Gaussian beam migration of common-shot records

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Summary

Gaussian beam migration (GBM) (Hill, 1990, 2001) is an elegant, accurate, and efficient depth migration method. It has the ability to image complicated geologic structures with fidelity exceeding that of single-arrival Kirchhoff migration and approaching that of wave-equation migration. In fact, its accuracy can exceed that of most wave-equation migrations in imaging very steep dips, especially in three dimensions and especially in the presence of anisotropy.

Poststack GBM is efficient, but a naïve implementation of prestack GBM can be relatively inefficient. Hill's (2001) implementation of prestack GBM takes advantage of symmetries available in common-offset, common-azimuth acquisition geometries to provide an extremely accurate method that is far more efficient than the naïve implementation. Unfortunately, different acquisition geometries, such as orthogonal land and marine bottom-cable geometries do not easily accommodate the requirements of common-azimuth migration. For these geometries, it might be more natural to perform migration on individual common-shot or common-receiver records.

In this abstract, I adapt Hill's (2001) method to common-shot migration. The common-shot migration described here loses some accuracy compared with Hill's method, in the sense that it doesn't handle multiple arrivals as well. It is possible to overcome this problem (Gray, 2005), but the method described here will be sufficient for the purposes of this abstract, which is to point out some advantages of migrating common-shot records. In practice, this slight loss of accuracy needs to be weighed against (1) the difficulties (in some geometries) of obtaining regularly sampled regularly sampled common-offset, common-azimuth records for migration, and (2) the difficulties in regularizing common-offset, common-azimuth data for the premigration slant stack when surface elevations and velocities vary rapidly.

Poststack Gaussian beam migration

GBM is a variant form of Kirchhoff migration that operates in the frequency-wavenumber, as well as the time-space, domains, and allows for multipathing in a natural way. Where Kirchhoff migration images a single input trace onto all image locations, GBM first groups a number of nearby input traces together by slant-stacking, then images each slant-stacked trace onto a restricted set of image locations, the image locations determined by a particular ray direction. The Gaussian beam raytracing method provides amplitude and phase information for these image locations. Figure 1 (Hale, 1992) illustrates the procedure for poststack migration. Input traces from CMP locations near 5 km (the beam center) have been slant stacked corresponding to a particular ray takeoff angle, then imaged according to amplitude and time information within the Gaussian beam for that ray angle, yielding a partial migration. To complete the image, accumulate all the partial migrations, summing over ray angles for each beam center location, then summing over beam center locations. Migrating a limited number of local slant stacks over a limited number of initial directions makes GBM efficient, and assuring that the number of directions is adequate to sample the entire wavefield makes it accurate.

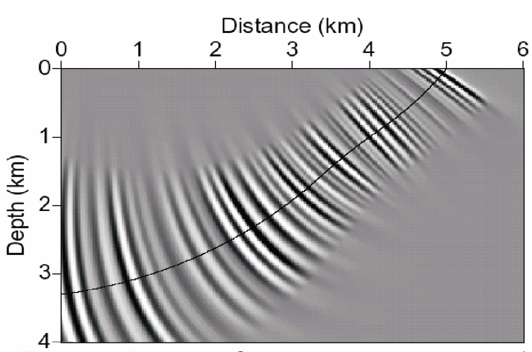

Figure 1. Poststack Gaussian beam migration of a single slant-stacked trace in a single direction. From Hale (1992).

Gaussian beams complicate the complete picture considerably. Gaussian beams have traveltimes and amplitudes that are both complex-valued. In fact, it is the imaginary part of the traveltime within the beam, not the taper applied to the local slant stack of the input traces, that causes the decay of the migrated image away from the central ray in Figure 1. The complex-valued amplitudes and traveltimes complicate the full description of GBM, but they also provide a regular migration Green's function. In contrast, the Green's functions used in standard Kirchhoff migration become singular at caustics, requiring *ad hoc* fixes. A wide latitude of choices exists for initial conditions for Gaussian beams; Hill's formulation of GBM uses Green's functions that are planar at their take-off points on the Earth's surface.

