some layer, we expect to see slower stacking velocity and vice versa. The thicker a specific layer is, the greater is its influence on the average value. This is true if the velocity anomaly is in a deep layer, but if the anomaly is shallow then the stacking velocity is "spoiled" – it does not correlate with average velocity at all. If we have a slow shallow velocity anomaly, the stacking velocity response may show slow velocity for shallow horizons (which corresponds to the idea that stacking velocity is kind of average velocity above the reflector), but fast velocity for deep horizons (which completely contradicts this idea).

## Some main questions about stacking velocity

As was described above, we often expect stacking velocity to be directly connected with some kind of average velocity over the reflector: "Because one wants the stacking velocities to approximate real velocities", Blackburn (1980, p. 1466). When we don't see this connection (and we don't see it when in the presence of shallow overburden velocity anomaly) we consider this as error in the stacking velocity. We can see this word "error" in the name of the papers (Blackburn, 1980) or in the papers: "In the presence of velocity anomalies, stacking velocities show systematic errors" Armstrong et al. (2001, p. 81). This "anomalous" behavior even leads to the statements, that "Stacking velocities, as derived from NMO correction of CDP gathers, need have no physical significance to the true velocity distribution below the gather location" Blackburn (1980, p.1466).

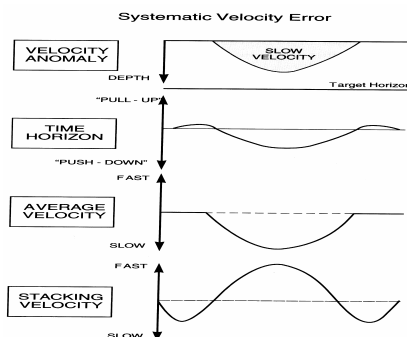


Fig.1 Problem with stacking velocity

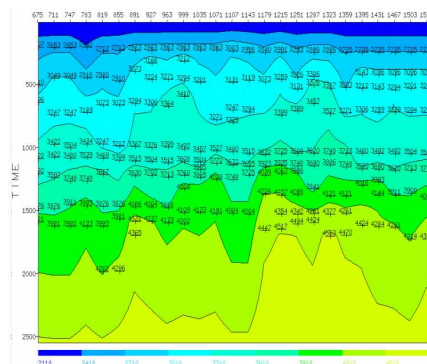


Fig. 2 a. Stacking velocity

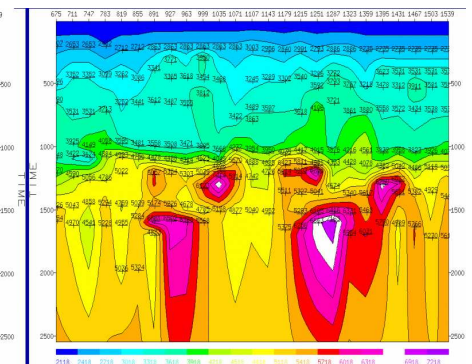


Fig. 2b. Dix's interval velocity

The connection between stacking and interval velocities has been considered over the last 50 years by many authors (the review of this problem can be found in Bergler et al, 2001) but the formulae (except for the medium with homogeneous horizontal layers) are quite complicated and require zero-offset ray parameters, which have to be numerically calculated. Fig. 1 is taken from the paper of Armstrong et al, (2002) and the text under this figure says: “A near-surface, slow-velocity anomaly introduces push-down in time at the target horizon on a CMP stack, but the stacking velocity response shows a fast velocity inside the anomaly and slow velocity on either side of the anomaly.” Looking at this picture, we can ask ourselves a question – why do we have this strange behaviour of stacking velocities, which seems not to correspond with average velocity at all? Fig. 2a shows typical behavior of stacking velocity in the presence of shallow anomalies. If we use Dix’s formula to calculate interval velocities, we will have the velocity grid, shown in the fig. 2b. It’s obvious that in this case we cannot use Dix’s formula. The question is “why”?

