Predicting stratigraphy with cepstral decomposition

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Abstract

Spectral decomposition is an established technique for stratigraphic analysis from seismic reflection data. Interpretation is usually purely qualitative, but in some circumstances the technique can also provide quantitative information about bed thickness, especially near or below seismic resolution. This paper describes a novel extension of the spectral decomposition technique that improves the accuracy of bed thickness estimation.

Thin beds produce periodic notches in the spectrum of the seismic trace. The spacing of these notches, which is inversely proportional to bed thickness, can be accurately measured by calculating the Fourier transform of the spectrum. This so-called cepstrum has local maxima corresponding to the thickness of the thin bed or beds. Since the technique does not rely on picking a single frequency, but instead detects tuning effects from the entire spectrum, it copes better with noise and is also able to resolve overlapping and interfering thin-bed effects.

In this paper, I present examples from one-dimensional synthetic seismic traces and two-dimensional synthetic seismic models. I show that bed thicknesses down to around 8 ms two-way time can be resolved to within 2 ms. By contrast, the classic spectral decomposition technique can result in systematic errors, commonly resulting in more than 10 ms of error. The classic technique is also unable to resolve tuning effects from compound or adjacent thin beds, whereas these may be roughly quantified with cepstral decomposition. Cepstral peaks are also a better predictor of thickness than the time difference between the top and base interpreted from the seismic trace.

Introduction

Spectral decomposition of seismic traces into the frequency domain is a well-established interpretation technique (Partyka et al. 1999). Typically, the Fourier transform spectra are interpreted geomorphologically as frequency slices. In some circumstances and with proper well calibration, however, quantitative interpretation is also possible. Specifically, the time thickness of a thin-bed giving rise to a seismic reflection can be calculated.

To do this, the frequency of the first local maximum in the amplitude spectrum (the first spectral peak) is doubled to estimate the period of the first harmonic, itself the inverse of bed thickness. This gives a measure of bed thickness that is mostly independent of phase and even of structural interpretation. Unfortunately, the first spectral peak frequency is prone to perturbation by noise and interference effects, and in my experience is often difficult to reconcile with bed thicknesses measured from wells without considerable support from modelling (eg Hall & Trouillot 2004).

My hypothesis was that it should be possible to measure the period of the notches in the spectrum by Fourier transform, instead of estimating using the first peak frequency. Researching this idea turned up the widespread use of the 'spectrum of the spectrum', or the cepstrum, in other fields of signal analysis such as voice recognition, mechanical vibration analysis, electrocardiogram interpretation, and echo detection. Note that 'cepstrum' is pronounced *kepstrum*; Tukey et al. (1963) invented this word, and all the other anagram-based terms (for example, quefrency and gamnitude) in the field of 'cepstral alanysis', only some of which are commonly used in the literature. They help to distinguish the cepstrum from the time domain, even though they have the same dimensions.

I have tested this idea on synthetic data and show in this paper that it is indeed possible to use cepstra as a superior measure of the thickness of simple thin beds, and that more complex stratigraphic arrangements can also be resolved, although possibly only qualitatively.

Method

The cepstrum can be defined in terms of the Fourier transform, FT, as follows (Tukey et al. 1963):

Mathematically cepstrum = FT(ln(FT(signal))) *Algorithmically* signal ⇨ FT ⇨ log ⇨ FT ⇨ cepstrum

Researchers should be aware when examining the literature of cepstral analysis that many sources define the cepstrum as the *inverse* FT of the log of the FT of the signal. This related function is used in other fields, including homomorphic deconvolution, which has applications in seismic processing (eg Ulrych 1971).

In practice, I found that taking the log of the spectrum sometimes has an adverse effect on the results. However, where resonant frequencies are very high, it is necessary to resolve the short quefrencies (that is, the thinnest beds). In this paper, I have used both FT(ln(FT(signal))) and FT(FT(signal)), taking the complex modulus of the complex spectrum at each step. The latter version, FT(FT(signal)), could be called the pseudo-cepstrum.

