Seismic Traveltime Inversion

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Introduction

Geophysical inversion can be defined as a procedure, which allows us to obtain a subsurface model, which fits observed data. In this procedure, we use forward modeling to establish the connection between the subsurface model and the observed data. If M is a subsurface model and D is observed data then we can write:

$$F(M) = D \qquad (1)$$

where F is a forward modeling operator. This operator calculates geophysical data D for a given subsurface model M. Then inversion can be described as solving equation (1) for unknown model M. As the inverse problem is usually ill conditioned, additional constraints are very useful and play an important role in traveltime inversion. Instead of solving equation (1), we minimize an objective function using L_2 (the least square method) or L_1 norm, which is more stable when there are spikes and large errors in traveltimes.

Traveltime Inversion Scheme

For seismic traveltime inversion, we can use the same model (1) where D is observed traveltimes, M is a depth velocity model and F is a forward modeling operator, which calculates reflection traveltimes. We will consider some specific features, which distinguish traveltime inversion from the other inversion problems, such as full waveform inversion, migration, AVO inversion etc. Seismic traveltime inversion can be considered as a procedure, which includes three main steps:

- **Traveltime determination**
- **Initial model building and editing**
- **Improving initial model (layered traveltime inversion or tomography)**

Each step has its own problems and requires automatic procedures along with manual work and geophysicist interference. Let's shortly consider the main problems in each step.

Traveltime determination.

a. To obtain observed reflected traveltimes D is a separate and not an easy problem. In theory, one can manually pick events on the gathers, but in practice, this is usually done by automatic velocity analysis (Taner and Koehler, 1969). The main reason for not using autopicker is that any auto-picker will meet difficulties working with prestack data because of noise. Even for poststack data, where the noise level is much less, auto-picking is a big problem and always needs manual editing and geophysicist interfering. For prestack data, the geophysicist interference in picking events will be much greater. Taking into account the size of the prestack data, in most cases it's just impossible to use this approach.

b. To obtain traveltimes, we usually use velocity analysis. This procedure is much more stable than event auto-picking and the main reason for this is that in the velocity analysis we mostly calculate one parameter (hyperbolic approximation coefficient $1/V_{Stack}^2$) or at most two, if we use a non-hyperbolic approximation. It can be shown that increasing the number of estimated parameters, even by one, makes the others much less reliable. In the stacking velocity estimation, we are not interested in AVO effects and we want not to be dependent on them. At the same time, if we have the Class II AVO response (Rutherford and Williams, 1989), when the events have polarity reversals, we have to take into account this AVO effect because we may have no response on velocity spectra. For the traveltime inversion, we need to know traveltimes for each CDP point. Then automatic high-density velocity analysis becomes an important tool for traveltime determination. This kind of analysis has been developed in Revolution Geoservices Inc. It takes into account horizons, interval velocity constraints and using Dix's formula generalization (Blias, 2003).

c. As we often have coherent noise on the prestack data, we have to know where to look for the primaries. It means that we have to have some a priory information about traveltime characteristics. In terms of the velocity analysis, it's usually information about stacking velocity ranges. If we don't have strong shallow velocity anomalies then we can consider stacking velocities close to some kind of average velocities. In the presence of overburden velocity anomalies, the difference between stacking and average velocity can be arbitrarily big and there should be special procedures for automatic velocity analysis. In this case, an analytical connection between stacking and interval velocities can be helpful (Blias, 1981, 2003b, 2005). Fig. 1 shows the result of the high-density automatic velocity analysis

