Simultaneous inversion of pre-stack seismic data

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Summary

In this paper, we present a new approach to the simultaneous pre-stack inversion of PP and, optionally, PS angle gathers for the estimation of P-impedance, S-impedance and density. Our algorithm is based on three assumptions. The first is that the linearized approximation for reflectivity holds. The second is that PP and PS reflectivity as a function of angle can be given by the Aki-Richards equations. The third is that there is a linear relationship between the logarithm of P-impedance and both S-impedance and density. Given these three assumptions, we show how a final estimate of P-impedance, S-impedance and density can be found by perturbing an initial P-impedance model. After a description of the algorithm, we then apply our method to both model and real data sets.

Introduction

The goal of pre-stack seismic inversion is to obtain reliable estimates of P-wave velocity (V_P), S-wave velocity (V_S), and density (ρ) from which to predict the fluid and lithology properties of the subsurface of the earth. This problem has been discussed by several authors. Simmons and Backus (1996) invert for linearized P-reflectivity (R_P), S-reflectivity (R_S) and density reflectivity (R_D), where

$$R_P = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho}\right] \quad (1)$$

$$R_S = \frac{1}{2}\left[\frac{\Delta V_S}{V_S} + \frac{\Delta \rho}{\rho}\right] \quad (2)$$

$$R_D = \frac{\Delta \rho}{\rho} \quad (3)$$

Simmons and Backus (1996) also make three other assumptions: that the reflectivity terms given in equations (1) through (3) can be estimated from the angle dependent reflectivity $R_{PP}(\theta)$ by the Aki-Richards linearized approximation (Richards and Frasier, 1976), that ρ and V_P are related by Gardner's relationship (Gardner et al. 19), given by

$$\frac{\Delta \rho}{\rho} = \frac{1}{4}\frac{\Delta V_P}{V_P}, \quad (4)$$

and that V_S and V_P are related by Castagna's equation (Castagna et al., 1985), given by

$$V_S = (V_P - 1360)/1.16. \quad (5)$$

The authors then use a linearized inversion approach to solve for the reflectivity terms given in equations (1) through (3).

Buland and Omre (2003) use a similar approach which they call Bayesian linearized AVO inversion. Unlike Simmons and Backus (1996), their method is parameterized by the three terms $\Delta V_P / V_P, \Delta V_S / V_S$, and $\Delta \rho / \rho$, again using the Aki-Richards approximation. The authors also use the small reflectivity approximation to relate these parameter changes to the original parameter itself. That is, for changes in P-wave velocity they write

$$\frac{\Delta V_P}{V_P} \approx \Delta \ln V_P, \quad (6)$$

where ln represents the natural logarithm. Similar terms are given for changes in both S-wave velocity and density. This logarithmic approximation allows Buland and Omre (2003) to invert for velocity and density, rather than reflectivity, as in the case of Simmons and Backus (1996). Unlike Simmons and Backus (1996), however, Buland and Omre (2003) do not build in any relationship between P and S-wave velocity, and P-wave velocity and density.

Post-stack inversion for P-impedance

In the present study, we extend the work of both Simmons and Backus (2003) and Buland and Omre (1996), and build a new approach that allows us to invert directly for P-impedance ($Z_P=\rho V_P$), S-impedance ($Z_P=\rho V_P$), and density through a small reflectivity approximation similar to that of Buland and Omre (2003), and using constraints similar to those used by Simmons and Backus (1996). It is also our goal to extend an earlier approach to post-stack impedance inversion (Hampson and Russell, 1991) so that this method can be seen as a generalization to the inversion of pre-stack data. We will therefore first review the principles of the post-stack inversion method. First, by combining equations (1) and (6), we can show that the small reflectivity approximation for the P-wave reflectivity is given by

$$R_{Pi} \approx \frac{1}{2}\Delta \ln Z_{Pi} = \frac{1}{2}\left[\ln Z_{Pi+1} - \ln Z_{Pi}\right], \tag{7}$$

where i represents the i^{th} interface between layers i and $i+1$. If we consider an N sample reflectivity, equation (7) can be written in matrix form as

$$\begin{bmatrix} R_{P1} \\ R_{P2} \\ \vdots \\ R_{PN} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 0 & \cdots \\ 0 & -1 & 1 & \ddots \\ 0 & 0 & -1 & 1 \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix}\begin{bmatrix} L_{P1} \\ L_{P2} \\ \vdots \\ L_{PN} \end{bmatrix}, \tag{8}$$

where $L_{Pi} = \ln(Z_{Pi})$. Next, if we represent the seismic trace as the convolution of the seismic wavelet with the earth's reflectivity, we can write the result in matrix form as