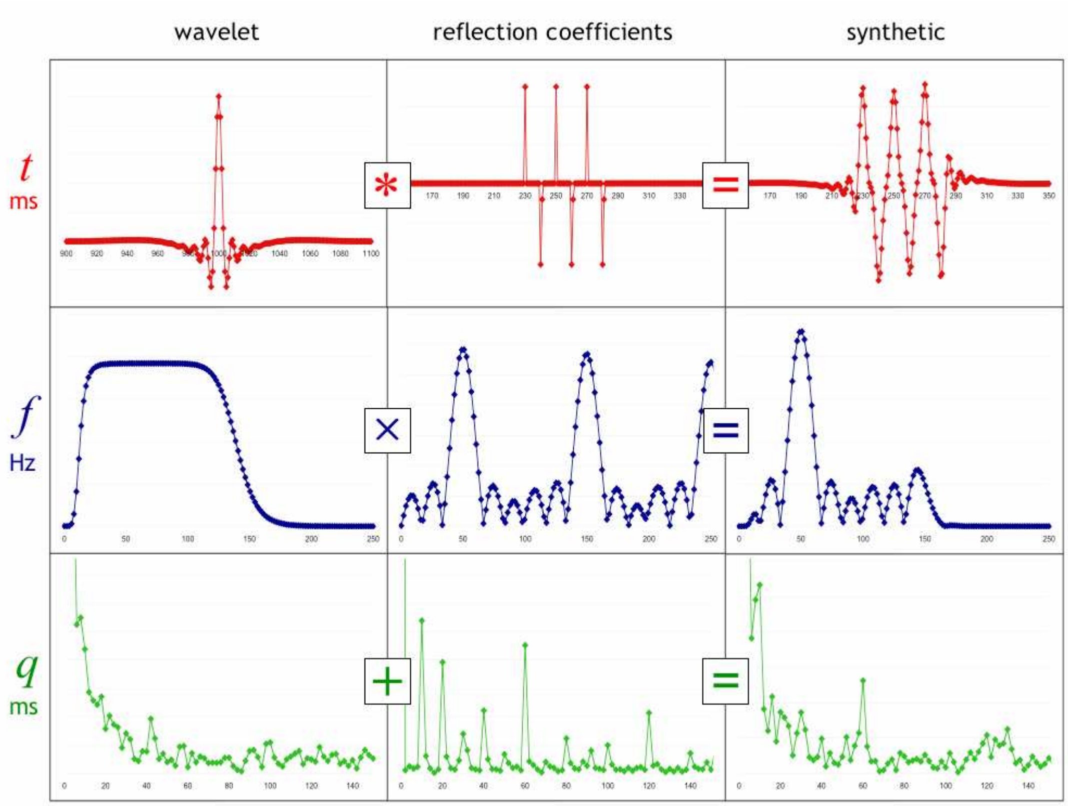

***Figure 1, One-dimensional synthetic thin-bed model**. **Top row:** a time series representation of a 50 ms-thick thin bed sequence of three 10-ms two-way time beds with 10-ms gaps. The x-axes are time, t, in milliseconds; the y-axes are relative amplitude. The middle column shows the reflection coefficient series convolved with the 6–15–120–150 Hz Butterworth filter shown in the left-hand column. **Middle row:** the complex moduli of the fast Fourier transforms of the time series, the spectra, showing the frequency domain. The x-axes are frequency, f, in Hertz; the y-axes are relative magnitude. The effect of the thin beds is apparent as periodic notches in the spectra of the RC series and the synthetic. The spacing of the notches is inversely proportional to the bed thickness. Importantly, using first peak frequency to estimate notch spacing would only resolve the total package, not the 10-ms subunits. Note that convolution in the time domain has transformed into multiplication in the frequency domain. **Bottom row:** the complex moduli of the fast Fourier transforms of the natural logs of the spectra, the cepstra, showing the quefrency domain. The x-axis is quefrency, q, in milliseconds; the y-axes are relative gamnitude. The thin beds are resolved as a cepstral peak at 10 ms, along with harmonics at integer multiples of 10 ms, and the 3-bed package shows up as another peak at 60 ms. Spectrum multiplication is addition in the quefrency domain.*

One-dimensional models

For the synthetic trace study shown in Figure 1, I used the Fourier Transform data analysis tool in Microsoft Excel. A compound thin-bed model synthetic trace was transformed to its amplitude spectrum, which reveals two sets of overlapping notches (see Partyka et al. 1999). The period of the notches with respect to frequency $P_f = 1/t$ = 20 Hz and 100 Hz, where t is the two-way time bed thickness. In a typical quantitative application of spectral decomposition, the first spectral peak frequency, in this case either around 8.5 Hz or 50 Hz (depending on how the first peak is selected), is measured by the software and then doubled to estimate P_f. This approach fails to separate the two scales of bedding, resolving either the thin beds themselves (the 50 Hz peak) or the package (the 17 Hz peak). Note that the thicker package is misinterpreted as almost 60 ms thick, as opposed to the actual 50 ms.

