The FOCI method versus the WLSQ and Hale's wavefield extrapolation methods

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Abstract

Recursive wavefield extrapolation methods are more powerful than ray theory based methods because of their great ability to handle strong lateral velocity variations. Wavefield extrapolation methods have two major problems (1) the extrapolator instability and (2) they are computationally expensive. The *forward operator and conjugate inverse* (FOCI) method is an appropriate method for designing accurate and efficient extrapolation operators that remain stable in a recursive algorithm. The FOCI's results are comparable with other results obtained with other known methods such as Hale's and the weighted least square (WLSQ) extrapolation methods. Further, the FOCI method is computationally more efficient than the other methods.

Introduction

There are different ways to design spatial convolution operators for recursive wavefield extrapolation. The most common approach is to design an operator that approximates the exact phase-shift operator in the frequency-wavenumber domain then transform it to the spatial domain. Hale (1991) introduced a method to calculate a stable explicit extrapolator. This method is based on the Taylor expansion of the exact phase-shift operator in the frequency-wavenumber domain and the use of basis functions. Hale's method can design short stable operators but can not handle high angles of propagation. Further, it is computationally expensive and requires the use of both symbolic and numerical mathematical software packages.

Thorbecke et al. (2004) have introduced a weighted least-squares method (WLSQ), which is not perfectly stable but has a controlled instability. The stability of the WLSQ method is sensitive to the parameters such as the velocity and spatial and vertical samplings.

Margrave et al. (2004) introduced a new method for designing spatial operators called the FOCI method. "FOCI" is an acronym for *forward operator and conjugate inverse*, which suggests the key concept in operator stabilization by Wiener filtering. However, there are three key innovations in the method with the other two being: (2) the use of dual operator tables to reduce evanescent filtering, and (3) spatial resampling of the lower frequencies to increase operator accuracy and decrease run times. In this paper, comparisons of the FOCI method versus Hale's and the WLSQ extrapolation methods are shown.

Theory

To design a wavefield extrapolator, we start with the 2-D scalar wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}. \qquad (1)$$

After 2-D Fourier transformation Equation (1) becomes

$$\frac{\partial^2\bar{\tilde{\psi}}}{\partial z^2}=-k_z{}^2\bar{\tilde{\psi}}, \qquad (2)$$

where

$$k_z{}^2=\frac{\omega^2}{v^2}-k_x{}^2. \qquad (3)$$

Equation 2 is just a 1D Helmholtz equation whose solution, for upgoing or downgoing waves, is

$$\bar{\bar{\psi}}(k_x, z, \omega) = \bar{\bar{\psi}}(k_x, z = 0, \omega)\, e^{ik_z z}. \qquad (4)$$

Note that ψ is the wavefield representing pressure, $\bar{\bar{\psi}}$ represents its 2-D Fourier transform, *t* is the two-way travel time, and *x* and *z* are the spatial and depth coordinates. Thus the wavefield at some depth *z*, $\bar{\bar{\psi}}(k_x, z, \omega)$, can be obtained by multiplying the recorded wavefield at the surface, $\bar{\bar{\psi}}(k_x, z = 0, \omega)$, by a phase shift operator, $e^{ik_z z}$, in a homogeneous medium (Gazdaq, 1978). The extrapolation methods such as Hale's, WLSQ, and FOCI try to design a stable operator in the spatial domain whose Fourier transform approximates the phase shift operator. The operator is then varied with the local velocity of the computation grid to handle lateral velocity variations (Holberg, 1988).

Discussion

Hale's method can design a stable explicit extrapolator but has some problems such as it is compuationally expensive to calculate the extrapolator and the extrapolator can not handle steeply dipping events. On the other hand, both the WLSQ and FOCI extrapolators have a controlled instability. However, they can handle higher angles of propagation than Hale's. Furthermore, it is compuationally more efficient to calculate WLSQ and FOCI extrapolators.

