A Generalized Deconvolution Approach for Local Radon Transforms

Mauricio D. Sacchi, Department of Physics, University of Alberta, Edmonton, Canada

2005 CSEG National Convention

Introduction

A chief problem in seismic data processing is the filtering of unwanted events like ground roll and multiples. Methods to deal with this problem often exploit moveout or curvature differences between offending events and the events one would like to preserve (primaries). In particular, removal of multiples based on moveout discrimination can be attained via parabolic and hyperbolic Radon transforms. In the parabolic transform, seismic data (after normal-moveout correction) are assumed to be composed of a superposition of parabolas; in the second case, the data are assumed to be a superposition of hyperbolas. Methods exist to enhance the resolution of both hyperbolic and parabolic Radon transforms (Thorson and Claerbout, 1985; Sacchi and Ulrych, 1995). In both cases, the operator capable of inverting the Radon transform is constructed in such a way that the Radon panel exhibits minimum entropy or maximum sparseness (synonymous used to describe a distribution of isolated events in the Radon panel). The sparseness assumption might not be optimal when there is a mismatch between the integration path of the Radon operator and the spatial-temporal signature of the seismic event. Amplitude variation with offset can further complicate the problem, as described by Spagnolini (1994). It is clear that assumption of sparseness or simplicity can lead to erroneous results when there is a mismatch between the operator and the data waveforms. The latter can be overcome by constructing local operators. In other words, we propose to use operators that match the structure of the wavefield on small spatio-temporal apertures. Alternatively, one could attempt a much more ambitious path where the operators are extracted from the data (data driven process). This paper describes a method to construct local Radon operators. We show that these operators can be designed with any integration path. This new class of Radon operators is implemented through a strategy that is based on Generalized Convolution (GC) and Generalized Deconvolution (GD) (Sacchi et. al, 2004). We describe this idea in the following section.

Generalized convolution and linear Local Wavefield Operators

In the classical Radon transform we attempt to represent the data with a finite number of waveforms defined over the data aperture by means of the following expansion:

$$D = \sum_k \alpha_k \Phi_k, \tag{1}$$

where $\Phi_k, k=1,N$ are the basis functions (waveforms with linear, hyperbolic or parabolic paths) and $\alpha_k, k=1,N$ are the coefficients of the expansion. In general, the coefficients $\Phi_k, k=1..N$ represent the non-zero coefficients of the Radon panel. In the new approach we propose to adopt basis functions that are local (waveforms that operate on a sub-aperture of the full data aperture). In this case we propose to represent the data using the following model (GC):

$$D = \sum_{k=1}^{N} F_k \otimes B_k\,, \tag{2}$$

In equation (2), the data are represented via the convolution of compact Local Wavefield Operators (LWO) B_k and an unknown suite of filters $F_k, k=1,N$. The symbol $\otimes$ represents multi-dimensional convolution. The problem reduces to finding the filters F_k given the data D and the operators B_k. It is clear that such a problem needs to be solved using an iterative method that takes advantage of fast convolvers. In our algorithm we have adopted a Conjugate Gradient method with optimized convolution operators computed via multi-dimensional FFTs. The Local Wavefield Operators (LWO) proposed by Sacchi et al. (2004) consists of waveforms of constant ray parameter defined on a small aperture (5-7 traces). A suite of *N=25* LWOs were numerically designed for the purpose of computing local linear Radon transforms (local slant stacks). The operators are shown in Figure 1. Each operator is parameterized with a ray parameter, a seismic wavelet and an operator aperture. The ensemble of operators was constructed with *N=25* ray parameters spanning the local dips in the data. In Figure 2 we examine the decomposition of a seismic record containing hyperbolic and linear events. Multi-dimensional generalized filters F_k are first estimated by inverting equation (2). Then, a subset of operators $B_k, k = kl,...kh$ is used to reconstruct the data. The reconstructed data are computed with the following expression:

$$\widehat{D} = \sum_{k=kl}^{kh} D_k, \quad D_k = F_k \otimes B_k \tag{3}$$

In our example*, kl=11, kh=15.* Each member of the sum in equation (3) (D_k) is called a mode. The *k*-mode is a panel of size equal to the size of the data; it captures waveforms *primarily and locally* modeled by the operator B_k . We have reconstructed the data using dips that locally model the hyperbolas. Residual energy from linear events leaks in the reconstructed model of hyperbolas. The signals are not orthogonal to each other and therefore, some degree of leakage is expected. Regularization strategies for inverse problems can be adopted to alleviate the aforementioned problem. This is discussed in the following section.