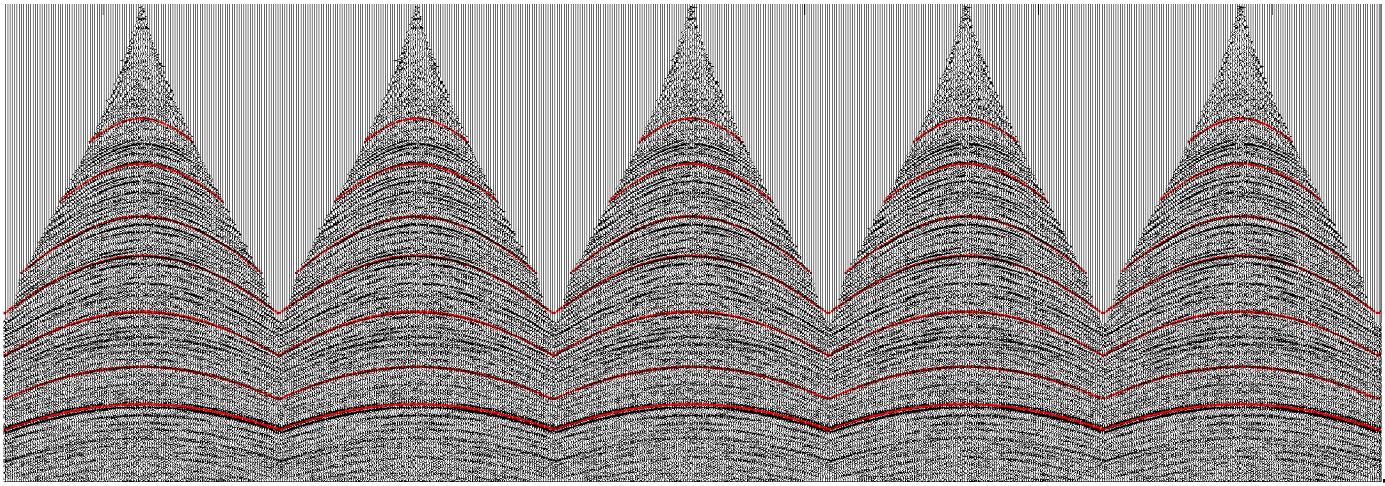

Fig. 1. High-density automatic traveltime picking

d. There is an important question about 3-term velocity analysis in order to decrease the bias of stacking velocities. Usually, the two-term velocity analysis is applied to prestack CDP gathers. If we have a long spreadlength (bigger than 1.5 reflector depth) we can try to determine the third term. For this we can use Malovichko's formula (Malovichko, 1978)

$$t(x) = t_0\left(1 - \frac{1}{S}\right) + \frac{1}{S}\sqrt{t_0^2 + S\frac{x^2}{V_{RMS}^2}}$$

where

$$S = \frac{\sum_{k=1}^{n} h_k / v_k \sum_{k=1}^{n} h_k v_k^3}{\left(\sum_{k=1}^{n} h_k v_k\right)^2}$$

For Western geophysicists, this formula is known as the shifted hyperbola approximation and was published by Castle in 1994 after translation of Malovichko's derivation of this formula. Using a statistical approach, it can be shown that simultaneous determination of stacking velocity and coefficient S leads to increasing standard deviation of V_{NMO} estimation by approximately six times. It implies that we should be doing three-term velocity analysis only for the data with big offset/depth ratio and low noise level.

Initial model determination

If we don't have shallow velocity anomalies, Dix's formula can be used for the interval velocity estimation. It means that we can use a local 1-D model to determine interval velocities. There are many factors that influence the result of this formula (Al-Chalabi, 1979), but the main is non-linear changes of the overburden velocities and dipping reflectors with angles more than 15^0. If there are no reflectors with big dips, the main factor, which can completely destroy Dix's interval velocity estimation, is nonlinear overburden velocity changes (Blias, 1981, 1988, 2003, 2005). In this case we have to use some generalizations of Dix's formula to obtain appropriate approximation to the depth velocity model (Krey and Hubral, 1980, Goldin, 1986, Blias, 2003). These formulas use a layer-by-layer approach, which has it's own pitfalls. It's interesting that for initial velocity depth model estimation we don't use equation (1). We can mention that the number of layers in the initial model depend on the number of reliable reflections that we can pick on poststack data and find them on prestack data, using velocity analysis. Some constraints should be used at this step. Interval velocities, obtained by using Dix's formula, are always biased because of non-hyperbolic NMO in layered ground. Their values depend on actual acquisition geometry. We can avoid this biasing, using an approach, suggested by Blias in 1983. To improve the initial depth velocity model, we can use iterative approach, described by Goldin (1986).