We can ask more questions, such as:

- (i) Should we consider this “strange” behaviour of stacking velocities as an error?
- (ii) If it’s not an error, what kind of connection can we expect between the stacking velocity and interval velocities.
- (iii) Is this behaviour really caused by non-hyperbolic moveouts, as we can find in some papers “This follows because the origin of the anomalous response is non-hyperbolic moveout within CMP gathers introduced by time delays caused by the velocity anomalies”, Armstrong et al. (2001, p. 82) or is there another reason?
- (iv) Can we use these “errors” to recover shallow velocity anomaly anomalies and to build a depth velocity model?

All these questions (and some others, related to this problem) have answers. One can try to find the answers through modeling different situations (Blackburn, 1980, Pickard, 1992) and make some general rules, but model studies show that this is not an easy task (Blackburn, 1980). To answer all these questions, we will use an explicit formula, which connects interval and stacking velocities when we have a lateral changes in the velocity. Using a method, developed by Blias (1981) and Blias (1987), the explicit formulae, connecting laterally changing interval velocities to stacking velocities, have been obtained. This formula is valid for an arbitrary number of anomaly layers, but for the sake of simplicity (and space) sake, we will write this formula when we have one layer with the velocity anomaly.

First let us know that we can use two descriptions of the lateral velocity anomaly. The first description uses two homogeneous layers, separated by a curvilinear boundary, as in fig. 2. We also can use a model with one layer with laterally changing velocity  $v(x)$ . We will consider both models. For the first model the explicit connection between NMO velocity and interval velocities is given by the formula (Blias, 1981):

$$1/V_{NMO}^2 = (1/V_{RMS}^2) [1 + (1/v_m - 1/v_{m+1}) F_m''(x) a_{mn}] \quad (1)$$

where

$$a_{mn} = \left( \sum_{i=m+1}^n h_i v_i \right)^2 / \left( \sum_{i=1}^n h_i v_i \right) \quad (2)$$

Here  $V_{NMO}$  is NMO velocity (parameter from hyperbolic approximation for relatively short offsets),  $F_m(x)$  is a curvilinear boundary between two layers, which creates shallow velocity anomaly,  $v_m$  is interval velocity and  $h_m$  is a thickness of  $m$ -th layer;  $n$  is a horizontal reflector number. The exact value of NMO velocity depends on the acquisition geometry.

For the second kind of shallow velocity anomaly description, we can derive an analogous formula:

$$1/V_{NMO}^2 = 1/V_{RMS}^2 (1 - h_m v_m'' b_{mn}/v_m^2) \quad (3)$$

where

$$b_{mn} \approx \left( \sum_{i=m}^n h_i v_i \right)^2 / \left( \sum_{i=1}^n h_i v_i \right) \quad (4)$$

These formulas allow us to answer most of the questions (i) – (iv) above. First of all, formulae (1) and (3) show that the difference between stacking and interval velocities depends on the non-linear lateral anomaly changes ( $v_m''$ ). That is, stacking velocities depend on the values of  $a_{mn}$  and  $b_{mn}$ . Scalars  $a_{mn}$  and  $b_{mn}$  are similar and have similar behavior – the value of these scalars depend on the position of this layer in the ground (number  $m$ ) with respect to the reflector (number  $n$ ). Formulae (2) and (4) show that with  $n$  increasing (reflector depth is increasing), the numerator increases as the second power of the sum and denominator only as the first power - that is more slower. It explains why the influence of the velocity anomaly increases with greater reflector depth. If the velocity anomaly is close to the reflector ( $m$  is close to  $n$ ) then the nominator in  $a_{mn}$  and  $b_{mn}$  are small and stacking velocity is close to the RMS velocity. With increasing the reflector depth with respect to the anomaly depth, numbers  $a_{mn}$  and  $b_{mn}$

also increase, which implies bigger difference between the stacking and RMS velocities. First scalar in (1) and (3) gives us RMS velocity, so when  $a_{mn}$  and  $b_{mn}$  are small (and they are small when the reflector depth is close to the anomaly depth), so stacking velocity lateral changes (behaviour) correspond to lateral average velocity changes: slow velocity anomaly produces slow stacking velocity. For the deep reflectors, lateral changes of the second scalar in (1) and (4) become more important than the first one, and stacking velocities repeat the second-order derivative of the anomaly. That is, positive second-order derivative in the centre of anomaly on fig. 2 produces positive stacking velocity anomaly. At the end of the shallow velocity anomaly, negative second-order derivative implies negative velocity anomaly for stacking velocities.