The procedure outlined above was also used to eliminate ground roll from a shot gather from the WCSB (Figure 3). In this example, N=41 LWOs were adopted for the generalized deconvolution. A subset of *11* modes was retained to reconstruct the data.

Parabolic Local Wavefield Operators

We adopt the same mathematical structure to model seismic data but now the LWOs represent waveforms with parabolic moveout. Each waveform is parameterized by a curvature. Figure 4 displays the synthetic seismic record used to test our algorithm. The goal is to separate the two events using generalized deconvolution. A suite of *11* LWOs with parabolic moveout is depicted in Figure 5. In Figure 6 we portrayed the filters F_k; the associated modes are portrayed in Figure 7. In this case the filters were computed using the least-squares method. The modal decomposition cannot capture the individual waveforms in the original data. The least squares method yields a solution where the energy is distributed over all the modes. We can circumvent the problem by introducing sparse regularization into the solution of equation (2) (Sacchi and Ulrych, 1995; Trad et al., 2003). Now, we observe that the ensemble of filters can capture the two signals quite well (Figure 8). The modal decomposition in Figure 9 has correctly identified the two waveforms. It is clear that the full reconstruction of the data (sum of all the modes) has provided the right reconstruction for both the Least Squares and the Sparse Least-Squares solution. The advantage of using a solver with sparseness constraints is quite evident: we have achieved simplicity in the filters and event separation in the modes.

Summary

We have presented a generalized convolution/deconvolution approach to solve the problem of waveform separation and filtering. The methodology is designed to represent seismic data in terms of Local Wavefield Operators. The ideas presented in this paper have numerous applications: random and coherent (aliased) noise attenuation, interpolation beyond aliasing, wavefield separation, filtering of diffracted multiples, etc. Similarly, these ideas can lead to interesting algorithms for migration velocity analysis where the focusing power of the filter ensemble may well be used for velocity estimation.

References

Sacchi, M.D., and Ulrych, T.J., 1995, High resolution velocity gathers and offset space reconstruction: Geophysics, **60**, 1169-1177.
Sacchi, M.D., Verschuur, D.J. and Zwartjes, P.M., 2004, Data reconstruction by generalized convolution: 74rd Ann. Internat. Mtg.: Soc. of Expl. Geophys., CDROM.
Spagnolini, U., 1994, Compound events decomposition and the interaction between AVO and velocity information: Geophys. Prosp., **42** ,241-259.
Thorson, J. R. and Claerbout, J. F., 1985, Velocity stack and slant stochastic inversion: Geophysics, **50,** 2727-2741.
Trad, D., Ulrych, T. and Sacchi, M., 2003, Latest views of the sparse Radon transform: Geophysics, **68,** 386-399.

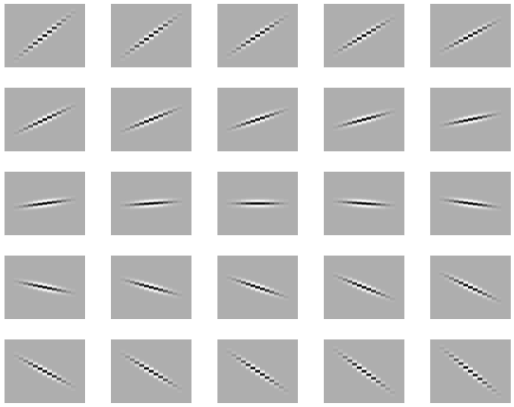


Figure 1. Linear Local Wavefield Operators (*N=25*).