The pseudo-cepstrum was calculated from the spectrum; it shows unambiguous peaks at 10 ms and 60 ms, representing the thin beds and the 3-bed package respectively. The cepstrum accurately reflects the notches the spectrum, but also gives the wrong answer for the total package thickness. It seems that only visual inspection of the spectrum can reveal the correct answer here.

Two-dimensional models

I have also applied the technique to a two-dimensional synthetic model. For this work, I used Landmark's Spectral Decomposition software, which shares its heritage with FreeUSP's tune3d tool (Partyka et al. 1999). I used the normalization function in each step; this makes features in the otherwise very weak high frequencies (and, in the cepstrum, quefrencies) resolvable. A simple wedge model, shown in Figure 2a, was decomposed using the discrete Fourier transform to give the spectrum (Figure 2b). This was then run through the software to give the cepstrum (Figure 2c).

Again, the cepstrum is considerably easier to interpret than the spectrum. Quantitatively, bed thickness can be calculated in three ways: from the time difference of the seismic events, from the first spectral peak frequency, and from the cepstral peak. A comparison of the thickness estimates from these three methods is shown in Table 1 and Figure 3. The systematic errors in the time-difference and first spectral peak estimates are apparent. In contrast, the cepstral peak gives good estimates even for very thin beds.

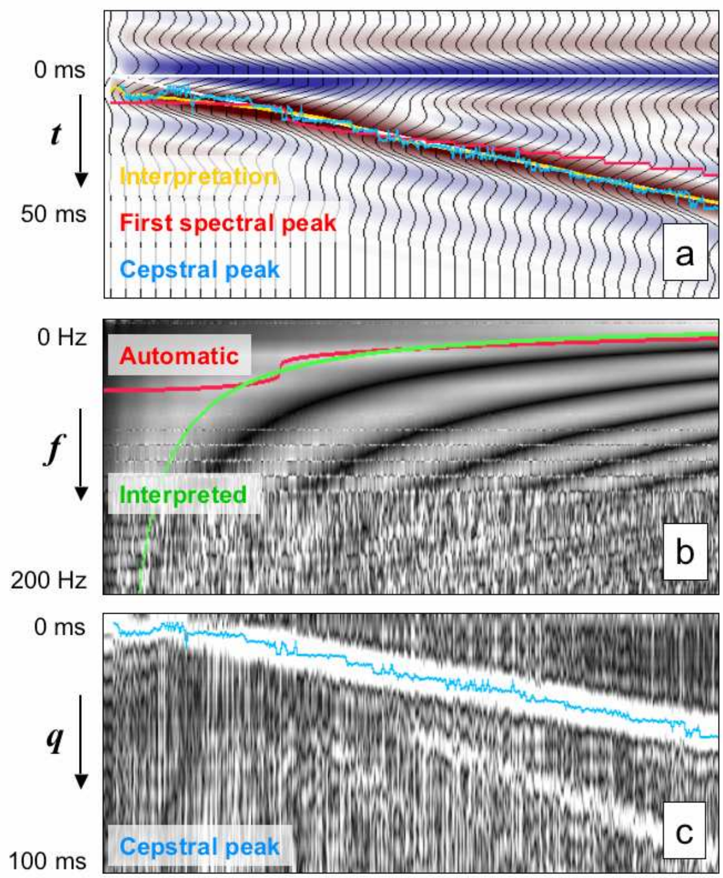

Figure 2, Two-dimensional wedge model. (a) *Signal; time domain, t. Wedge model comprising equal but opposite reflection coefficients convolved with a Ricker filter with 40 Hz peak frequency. A peak represents a downwards increase in acoustic impedance. The wedge increases in thickness from 0 ms on the left to 50 ms on the right. The yellow line represents the interpreted trough at the base of the wedge. The red and blue lines represent time thickness calculated from the first spectral peak and the cepstral peak respectively.* ***(b)*** *Spectrum, frequency domain, f. The result of a discrete Fourier transform of the wedge model, calculated for a 100 ms window centered on 0 ms. A Gaussian window (taper) was applied to the data, and the frequency slices were normalized as described by Partyka et al. (1999). The red line represents an automatic measure of the first spectral peak frequency. The green line is a manual interpretation of the first spectral peak; it is not part of the analysis.* ***(c)*** *Pseudo-cepstrum, quefrency domain, q. The result of a discrete Fourier transform of the spectrum, with no window. Normalization of the quefrency slices was performed as before. The periodic notches in the spectrum have been transformed into a strong peak in the cepstrum at 0–50 ms. The cepstral peak has been detected automatically.*

