The Exact Elastic Impedance

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Abstract

A new elastic parameter, referred to as the Zoeppritz elastic impedance (ZEI), is derived by extending the concept of ray-path elastic impedance. Unlike the existing acoustic impedance (AI), elastic impedance (EI), and reflectivity impedance (RI) measures, ZEI gives an exact representation of the *P*-wave reflectivity at all angles and velocity/density contrasts, and is given by a recursive formula suitable for conventional impedance inversion. Especially, the ZEI is exact even beyond the critical angle which might give us a new approach to analyze seismic attribute over critical angle. ZEI can be used in the same style and context of AI,EI,or RI inversion and interpretation. In several end-member limiting cases, ZEI reduces to AI, EI, RI, and other approximate definitions of elastic impedance. Similarly to all other forms of impedance, ZEI requires normalization for comparison with rock properties. Along with facilitating accurate inversion, the exact ZEI also helps in the analysis of the non-uniqueness of the traditional AI or EI inversion. From the seismic reflectivity data, ZEI can be obtained either by creating common-ray parameter gathers or from τ-p transformed CMP gathers. By combining the AVA information at all ranges of ray parameters or reflection angles and correlating them with rock properties, ZEI inversion leads to an improved and reliable lithological information.

Introduction

Conventional acoustic impedance (AI) inversion is based on an assumption of a *P* wave striking subsurface interfaces at normal incidence (Lindseth, 1979). When the offset range in a CDP gather is small and the reservoir is deep, such an assumption is approximately satisfied and the inversion produces reliable results. However, in the presence of amplitude variation with offset (AVO), and particularly at large offsets, reflection coefficients and polarities may differ significantly from those at normal incidence. In such cases, conventional seismic trace inversion needs to be modified by taking the AVO effects into consideration.

Connolly (1999) put forward the concept of Elastic Impedance (EI), which takes into account the dependence of the *P*-wave reflectivity on the incident angles. Poststack EI inversion results in an impedance section representing reflectivity at a constant incidence angle:

$$EI(\rho,\alpha,\beta,p)=\rho^{1-4K\sin^2\theta}\alpha^{1+\tan^2\theta}\beta^{-8K\sin^2\theta}, \qquad (1)$$

Although useful and efficient for capturing the AVO effects, this method relies on an unrealistic assumption of the *S*- to *P*-wave velocity ratio, $K=\beta/\alpha$, staying constant throughout the depth column. Also, because seismic velocity generally increases with depth, EI integration tends to utilize larger angles than involved in the actual reflection paths at shallow depths, and the resulting EI cannot be interpreted as a physical measure of impedance to wave propagation along any path. Furthermore, EI requires normalization that may not be easy to control (Whitcombe, 2002). These limitations restrict the applicability of EI.

In order to overcome the above limitations, Ma (2003) and Santos and Tygel (2004) proposed a generalized, or Reflection Impedance (RI) derived by integrating the reflectivity along ray path. The β/α ratio needed no longer remain constant; however, the density was assumed to be functionally related to the *S*-wave velocity (Potter et al., 1998). In particular, the dependence $\rho=\rho_0\beta^k$ led to simple functional forms (Ma, 2003; Santos and Tygel, 2004):

$$RI(\rho,\alpha,\beta,p)=\frac{\rho\alpha}{\sqrt{1-\alpha^2 p^2}}.(1-\beta^2 p^2)^{2(k+2)} \tag{2}$$

(also denoted GEI in Ma, 2003, and RI in Ma and Morozov, 2004), and:

$$RI(\rho,\alpha,\beta,p)=\frac{\rho\alpha}{\sqrt{1-\alpha^2 p^2}}.\exp(-2(\mathrm{k}+2)\beta^2 p^2) \tag{3}$$

(Santos and Tygel, 2004). RI represents the impedance as a function of the ray parameter, *p*, and *P*- and *S*-wave velocities (α and β, respectively), with density related to β. Note that both forms (2) and (3) were derived in the approximation of small *S*-wave conversion angles, $\rho\beta$<<1 (Ma 2003; Santos and Tygel, 2004), and thus a Taylor expansion of these forms yields yet another functional form for RI:

$$RI(\rho,\alpha,\beta,p)=\frac{\rho\alpha}{\sqrt{1-\alpha^2 p^2}}.\left(1-2(2+\mathrm{k})\beta^2 p^2\right). \tag{4}$$

Multiplicity of functional representations (1-4, with numerous other possible approximations) reflects the fundamental ambivalence of the concept of the impedance at non-normal incidence. Such functional dependencies of the form $I(\rho,\alpha,\beta,p)$ are possible only when the number of independent parameters is reduced to two by either assuming $\beta=K\alpha$ (Connolly, 1999) or $\rho=\rho(\beta)$ (Ma 2003; Santos and Tygel, 2004) throughout the entire depth interval of interest. Both of these assumptions are quite restrictive and their validity is difficult to ascertain. In particular, neither EI nor the various forms of RI are suitable for calculating postcritical reflectivity.