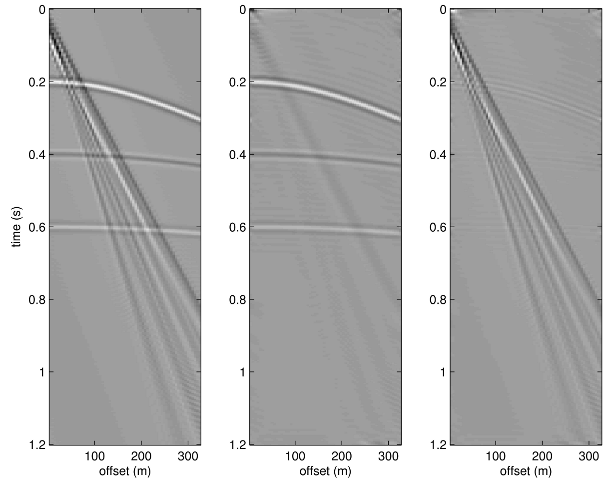


Figure 2. Synthetic shot gather (left). Reconstruction using modes *k=11…14* associated to the Local Wavefield Operators in Figure 1 (center). Residual panel (right).

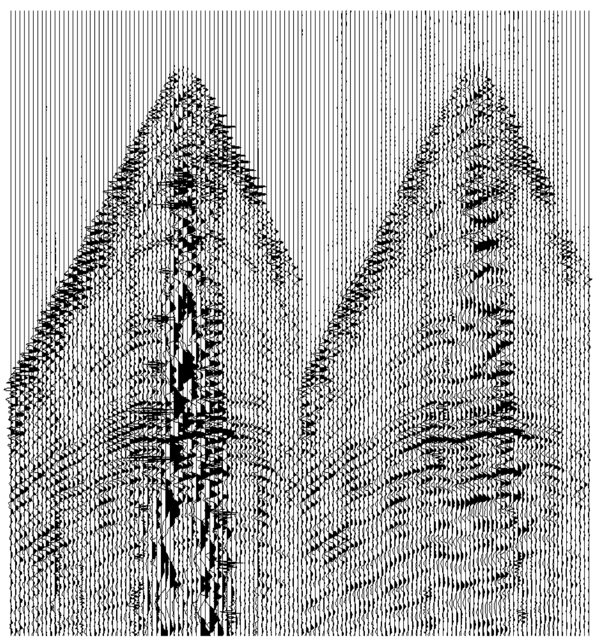


Figure 3. Linear noise removal using Generalized Deconvolution. Linear Local Wavefield Operators were deconvolved from the data. The modes capturing dips associated to the ground roll were eliminated from the data (right).

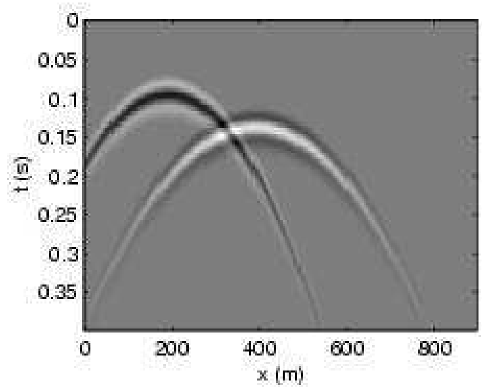


Figure 4. Synthetic example used to test the decomposition with parabolic Local Wavefield Operators.

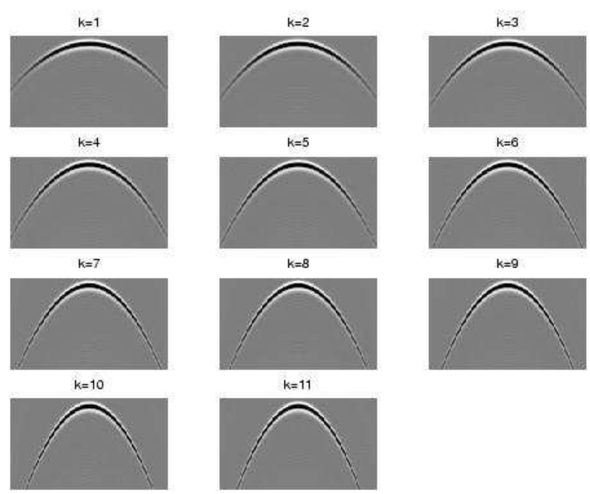
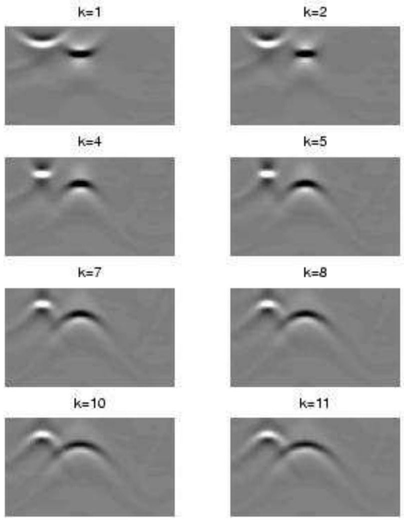