Figure 1a shows a comparison among the three extrapolators. The FOCI extrapolator exhibits a better stability than the WLSQ but it is less stable than Hale's with a broader amplitude spectrum, which means that the FOCI extrapolator is more effective in handling the high angles of propagation than Hale's. When increasing the depth step size from 2 m to 10 m, the stability of the FOCI extrapolator does not change as much as the WLSQ extrapolator (Figure 1b).

The impulse responses of the three extrapolators are used to analyze their accuracies. The zero-offset experiment is done with an operator length of 31 points in a homogenous medium, a receiver spread of 1280 m, a maximum extrapolation depth of 1280 m, a velocity of 2000 m/s, a spatial spacing of $\Delta x = 10$ m, and a vertical spacing of $\Delta z = 10$ m. The trace in the center of the zero-offset section contains five Ricker wavelets at 0.0600, 0.1240, 0.1880, 0.2520, and 0.3160 seconds. The sample rate is 4 ms and the dominant frequency of the Ricker wavelet is 30 Hz. Figure 2 shows the impulse responses of the WLSQ, FOCI, and Hale's extrapolators compared with the result from phase-shift migration. Whilst Hale's extrapolator could not migrate the high angles of propagation, the WLSQ and FOCI extrapolators show that they can better handle the high angles of progagation.

The Marmousi dataset is usually used to test the accuracy of migration algorithms because of the strong lateral velocity variations and steep dips. These data will be used as a second test to further analyze Hale's, WLSQ, and FOCI extrapolators in the presence of strong lateral velocity variations and steep dips. Figures 3a shows the velocity model of the Marmousi dataset. Figures 3b and 3c show the migration results using WLSQ and FOCI extrapolators, respectively. The operator length used for both is 69 points and the images have $\Delta x = 12.5$ m and $\Delta z = 12.5$ m spacing. The run times on a standard PC were 23.7 and 15.8 hours for the WLSQ and FOCI results, respectively. Both of these methods handled the strong lateral velocity variations and the steeply dipping events. However, due to spatial resampling, the FOCI method is more efficient than WLSQ.

Conclusions

Unlike Hale's extrapolator, both the WLSQ and FOCI extrapolators are not perfectly stable but have controllable instabilities. However, they can handle higher angles of propagation than Hale's. The stability of the WLSQ extrapolator is more sensitive to the size of the depth step and operator length than FOCI extrapolator. Calculating tables of extrapolators using the WLSQ and FOCI methods is computationally more efficient than using Hale's method. Further, the FOCI method with spatial resampling is computationally less expensive than the other two methods and this can make a big difference in 3-D prestack depth migration. Despite the fact that FOCI is a new method, its results are already comparable with other standard methods' results. This shows that FOCI is a promising package for seismic imaging that combining both efficiency and accuracy.