Let us shortly describe the implications of the formulae (1) – (4) and answers for all the questions. The answer for the first question (Should we consider this “strange” behaviour of stacking velocities as an error?) in **no**. This follows from formulae (1) and (3) and the above paragraph. It means that we should not expect direct connection between stacking and average velocities, because of the second term in the formulae (1) and (3), which contain the second-order derivatives of the velocity anomaly. We can illustrate them with fig. 3a-e. Fig. 3a shows a velocity model with two shallow anomalies. The first anomaly is described with the curvilinear boundary (black lines), for the second one we use lateral changes in the first interval velocity (red lines). Question (ii) is also answered. The answer for the question (iii) depends on the relation between the velocity anomaly and the spread length. If the spread length is less than the shallow velocity anomaly length then we have non-hyperbolic NMO moveout but this is not the reason, contrary to what is claimed in Armstrong et al. (2001, p. 82). Fig. 3 and 6 confirm this.

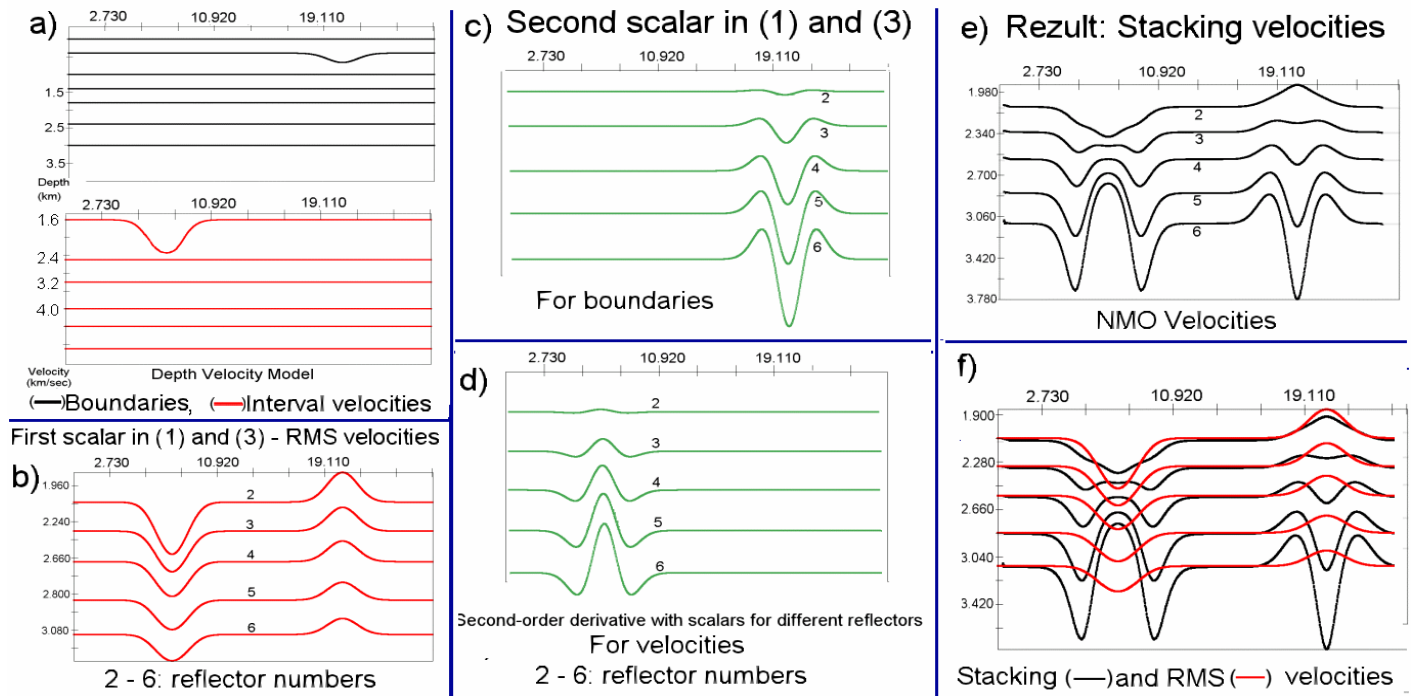


Fig. 3. Illustration to the formulae (1) and (3)

Fig 3b shows RMS velocities, which correspond with average velocity. Fig. 3c plots second scalars in the formulae (1) and (3) for each reflector, which are the second-order derivatives of the shallow interval velocity with scalars  $a_{mn}$  and  $b_{mn}$ . Fig. 3e shows stacking velocities: the result of the formulae (1) and (3) – the product of the first (RMS) and the second (second-order derivatives) scalars in (1) and (3) and their sum. For the shallow reflector the scalars  $a_{mn}$  and  $b_{mn}$  are small and stacking velocity correlates with the RMS velocity. For deeper reflectors the values of  $a_{mn}$  and  $b_{mn}$  increase, which leads to the oscillation of stacking velocities and these oscillations, correspond to the second-order derivative of the shallow anomaly. Formulae (1)- (4) also explain why we have inversion of stacking velocities while going from the shallow reflectors to the deep ones. For a shallow reflector the second term is small and stacking velocities correlate with the RMS, that is, average velocity. Slow shallow anomaly corresponds to slow stacking velocity. The deeper we go, the less the influence of the average velocity (see Fig. 3b) and the bigger is the role of the second term (fig. 3b,d). But the maximum of the shallow velocity anomaly corresponds to the minimum of its second derivative.

