Computations of Circular Wavefront for Gridded Traveltime

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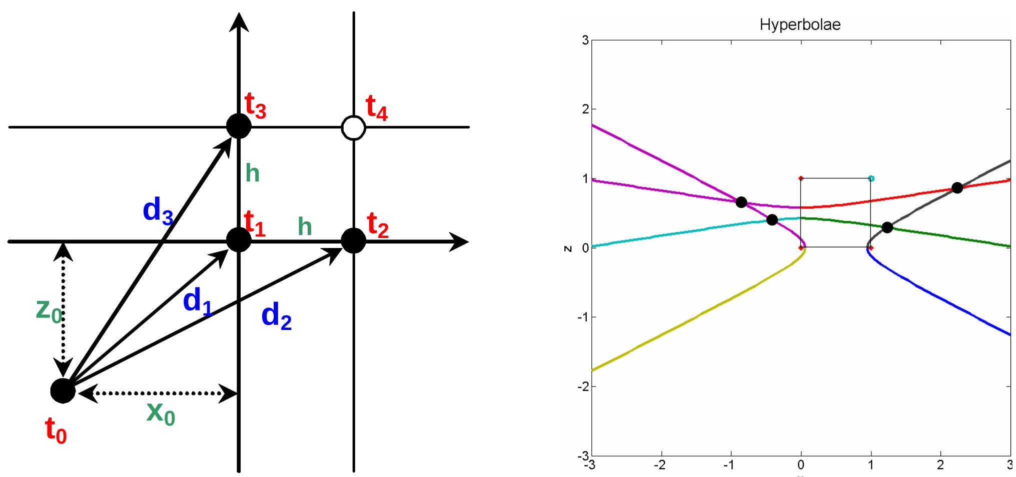
Introduction

Traveltime computations are an integral part of modelling and imaging seismic data by providing efficient kinematic information on the location of propagated energy. The traveltimes may be computed analytically using simplifying assumptions, or may be computed on a complex geological structure using raytracing or gridded traveltimes. A basic requirement for the propagation of gridded traveltimes is the estimation of one point on a corner of a square when given the traveltimes of the other three points on the square. A number of solutions are available to solve for the unknown time and are based on either a plane-wave assumption, a finite difference solution to the Eikonal equation, or an assumption that the wavefront at the square is curved. A known solution for estimating the center of curvature for a curved wavefront requires solving a quartic equation and choosing one of four possible solutions. An alternate method is presented to estimate the center of curvature using an iterative procedure that does not require solving the quartic equation.

Principle

The problem of estimating the center of curvature is illustrated in Figure 1a that shows one square of the grid with time t_1 at the origin, and the center of an equivalent circular wavefront at (x_0, z_0, t_0). Each side of the square has a dimension h, and the velocity v that is unique to this square, is assumed to occupy the entire space that includes the center of curvature. We desire to know the traveltime at t_4, which requires an estimation of t_0 and location of the source. Note that t_0 is not necessarily zero in areas with a complex wavefront due to varying velocities. Traveltime differences ($t_{21} = |t_2 - t_1|$) and ($t_{31} = |t_3 - t_1|$) may be used to define two sets of hyperbolas that are symmetric about the $z = 0$ and $x = 0$ axis. These hyperbolae are shown in Figure 1b along with the corners of the square and four intersections, all of which are possible source locations. A well known solution to this problem (Vidale 1988) requires solving a quartic equation and choosing one solution from the four possible choices.



a) b)

Figure 1 Three point problem showing in a) the geometry of an assumed source and b) the two sets of hyperbolae that result from two time differences.

Solving the quartic equation can be difficult, but four straight forward analytic solutions can be obtained using symbolic math in Mathematica. These equations occupy 100 lines of text, (simplifications are possible) that require significant computation time, and involve complex arithmetic that also slows the speed of the computation.

The new approach uses the three distances d_1, d_2, and d_3 from the geometry of Figure 1a, to define an equation that contains only one variable t_0. This equation

$$t_0 = t_1 - \frac{1}{v}\sqrt{\left\{\frac{v^2}{2h}\left[2t_0\left(t_2 - t_1\right) - t_2^2 + t_1^2\right] + \frac{h}{2}\right\}^2 + \left\{\frac{v^2}{2h}\left[2t_0\left(t_3 - t_1\right) - t_3^2 + t_1^2\right] + \frac{h}{2}\right\}^2},$$

can be solved using an iterative technique based on the Newton-Raphson method. Once the value of t_0 is estimated, the estimated center of curvature (x_0, z_0) is computed. Applications of this three point solution usually assume that the center of curvature is located in the third quadrant of Figure 1a, where this iterative technique provides a single solution.