Acknowledgments

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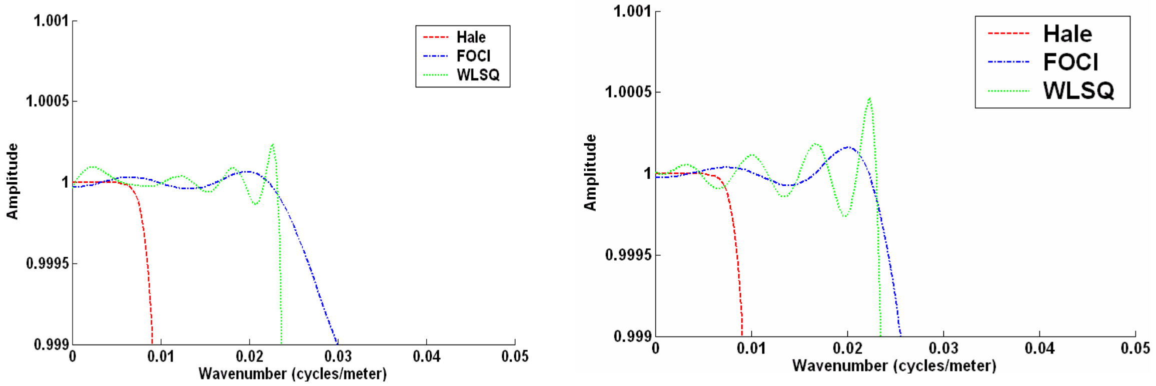
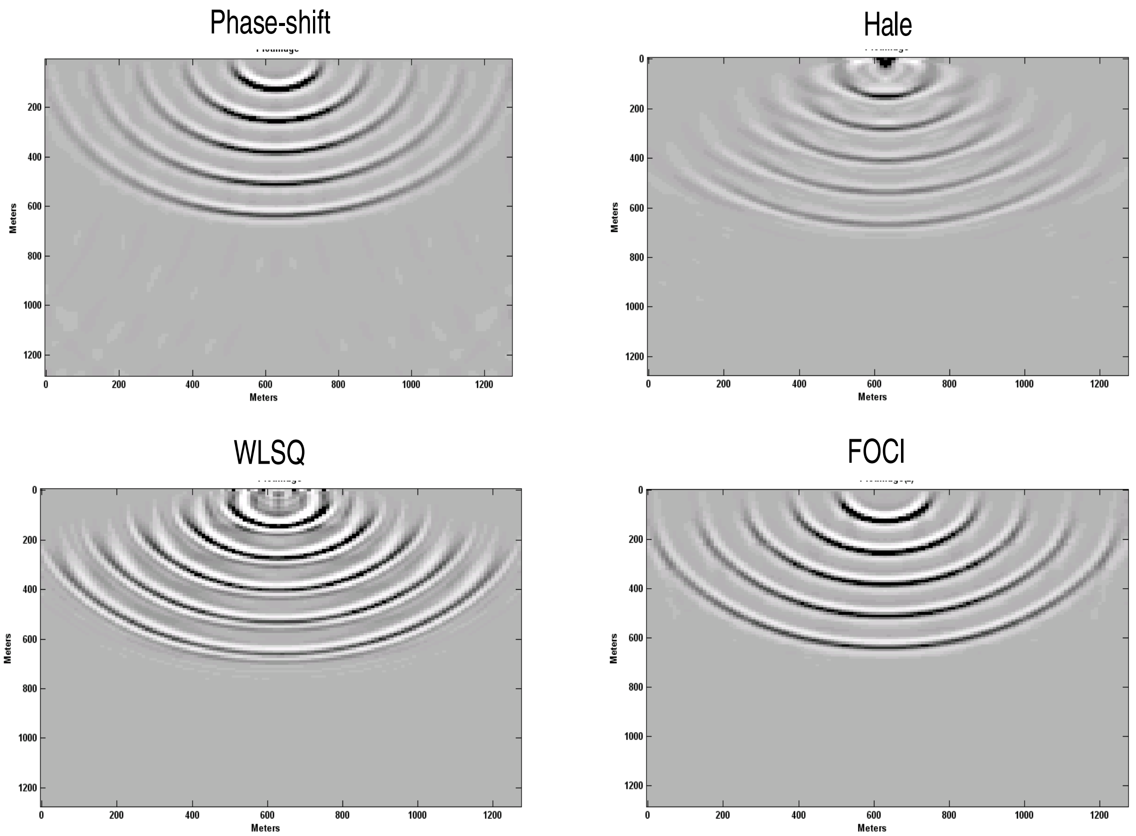


Figure 1. A comparison among the amplitudes of Hale's, FOCI, and WLSQ extrapolators. (a) $\Delta x = 10$ m and $\Delta z = 2$ m, operator length=25 points, velocity=2000m/s, and frequency=50 Hz and (b) same parameters as in (a) but $\Delta z = 6$ m.


Phase-shift

Hale

WLSQ

FOCI

Figure 2. Impulse responses of Hale's, WLSQ, and FOCI extrapolators compared with the result from phase-shift migration. The velocity is 2000 m/s, Δx =10 m, Δz =10 m, and the operator length is 31 points.