In addition to the complex-valued ray quantities, the traveltime tables used in GBM are considerably different from those used in standard Kirchhoff migration. Standard Kirchhoff migration uses tables of traveltimes from point sources. Each table contains the traveltimes from a station location to all image locations. In a constant-velocity Earth, contours of equal traveltime sweep out spherical shells in three dimensions. In an Earth with considerable velocity variation, single-arrival traveltime tables cannot accommodate energy that travels to an image location along more than one path. In GBM, the point-source response is built as a superposition of plane-wave responses, and the plane-wave traveltime tables are collimated, as in Figure 1. If several beams from the same CMP location strike the same image location, GBM accumulates several arrivals into that location in a natural fashion.

Poststack GBM is thus a generalization of Kirchhoff migration, and it improves upon Kirchhoff migration in its natural handling of both wavefield caustics and multi-valued traveltimes.

Prestack Gaussian beam migration

Once we understand the mechanics of poststack GBM, we can think of a naïve generalization to prestack migration very easily. For a given gather of input traces, perform local slant stacks and then image the local slant stacks along beams from the source and receiver locations. In a common-shot gather, for example, the beam center locations are a subset of all the receiver locations, and the ray parameters correspond to ray directions from each beam center location. The source location is also a beam center location, and each beam from the source location and each beam from each of the receiver beam center locations contributes part of the final image. For each receiver beam center location, adding all these angular contributions gives the total image from the location, and adding the contributions from all the receiver beam center locations gives the migrated image of the shot record. The computational structure for this prestack migration method is:

```
For each receiver beam center location

   Tapered local slant stacks (using local surface velocity)
      of input traces over several ray parameters
   For each ray parameter from the receiver beam center
      For each ray parameter from the source location
         For all image points in the aperture
            Accumulate the slant-stacked input trace into the
               image
         End loop over image points
      End loop over source ray parameters
   End loop over receiver ray parameters
End loop over receiver beam centers
```

This prestack migration algorithm, while extremely accurate, is not particularly efficient. By performing a saddle point analysis of the behavior of the downward continued source and receiver wavefields at image points, Hill (2001) has derived a very efficient version of this algorithm that collapses some of the computational loops (integrals in the continuous formulation). The saddle point analysis relies on the fact that the strength of the Gaussian beam wavefield associated with a raypath decays exponentially away from the raypath, as in Figure 1. The analysis confirms the intuitive observation that raypaths from the source and receiver locations of a beam center will contribute significantly to the migrated image at a point only if both raypaths pass close to that point. The exponential decay of the wavefield strength is determined by the imaginary part of the traveltime within the beam, so raypaths whose total (source plus receiver) imaginary traveltimes are small (i.e., small total decay) will contribute the dominant part of the migrated image. In common-offset migration, the ray parameters p_s and p_r from the source and receiver locations of a beam center are combined as the midpoint ray parameter $p_m = p_r + p_s$, so these dominant contributions can be found by scanning all values of offset ray parameter $p_h = p_r – p_s$ to obtain the combination of p_s and p_r that minimizes the imaginary part of the total traveltime. This scan can be performed over a coarse-grid subset of the migration grid, with values interpolated onto the migration grid, because the traveltime tables are well-behaved (again, see Figure 1). With the (efficient) scan replacing the innermost loop of the migration program, the computational structure becomes

For each *midpoint* beam center location
   Tapered local slant stacks
   For each *midpoint* ray parameter
      For each coarse-grid image point
         Scan over *offset* ray parameters for the minimum
            value of imaginary time
      End loop over coarse grid of image points
      For all image points in the aperture

         Accumulate the slant-stacked input trace into the
            image using times obtained from the scan
      End loop over image points
   End loop over *midpoint* ray parameters
End loop over *midpoint* beam centers

This structure differs from the preceding one in that its innermost loop, the scan, is performed only over a coarse-grid subset of the entire migration aperture. It is therefore much more efficient and, as Hill has shown, very accurate.