Below, we propose a new elastic impedance measure (hereafter referred to as ZEI) based solely on the available sonic and seismic data and utilizing the exact Zoeppritz equations. While removing the approximations and ambiguities of the previous definitions, the method operates in a similar way and in the same context as the EI and RI approaches.

Zoeppritz Elastic Impedance as a ray-path functional

Elastic impedance, $I(\rho,\alpha,\beta,p)$, is a quantity arising in reflection seismic trace inversion and is defined through its recursive relation to the *P*-wave reflectivity:

$$R_{PP_i}(p)=\frac{I(\alpha_{i+1},\beta_{i+1},\rho_{i+1},p)-I(\alpha_i,\beta_i,\rho_i,p)}{I(\alpha_{i+1},\beta_{i+1},\rho_{i+1},p)+I(\alpha_i,\beta_i,\rho_i,p)}. \tag{5}$$

Here, R_{PP} is the exact reflectivity at the contact of two layers numbered *i* and *i*+1. By inverting the full recursive equation (5), for *I*(…), the exact Zoeppritz Elastic Impedance (ZEI) becomes:

$$ZEI[\mathscr{R}]=ZEI(\alpha_0,\beta_0,\rho_0,p)\prod_{i=0}^{N-1}\frac{1+R_{PP,i}(p)}{1-R_{PP,i}(p)}, \tag{6}$$

where *N* is the number of layers, and the normalization factor is defined as the impedance of the top layer.

The meaning of ZEI defined here is similar to that of AI but applicable to non-normal incidence, ray-path seismic trace inversion. At normal incidence (*p* = 0), ZEI becomes the acoustic impedance (AI) and equation (5) becomes the reflectivity at normal incidence. Therefore, all the modeling and inversion methods based on AI are applicable to ZEI as well. Also, for small velocity contrasts, ray parameters, and additional constraints, ZEI reduces to the approximate forms (1-4) above. Figure 1 compares these approximations with the accurate Zoeppritz reflection coefficients in the fluid substitution model (Hilterman, 2001).

ZEI Eq. (3) is more accurate than any other approximations, and its computer implementation or interpretation are no more difficult than those of Eqs. (2-4). Although the different approximations are relatively accurate within parts of the incident angle range, the use of ZEI would guarantee the accuracy at all (including postcritical) angles. For example, considering only the gas cases, the use of different approximations could lead to erroneous zero amplitudes and polarity changes. In addition, the use of Eq. (3) could also lead to confusing the water-saturated with oil-saturated sands AVO response resulting in erroneous water substitution prediction. Eq. (1) is accurate at small incidence angles when using correct $K=\beta/\alpha$ values. However, for multiple reflection layers, its constant *S*- to *P*-wave velocity ratio assumption *K*=const could no longer be appropriate for predicting AVO anomalies or reservoir for all layers of interest.

Because ZEI or RI attributes are computed in the ray-parameter, *p*, domain, they can be naturally presented in the form of *t*-*p* panels giving complete description of the AVA properties of the model. However, they could also be presented in the form of common-reflection angle columns as the EI is commonly displayed (Connolly, 1999).

ZEI Normalization and Comparison EI and RI

When calculating ZEI using equation (6) or integration method, one needs to specify the first value $ZEI(\alpha_0,\beta_0,\rho_0,p)$ of the ZEI series. This parameter is not constrained by the elastic impedance definition (5) and could be chosen so that ZEI approaches AI, EI, or RI (Eqs. 2-4) in the corresponding limiting cases. Generally, we can calibrate $ZEI(\alpha_0,\beta_0,\rho_0,p)$ by the values of AI_0, $EI(\alpha_0,\beta_0,\rho_0,p)$, or $RI(\alpha_0,\beta_0,\rho_0,p)$, or by the corresponding average values within a range of reflection angles or ray parameters.

Comparison of the ZEI and EI also highlights a somewhat subtle inconsistency in the conventional EI or RI inversion. Conventional EI inversion combines the low-frequency component computed from the EI (or RI) approximation and a high-frequency component computed from seismic data. However, the high-frequency impedance computed from seismic data should in fact correspond to the accurate, ZEI model. At the same time, the low-frequency component of EI strongly deviates from ZEI, particularly at the intermediate reflection angles. Therefore, in EI (as well as AI or RI) inversion, the low-frequency model does not match the high- frequency component of the inverted EI. The concept of ZEI offers a consistent approach to resolving this contradiction: with the use of ZEI, acoustic logs and both the low- and seismic-frequency data would be treated in a uniform and accurate manner.

Because ZEI is derived from the accurate, plane-wave Zeoppritz equation, it reflects rock properties better than any other definition of elastic impedance. When applying this method to the real data, we do not need to constrain the *P*- and *S*-wave velocities and density across the interfaces. Therefore, ZEI could be used to invert any type of rocks with large velocity contrasts across the interfaces, such as volcanic rocks. *P*-wave ZEI concept can also be extended to the P-SV wave conversion case.