Improving initial model

After traveltime determination and building the initial depth velocity model, we can solve equation (1). We may change the parameterization of the model, if we use tomographic approach, or keep it the same if we use basic functions to describe reflection boundaries and interval velocities between these boundaries (Goldin, 1986, Blias and Khatchatrian, 2003). In the presence of shallow strong velocity anomalies, stacking velocities (and, consequently, traveltimes) depend on the second-order derivatives of the anomalies (Blias, 1981, 1988, 2003, 2005a, 2005b). This implies a smoothing problem (Blias and Gritsenko, 2003) for the curvilinear boundaries and laterally changing interval velocities as well. While raytracing, we have to take care not only about the boundaries and interval velocities but also about their second-order derivatives. To improve the initial model, we minimize the difference between the observed and calculated traveltimes (Goldin, 1986). In traveltime inversion we are looking for the lateral velocity changes only between the reflectors, that is, for interval velocities. In the presence of shallow velocity anomalies, some other problems arise (Blias, 1988, 2005a, 2005b)

Reflection tomography (Bishop et al., 1985, Stork and Clayton, 1991, 1992) utilizes another approach for improving reference depth velocity model. Can it improve the vertical resolution? Does the tomography give something extra compared to the layered traveltime inversion? Formally speaking "YES", but essentially "NO". By tomography we understand cell-based parameterization of a velocity distribution v(x,z) or using some functions with vertical changes between the reflectors. Reflectors are parameterized independently from the cells (Stork and Clayton, 1991). In the same paper, the authors state that the problem is "ill conditioned because several aspects of the model cannot be resolved with the given data… The poor behavior generally comes from instabilities of using a gradient approach with the given starting model. All the solutions should be considered suspect. Inversion will not replace the need for the user to make subjective decision", p. 485.

This does not sound very encouraging and at the same time does not tell us the real reason of the poor behaviour of our tomographic solution. The reason for this poor behaviour is that, in the tomographic approach, we try to use a velocity grid, which is not directly connected with the reflectors ("The velocity field is parameterized independently of the reflector positions as an effective continuum of desired accuracy. This parameterization places no inherent restrictions on the structure the velocity field can take on.", Stork and Clayton, 1991, p. 485). Generally speaking (with some exclusions), we should not expect to reliably determine any vertical velocity changes between the reflectors. When we use the cells between the reflectors or any other velocity parameterization, which is not connected with reflectors (includes vertical velocity changes between the reflectors), we just make the matrix very ill-conditioned and only hide a real problem but do not solve it. I will theoretically prove this statement for the laterally homogeneous ground and will confirm by a simple numerical example. It implies that we should not vertically divide layers between the reflectors, using cells, and try to find the velocity in the cells. Instead of this, we should consider layers as vertically homogeneous. What velocity we actually find, is another question. For the laterally homogeneous medium, we find RMS velocity in each layer. For laterally changing layer, it's hard to answer the question but, anyway, mostly we can determine only lateral changes in the interval velocities. It implies that the vertical resolution in our depth velocity model estimation depends on the number of reflectors (corresponding traveltimes) that we can define from the data.

The reason why reflection tomography cannot improve vertical resolution compare to traveltime inversion is the fact that some vertical changes in the velocity imply the second-order changes in the traveltime. To show this, let us consider a medium with two horizontal layers. We can write parametric traveltime equations with parameter λ (Taner and Koehler, 1969):

$$t(\lambda) = 2\sum_{k=1}^{2} \frac{h_k}{v_k\sqrt{1-\lambda^2 v_k^2}}$$

$$x(\lambda) = 2\lambda\sum_{k=1}^{2} \frac{h_k v_k}{\sqrt{1-\lambda^2 v_k^2}} \qquad (2)$$

Instead of the velocities v_1 and v_2 let us consider two parameters v and β:

$$v_1 = v(1-\beta), \qquad v_2 = v(1+\beta) \qquad (3)$$

where

$$v = ½\,(v_1 + v_2), \qquad \beta = (v_2 - v_1)/(v_1 + v_2) \qquad (4)$$

Then equations (2) can be written this way:

$$t(\lambda) = 2\frac{h}{v}\left(\frac{1}{(1+\beta)\sqrt{1-\lambda^2 v^2 (1+\beta)^2}}\right) + \left(\frac{1}{(1-\beta)\sqrt{1-\lambda^2 v^2 (1-\beta)^2}}\right)$$

$$x(\lambda) = 2\lambda v\left(\frac{1+\beta}{\sqrt{1-\lambda^2 v^2 (1+\beta)^2}}\right) + \left(\frac{1-\beta}{\sqrt{1-\lambda^2 v^2 (1-\beta)^2}}\right) \qquad (5)$$

Equations (5) show that the time t is an even function of the parameter β. Then in the Taylor series in the power of β, we have

$$t(x,\beta) = t_0(x) + \tfrac{1}{2} t_1(x)\beta^2 + t_1(x,0)\beta^4 + \ldots \quad (6)$$