Common-shot Gaussian beam migration

The efficient migration method just described is very well suited for input data sorted into common-offset, common-azimuth volumes. It doesn't appear to be as well suited for different types of input data volume, such as wide-azimuth surveys recorded by ocean bottom sensors. For such data, it might be more efficient to migrate these wide-azimuth data volumes directly, without an intermediate sort to sparsely populated common-offset, common-azimuth bins.

A separate issue arises with prestack GBM of land data where velocity and/or topography vary rapidly along the recording surface. The local slant stack of a common-shot record with a beam center at a particular receiver location depends on values of the velocities near that location. By contrast, the local slant stack of a common-offset record with a beam center at a particular midpoint location depends on velocity values near both the source and receiver locations. It will then be harder to control the slant stack of the common-offset record to behave as desired, especially when the velocities near the source and receiver locations are very different. Likewise, it is harder to perform suitable elevation corrections, before slant stack, for traces in a common-offset gather than for traces in a common-shot gather.

The issue of velocity and topography is not insoluble for common-offset migration, but the solutions are not straightforward, and they present technical compromises that may reduce the quality of the final migrated image. Similarly, the common-shot algorithm whose structure I describe next presents a technical compromise. In most cases the compromise is minor, and in most cases the algorithm produces depth-migrated images whose quality is comparable to that of common-offset migration.

To adapt Hill's efficient common-offset migration to common-shot records, I replace *midpoint* with *receiver*, and *offset* with *source*, in the computational structure:

```
For each receiver beam center location
  Tapered local slant stacks
  For each receiver ray parameter
    For each coarse-grid image point
      Scan over source ray parameters for the minimum
        value of imaginary time
    End loop over coarse grid of image points
    For all image points in the aperture
      Accumulate the slant-stacked input trace into the
        image using times obtained from the scan
    End loop over image points
  End loop over receiver ray parameters
End loop over receiver beam centers
```

This computational structure is an exact analogy to the one for efficient common-offset migration, but it has a disadvantage in its ability to handle multipathing. In the scan, a particular receiver beam is fixed, and it searches for the source beam with the minimum value of imaginary time. Moving from one image location to another within a given receiver beam can result in jumps in selected source beam, sometimes with associated jumps in the real part of source traveltime. Since the receiver beam direction is fixed, the total (source plus receiver) traveltime can jump, causing the same sort of branch jumping that we often see with single-arrival Kirchhoff migration. For common-offset migration, both source and receiver beam directions change as we move over image locations during the scan, and changes in the real part of source traveltime are likely to be compensated by changes in the real part of receiver traveltime.

This common-shot implementation thus handles multiple arrivals correctly from the receiver side, and sometimes incorrectly from the source side. So we can expect its quality to lie between that of (single-arrival) Kirchhoff migration and wave-equation migration, except in situations presenting very steep dips or anisotropy. Typically, however, its quality is closer to that of common-offset GBM than to that of Kirchhoff migration, and the example below illustrates its superiority over Kirchhoff migration.

An example

Figure 2 shows a 2-D structural cross section typical of the Foothills of the Canadian Rockies. Topography varies approximately 900 m along the cross section; high-velocity thrust sheets pierce the surface next to low-velocity layers, with a total velocity contrast of 6000 / 2500; and the subsurface is structurally complex with deep hydrocarbon targets. Figure 3 shows an imaging comparison between Kirchhoff migration and GBM of shot records. The Kirchhoff migrated image is adequate. Still, the GBM image is clearly superior. Overall, it has a cleaner appearance, which is characteristic of images from wave-equation migration. Figure 4 gives a partial explanation for the imaging differences. This figure shows impulse responses of both migration operators, for a single input spike on a near-offset trace whose surface location is between the high-velocity thrust sheets. Where the single-arrival Kirchhoff operator has run into difficulties near the bottom of the migration smile, the GBM operator has placed overlapping arrivals. (The low-amplitude arrivals are due to the input trace contributing to a few overlapping beam centers.) Also, the amplitudes are fairly uniform along the Kirchhoff impulse response, while the GBM operator has placed less energy along certain parts of the curve. The branch jumping of the Kirchhoff operator tends to add random noise to the image, and the uniform amplitudes of the Kirchhoff operator tend to produce migration smiles.