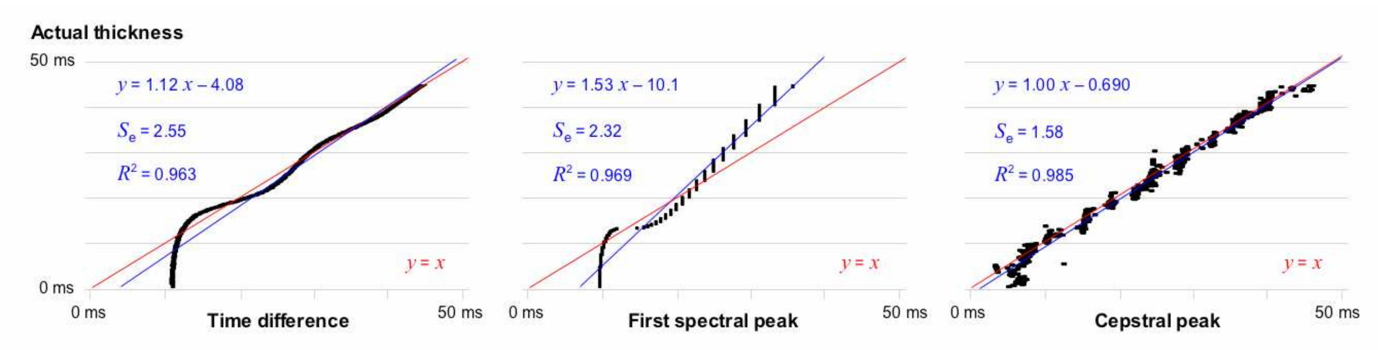

Figure 3, Thickness predictions for the wedge model. *Comparison of three predictions of thickness for the wedge model: time-difference from interpretation of the top and base of the wedge, calculation from first spectral peak frequency, and the first cepstral peak quefrency. Blue lines are linear regression models, with model equations, standard errors of estimation,* S_e*, and the coefficients of determination,* R^2*. The ideal model,* $y = x$*, is also shown for each plot, to show how close the prediction is in each case. Note that the standard errors compare the data with the regression model, not with the ideal model. Of the three, cepstral peak gives the most robust prediction. See also Table 1.*

	Time difference			**First spectral peak**			**Cepstral peak**		
	Thickness	Residual	% error	Thickness	Residual	% error	Thickness	Residual	% error
1 ms	10.8	9.8	980	9.6	8.6	860	7.0	6.0	600
5 ms	10.9	5.9	118	9.8	4.8	96	5.4	0.4	8.0
10 ms	11.5	1.5	15	10.3	0.3	3.0	10.0	0.0	0.0
20 ms	21.4	1.4	7.0	21.3	1.3	6.5	19.9	0.1	5.0
30 ms	29.6	0.4	1.3	27.0	3.0	10	29.9	0.1	0.3
40 ms	40.7	0.7	1.8	32.0	8.0	20	41.2	1.1	2.8

Table 1, Thickness predictions for the wedge model. *For each method, the first column gives the two-way time thickness prediction in ms, the second gives the difference between the prediction and the actual value in ms, and the third gives the percentage error. Time difference from interpreted horizons has a progressively smaller error, below 5% for a wedge thicker than about 27 ms. The first spectral peak gives estimates within 5% of actual thickness for thicknesses around 12 ms and 25 ms, but is increasingly inaccurate elsewhere. In contrast, the cepstral peak method may be capable of estimates within 5% for all thicknesses above about 8 ms.*

Conclusions

Cepstral decomposition has the potential to significantly improve the accuracy of bed thickness estimation from seismic. In the one- and two-dimensional models presented in this paper, estimations are much more accurate than those from horizon interpretation alone, or from traditional spectral decomposition workflows. In addition, the technique is able to resolve multiple beds of various thicknesses from the same window, effectively sidestepping one of the limitations of spectral decomposition: the analysis window must be long enough to give reasonable frequency resolution, but short enough to isolate the stratigraphic interval of interest. As always, indiscriminate and offhand application could result in errors; pitfalls include having insufficient bandwidth, or a window of inappropriate length. But as a companion to these other techniques, cepstral decomposition promises to be a valuable stratigraphic analysis tool.

References

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