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} w_1 & 0 & 0 & \cdots \\ w_2 & w_1 & 0 & \ddots \\ w_3 & w_2 & w_1 & \ddots \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix}\begin{bmatrix} R_{P1} \\ R_{P2} \\ \vdots \\ R_{PN} \end{bmatrix}, \tag{9}$$

where T_i represents the i^{th} sample of the seismic trace and w_j represents the j^{th} term of an extracted seismic wavelet. Combining equations (8) and (9) gives us the forward model which relates the seismic trace to the logarithm of P-impedance:

$$T = (1/2)WDL_P, \tag{10}$$

where W is the wavelet matrix given in equation (9) and D is the derivative matrix given in equation (8). If equation (10) is inverted using a standard matrix inversion technique to give an estmate of L_P from a knowledge of T and W, there are two problems. First, the matrix inversion is both costly and potentially unstable. More importantly, a matrix inversion will not recover the low frequency component of the impedance. An alternate strategy, and the one adopted in our implementation of equation (10), is to build an initial guess impedance model and then iterate towards a solution using the method of conjugate gradients.

Extension to pre-stack inversion

We can now extend the theory to the pre-stack inversion case. The Aki-Richards equation was re-expressed by Fatti et al. (1994) as

$$R_{PP}(\theta) = c_1 R_P + c_2 R_S + c_3 R_D, \tag{11}$$

where $c_1 = 1 + \tan^2\theta$, $c_2 = -8\gamma^2 \tan^2\theta$, $c_3 = -0.5\tan^2\theta + 2\gamma^2 \sin^2\theta$, $\gamma = V_S / V_P$, and the three reflectivity terms are as given by equations (1) through (3).

For a given angle trace $T(\theta)$ we can therefore extend the zero offset (or angle) trace given in equation (10) by combining it with equation (11) to get

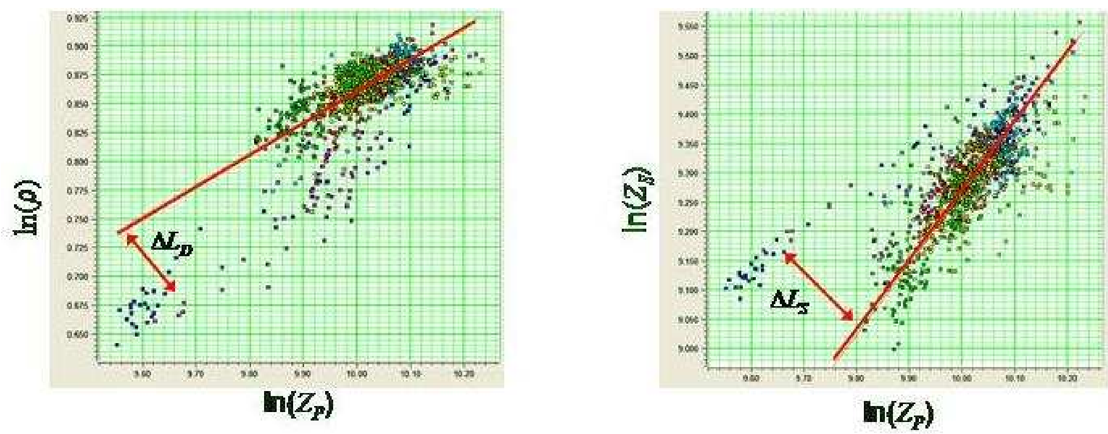
$$T(\theta) = (1/2)c_1 W(\theta) D L_P + (1/2)c_2 W(\theta) D L_S + W(\theta) c_3 D L_D \, , \tag{12}$$

where $L_S = \ln(Z_S)$ and $L_D = \ln(\rho)$. Note that the wavelet in now dependent on angle. Equation (12) could be used for inversion, except that it ignores the fact that there is a relationship between L_P and L_S and between L_P and L_D. Because we are dealing with impedance rather than velocity, and have taken logarithms, our relationships are different than those given by Simmons and Backus (1996) and are given by

$$\ln(Z_S) = k \ln(Z_P) + k_c + \Delta L_S \, , \tag{13}$$

$$\ln(Z_D) = m \ln(Z_P) + m_c + \Delta L_D \, . \tag{14}$$

That is, we are looking for deviations away from a linear fit in logarithmic space. This is illustrated in Figure 1.



(a) (b)

Figure 1. Crossplots of (a) ln(Z_D) vs ln(Z_P) and (b) ln(Z_S) vs ln(Z_P) where, in both cases, a best straight line fit has been added. The deviations away from this straight line, ΔL_D and ΔL_S, are the desired fluid anomalies.