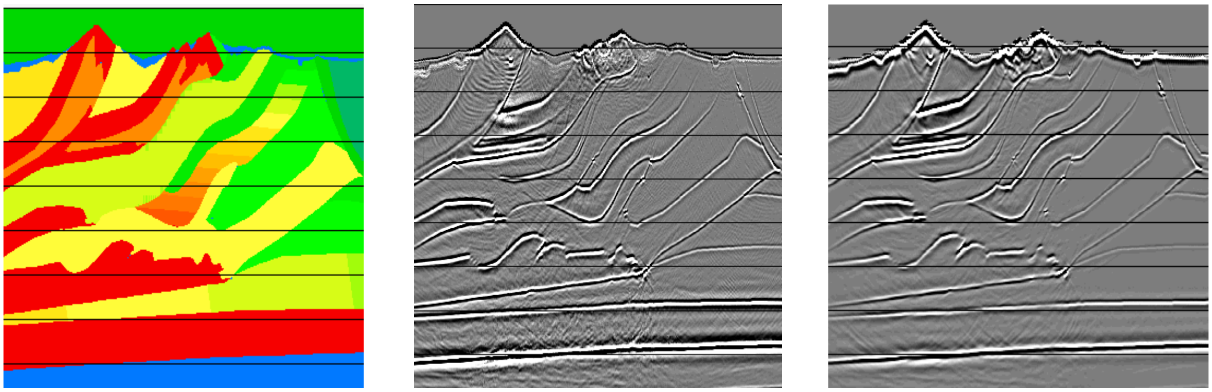
a b c

Figure 2. (a) Portion of a Canadian Foothills structural cross section, showing significant topographic variations, rapid near-surface velocity variations, and significant subsurface structural complexity. (b) Kirchhoff migrated image. (c) Common-shot Gaussian beam image.

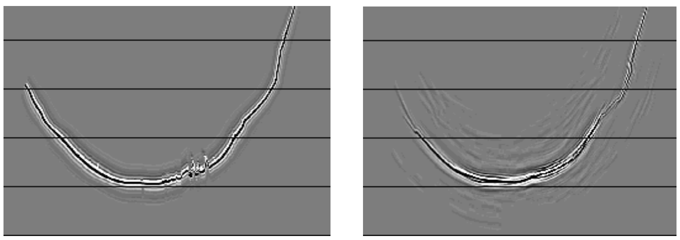
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Figure 3. Migration impulse responses: (a) Kirchhoff migration; (b) Common-shot Gaussian beam migration.

Conclusions

The common-offset GBM method presented by Hill (2001) is ideal for marine streamer data, but it is less than ideal for other recording geometries containing a wide range of azimuths, or with a wide range of surface elevations and rapid near-surface velocity variations. Here I have shown that common-shot GBM can be applied to data recorded within these other geometries.

This method is a straightforward adaptation to Hill's method to common-shot records; as such, it doesn't handle multiple arrivals as well as its common-offset counterpart. This shortcoming can be overcome with some modifications to the algorithm. Rather than describe these in detail, I have tried to describe GBM in general terms and to show that it is feasible, and sometimes desirable, to perform GBM on common-shot records.

References

Gray, S. H., 2005, Gaussian beam migration of common shot records (to appear in Geophysics).
Hale, D., 1992, Migration by the Kirchhoff, slant stack, and Gaussian beam methods: Colorado School of Mines Center for Wave Phenomena Report 121.
Hill, N. R., 1990, Gaussian beam migration: Geophysics, 55, 1416-1428.
Hill, N. R., 2001, Prestack Gaussian-beam depth migration: Geophysics, 66, 1240-1250.