The last question has the answer “yes” but this goes beyond the scope of the paper. I can only mention, that Dix’s formula gives interval velocity with high accuracy if the velocity anomaly is in the estimated layer itself. This directly follows from formulas (2) and

(4). If  $m = n$  (anomaly is in the estimated layer) then  $a_{mn}$  and  $b_{mn}$  are small and the second term in (1) and (3) is close to (1). This implies that stacking velocity is very close to the RMS velocity and we can apply Dix's formula. If the estimated layer is much deeper than the velocity anomaly ( $n \gg m$ ) then the coefficients  $a_{mn}$  and  $b_{mn}$  are big and the second term in (1) and (3) differs from 1. It implies that the stacking velocity are far from the RMS velocities and Dix' formula gives big errors.

Using formulas (1) – (4), it can be shown (Blais, 1988) that in the case when the anomaly is described with a laterally varying velocity  $v_m$ , Dix's formula gives the value  $w_n$ :

$$w_n = v_n \left[ 1 + h_m v_m'' \left( \sum_{i=m+1}^n h_i v_i \right) / v_m^2 \right] \quad (5)$$

Formula (5) shows that the difference between interval velocity  $v_n$  and its Dix's estimation  $w_n$  depends on the non-linear anomaly changes (the second order anomaly derivative  $v_m''$ ), anomaly thickness  $h_m$  and the distance between the anomaly and the reflector ( $h_{m+1}v_{m+1} + h_{m+2}v_{m+2} + \dots + h_n v_n$ ). With increasing the vertical distance between the anomaly and the reflector, the sum in the parentheses increases, which implies bigger difference between the interval velocity  $v_n$  and it's Dix's estimation  $w_n$ . Figures 4 - 6 illustrate this connection. Fig. 6b shows the difference between NMO function and its hyperbolic approximation. For deep boundaries it is less than 2 ms but stacking velocity for these reflectors has the biggest oscillation. It means that anomalous behavior of stacking velocities is not caused by non-hyperbolic moveout but by non-linear velocity anomaly changes.