Figure 5. Local Wavefield Operators with parabolic moveout.



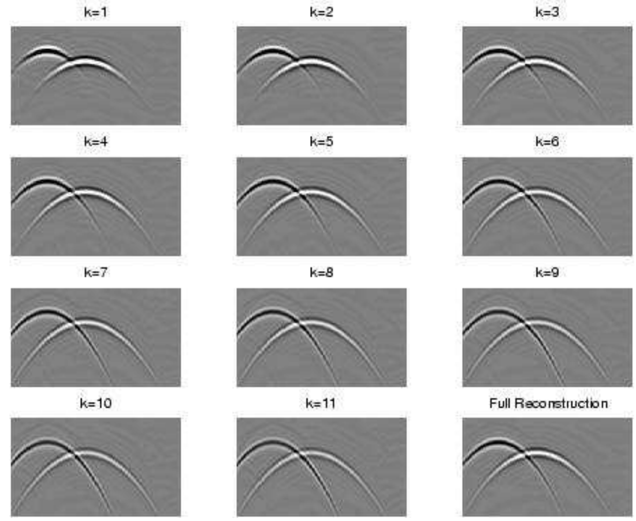
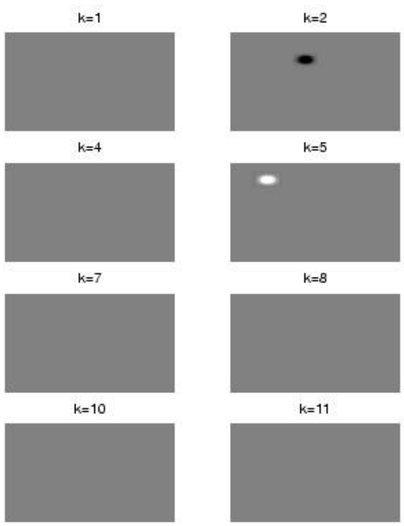


Figure 6. Filters computed using the least-squares method with no regularization.

Figure 7. Modes obtained by convolving the filters from Figure 6 with parabolic Local Wavefield Operators. The last sub panel is the full data reconstruction (sum of all modes).



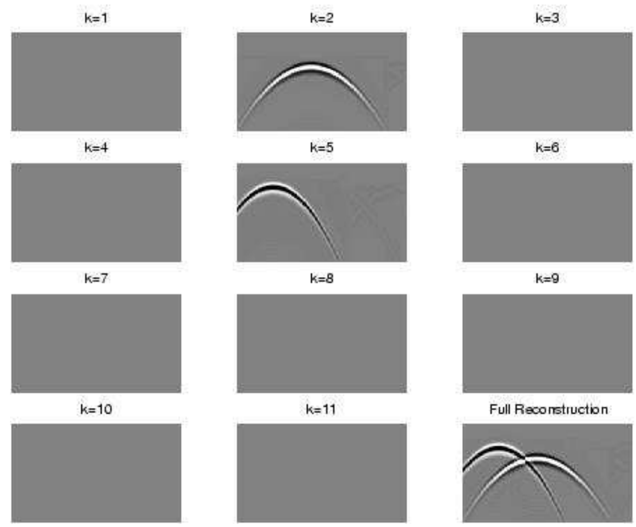


Figure 8. Filters computed using the least-squares method with sparseness constraint regularization.

Figure 9. Modes obtained by convolving the filters from Figure 8 with parabolic Local Wavefield Operators. The last sub panel is the full data reconstruction (sum of all modes).