Three methods for estimating t_4 are compared in Figure 2. A grid of spherical source points were defined in the third quadrant, and the three traveltimes t_1, t_2, and t_3 computed. The errors in estimating t_4 are compared using (a) the plane-wave assumption, (b) the Vidale finite difference method, and (c) the new iterative method. The source points are only located up to five square dimensions 5h away, and the maximum error is limited to ten percent. As expected in this region, the first two methods show significant error when compared with the iterative method.

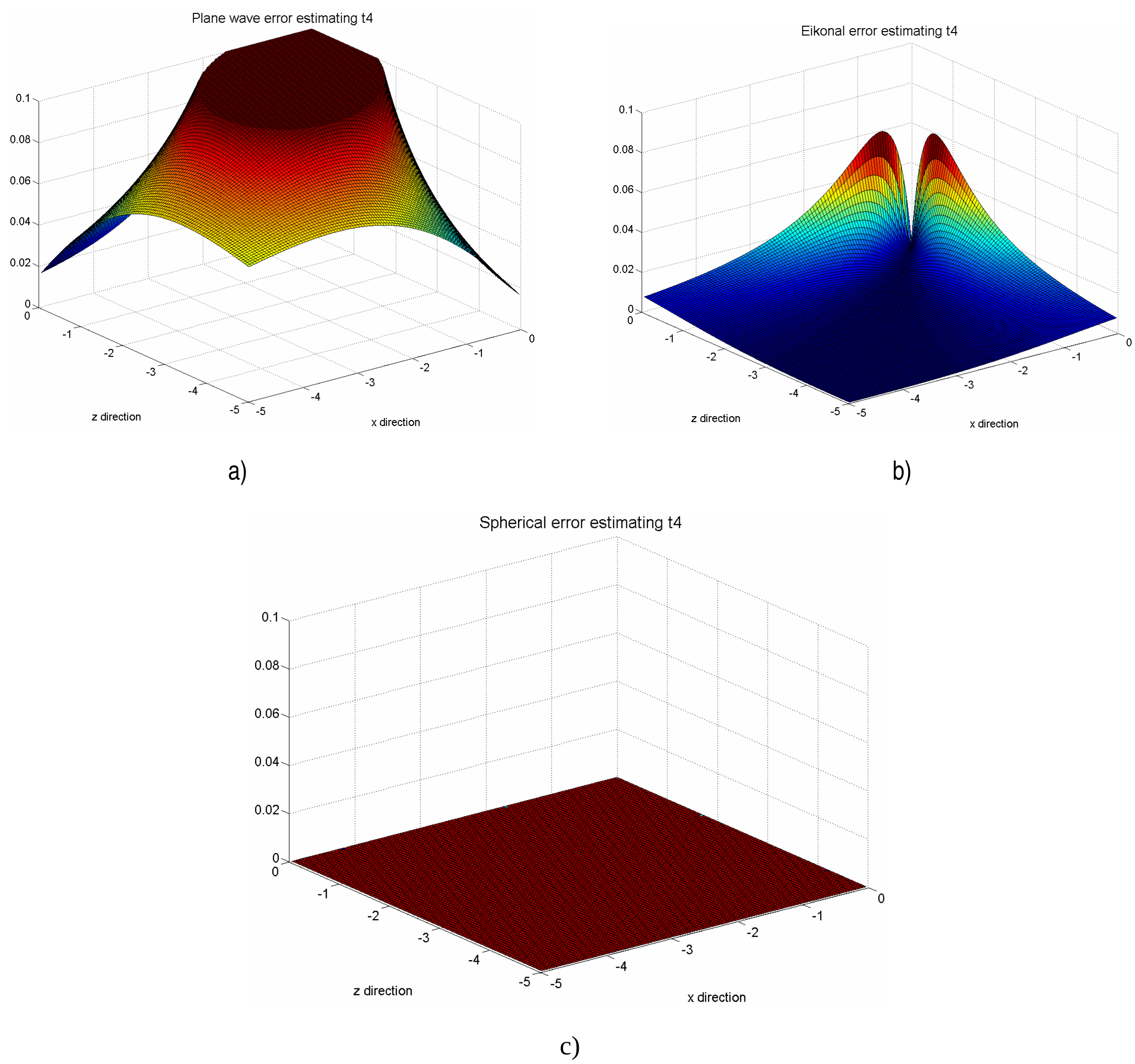


Figure 3 Comparison of errors in estimating the time t_4 using a) a plane wave assumption, b) Vidale's method, and c) the new iterative method.

Reference

Vidale, J., 1988, Finite-difference calculation of travel times, Bulletin of Seismological Society of America, Vol. 78, No. 6, pp 2062-2076