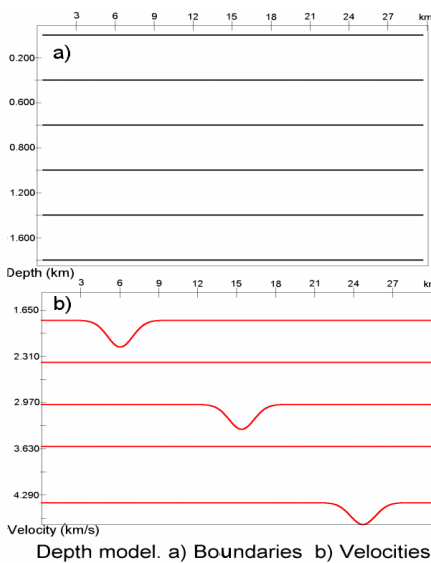


Fig. 4. Boundaries (a) and interval velocities (b)

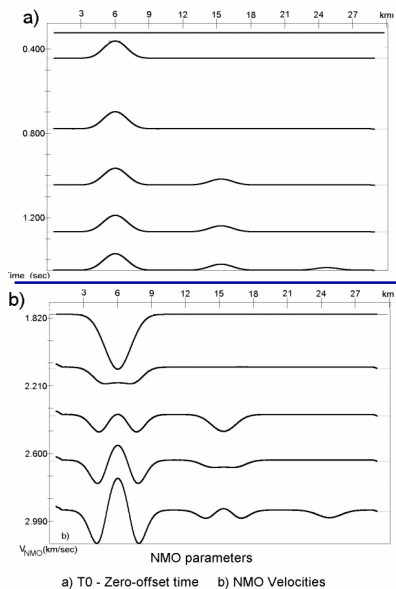


Fig. 5. Zero-offset time (a) and NMO velocity (b)

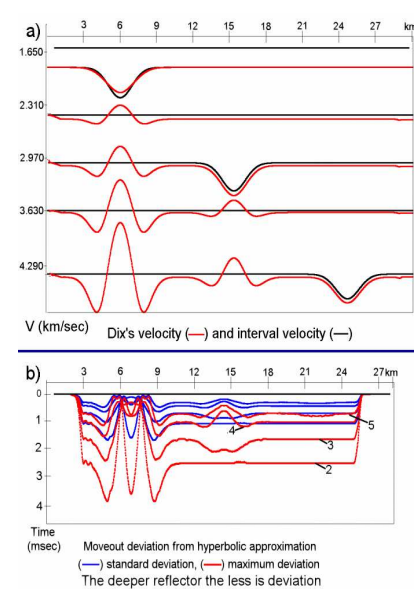


Fig. 6. Interval and Dix's velocities (a) Non-hyperbolic difference (b)

## Conclusions.

Formulae (1) – (4) give an explicit connection between stacking and RMS velocities. They explain anomalous (far from expected average) behavior of stacking velocity in the presence of overburden velocity anomalies. The influence of the velocity anomaly depends on its depth, the reflector depth, which define the numbers  $a_{mn}$  and  $b_{mn}$  in the formulae (1) – (4) and non-linear lateral changes. These non-linear changes (the second-order derivative with scalars, increasing with reflector depth) in the second term in formulae (1) and (3) just produce “strange” behaviour of stacking velocities. For the deep reflectors, stacking velocities repeat the second-order derivative of the shallow anomaly because of big values for the scalars  $a_{mn}$  and  $b_{mn}$ . For the shallow reflectors, these scalars are small and stacking velocities are close to RMS velocities. From (1) – (4) it also follows that to calculate accurate interval velocities in deep layers, we have to know not only local shallow velocity anomalies but also to the **second-order derivatives** with high accuracy and this is not an easy problem. Let us mention that for correct time-to-depth transformation it's enough to know anomaly velocity lateral changes and not their second derivatives.

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## References.

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