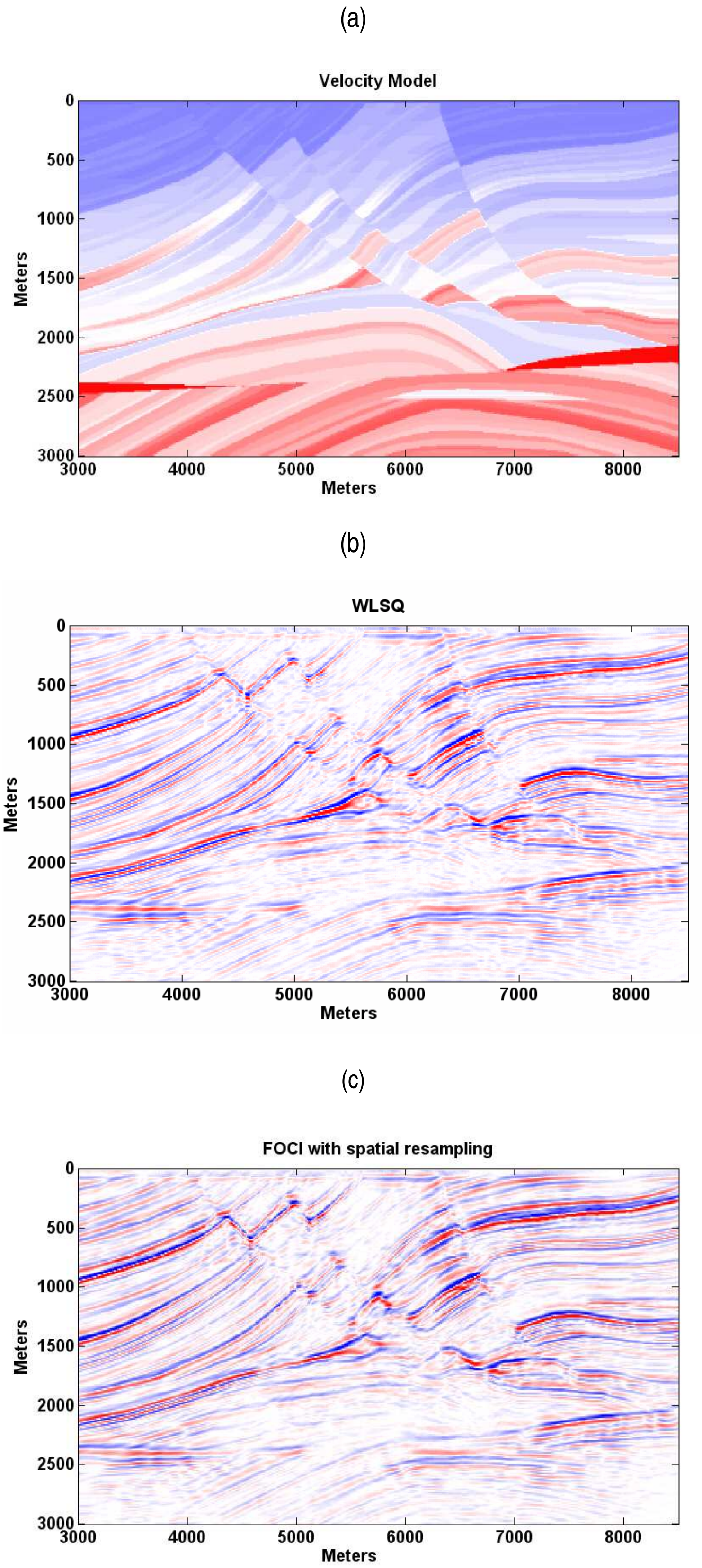


FIG 3. (a) Showing the velocity model of Marmousi data, (b) is the WLSQ prestack depth migration result with run time=23.7 hours, and (c) is the FOCI prestack depth migration result with run time=15.8 hours where a 69-point operator was used in both results.