Conclusions

Zoeppritz Elastic Impedance (ZEI) reproduces accurate oblique-incidence reflection coefficient series in complex layered models and thus offers an improved alternative to the several forms of elastic impedance (EI) or reflection (ray-path) impedance (RI), as well as to the normal-incidence acoustic impedance (AI). By removing the limitations of a small *P*- and S-wave velocity and density contrasts, and the requirements for constant *P*- to *S*-wave velocity ratios or $\rho\sim\beta^k$ relations assumed in EI or RI, respectively, the ZEI formulation becomes significantly more general and accurate.

The physical meaning of ZEI is that of impedance exerted on a plane *P* wave propagating through the layered medium. Using the ZEI concept, one can accurately solve the nonzero-offset seismic trace inversion problem and reduce the non-uniqueness of the traditional inversion based on other impedance definitions. By contrast to the traditional seismic trace inversion, ZEI method would be suited to the inversion of seismic data at any offset within the critical angle.

ZEI formulation opens a number of venues for application of EI in seismic interpretation and inversion. Due to its relation to the ray-parameter domain, ZEI can be derived directly from τ-p stacks either using or bypassing the AVA stacks. The ray-parameter formulation could lead to a straightforward extension of ZEI to the cases of dipping structures in 3-D. ZEI can also be calibrated to any desired rock property with which it correlates. Because ZEI represents the most accurate representation of the relation of seismic responses and rock properties, it might be useful in the prediction of the fluid state of reservoirs and in time-lapse seismic observations.

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Table 1. Fluid Substitution Modeling (Hilterman, 2001)

	Vp (m/s)	Vs (km/s)	Density (gm/cm^3)	Sw	φ
Water Saturated Sand (Sand=85%,Shale=15%)	3470	2010	2.29	100%	22%
Oil substitution	3362	2042	2.218	30%	22%
Gas substitution	3290	2080	2.14	30%	22%

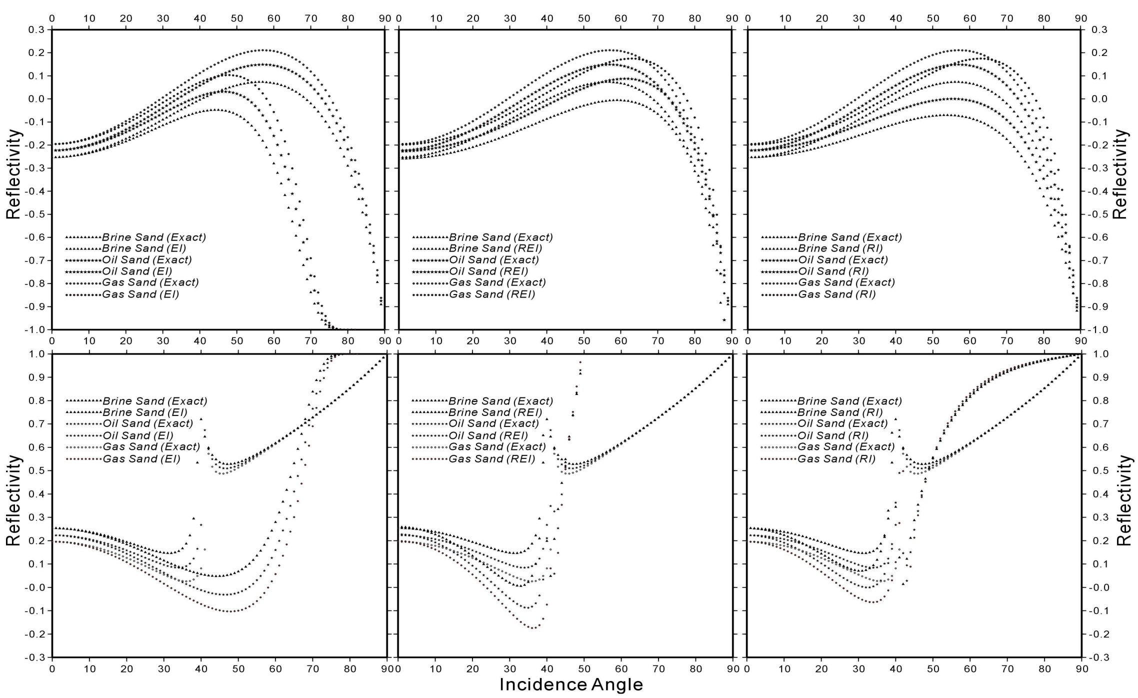

Figure 1. Fluid substitution model AVO responses of different reflectivity approximations are compared with the exact reflectivity. Water-saturated sand (brine sand), oil-saturated sand and gas-saturated sand parameters are from Table 1 (Hilterman,2001). Upper three figures show shale-sand interface AVO responses.Bottom three figures show sand-shale interface AVO responses.Shale parameters are α_1=2191.56m/s, β_1=818.1m/s,ρ_1=2.16 g/cm^3.The exact reflectivity is corresponding to Zoeppritz equation; EI corresponds to eq. (1), where K is selected as $((\beta_2/\alpha_2)^2+(\beta_1/\alpha_1)^2)/2$;REI and RI correspond to eq. (2) and eq. (3), with $k = \ln(\rho_2/\rho_1)/\ln(\beta_2/\beta_1)$.