Combining equations (12) through (14), we get

$$T(\theta) = \tilde{c}_1 W(\theta) D L_P + \tilde{c}_2 W(\theta) D \Delta L_S + W(\theta) c_3 D \Delta L_D \, , \tag{15}$$

where $\tilde{c}_1 = (1/2)c_1 + (1/2)kc_2 + mc_3$ and $\tilde{c}_2 = (1/2)c_2$. Equation (15) can be implemented in matrix form as

$$\begin{bmatrix} T(\theta_1) \\ T(\theta_2) \\ \vdots \\ T(\theta_N) \end{bmatrix} = \begin{bmatrix} \tilde{c}_1(\theta_1)W(\theta_1)D & \tilde{c}_2(\theta_1)W(\theta_1)D & c_3(\theta_1)W(\theta_1)D \\ \tilde{c}_1(\theta_2)W(\theta_2)D & \tilde{c}_2(\theta_2)W(\theta_2)D & c_3(\theta_2)W(\theta_2)D \\ \vdots & \vdots & \vdots \\ \tilde{c}_1(\theta_N)W(\theta_N)D & \tilde{c}_2(\theta_N)W(\theta_N)D & c_3(\theta_N)W(\theta_N)D \end{bmatrix} \begin{bmatrix} L_P \\ \Delta L_S \\ \Delta L_D \end{bmatrix} . \tag{16}$$

If equation (16) is solved by matrix inversion methods, we again run into the problem that the low frequency content cannot be resolved. So, a practical approach is to initialize the solution to $\begin{bmatrix} L_P & \Delta L_S & \Delta L_D \end{bmatrix}^T = \begin{bmatrix} \ln(Z_{P0}) & 0 & 0 \end{bmatrix}^T$, where Z_{P0} is the initial impedance model, and then to iterate towards a solution using the method of conjugate gradients. We will also show how to extend the theory in equations (15) and (16) by including PS angle gathers as well as PP angle gathers.

Model example

We will now apply this method to a model example. Figure 2(a) shows the well log curves for a gas sand on the left, with the initial guess curves (in red) set to be extremely smooth so as not to bias the solution. On the right are shown the input modelled gather from the full well log curves, the model from the initial guess, and the error, which is almost identical to the input. Figure 2(b) then shows the same displays after 20 iterations through the conjugate gradient inversion process. Note that the final estimates of the well log curves match the initial curves quite well, especially for the P-impedance, Z_P, and the Poisson's ratio (σ). The density (ρ) shows a bit of "overshoot" above the gas sand (which is at 3450 ms), but agrees with the correct result within the gas sand. The V_P and V_S curves also show some mismatch with the original curves, since they were derived by dividing the P and S-impedances by the density.

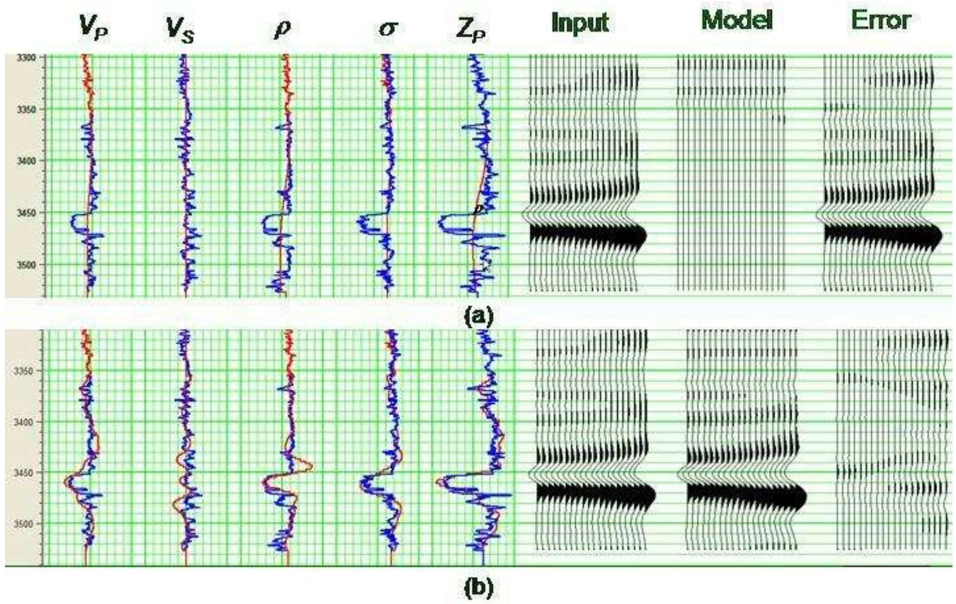


Figure 2. The results of inverting a gas sand model, where (a) shows the initial model before inversion, and (b) shows the results after inversion. In both plots, the original well log curves are in blue and the final estimated curves from the inversion are in red. The seismic gathers on the right hand side of the plot show, from left to right, the input model, the inverted result, and the difference between the input and inverted results.

When we look at the model results on the right of Figure 2(b), note that the inversion has done a good job in re-creating the original model, and that the error is very small.

Conclusions

We have presented a new approach to the simultaneous inversion of pre-stack seismic data which produces estimates of P-impedance, S-Impedance and density. The method is based on three assumptions: that the linearized approximation for reflectivity holds, that PP and PS reflectivity as a function of angle can be given by the Aki-Richards equations, and that there is a linear relationship between the logarithm of P-impedance and both S-impedance and density. Our approach was shown to work well for a modelled gas sand.

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