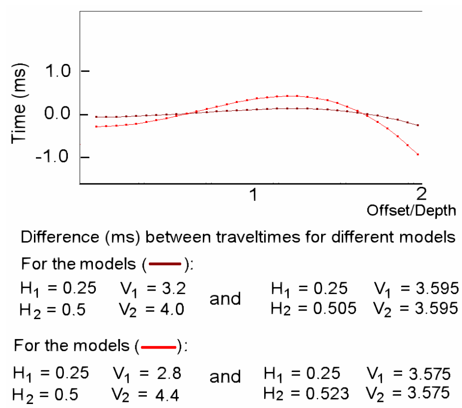

Fig. 2. Traveltimes difference for different depth velocity models

Function $t_0(x)$ corresponds to homogeneous layer and is a hyperbola. Function $t_1(x)$ is the second-order derivative with respect to β and can be obtained from parametric equations (5) or from Malovichko's formula. The explicit expression for this term is quite complicated and is not suitable for visual analysis.

This equation shows that traveltime t slightly depends on the parameter β, that is, on the small changes in the interval velocities v_1 and v_2. Equation (6) implies that if we have a layer, divided by an additional horizontal boundary into two horizontal layers, we cannot have reliable estimation of the interval velocities above and below the additional boundary, using only traveltime function from the bottom of the layer. This means that we cannot find feasible estimation of vertical velocity changes without additional (a priory) information and therefore should consider this layer as vertically homogeneous. It also implies that the velocity parameterization should not treat the velocity component and the reflector component as separate classes. We should consider interval velocities between the reflections and to improve vertical resolution of the depth velocity model, we have to pick more reflections.

Here is a simple numerical example, which shows that we cannot determine vertical changes even when they are significant. Let us consider a homogeneous horizontal layer. Reflected traveltimes have been calculated for the bottom of the layer. Fig. 2 shows the difference between traveltimes for the initial homogeneous layer and the two-layered model with different velocities. From it we can see that even for the model with the velocities 2.8 km/s and 4.2 km/s (50% difference!) and Offset/Depth ratio equal 2, we have almost the same traveltime function as for the homogeneous layer. This example, with the above consideration, confirms that in most cases we can only determine velocities between the reflectors. In other words, we should divide the ground into several layers according to picked horizons and consider determination of these layer velocities. With shallow velocity anomalies, the situation is different (because of their strong influence on stacking velocities from deep layers) and we can determine them from reflection using some additional constraints and assumptions.

Conclusions.

Seismic traveltime inversion scheme has been considered. It is composed of three main steps. High-density automatic velocity analysis has been developed in order to prepare data for traveltime inversion. Constraints are included in this analysis as well as Dix's formula generalization. To obtain an initial depth velocity model, we can use generalization of Dix's formula and an iterative algorithm. We can expect to find lateral changes in the interval velocities between the reflectors. Using reflection tomography, that is putting additional cells (or boundaries) between the reflectors, makes the inverse problem very ill-conditioned. In most cases, the tomographic approach does not allow us to improve vertical resolution compared to a layered traveltime inversion scheme when we consider vertical homogeneous layers between the reflectors.

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References

Al-Chalabi, M., 1979, Velocity determination from seismic reflection data: Development in Geophysical Exploration Methods, Vol.1, Applied Science Publishers

Bishop, T. N., Bube K., P., Cutler, R. t., Langan, R., T., Love, P., L., Resnick, J., R., Shuey, R., T., Spindler, D., A. and Wyld, H., W., 1985, Tomographic determination of velocity and depth in laterally varying media: Geophysics, **50**, p. 903-923

Blias E., 1981. Moveout approximation for a medium with curvilinear interfaces and variable interval velocities: Soviet Geology and Geophysics, **11**, p. 88 – 96

Blias E., 1982, Calculation of interval velocities for layered medium through hyperbolic traveltime approximation: Soviet Geology and Geophysics, **4**, p. 66 - 71.

Blias E., 1988 Reflected wave's traveltime in 3D layered media with curvilinear boundaries: Mathematic Problems of Seismic data interpretation. Novosibirsk (in Russian)

Blias, E., 2003a, Dix's type formulae for a medium with laterally changing velocity: SEG 73 Annual Meeting, Dallas, SEG Expanded abstracts.

Blias, E., 2003b, Stacking and Interval Velocities in a Medium with Laterally Inhomogeneous Layers: SEG Convention, Dallas, Expanded abstracts

Blias, E., 2005a, De-mystification of "errors" in stacking velocities. Classification, analysis and improvement of understanding: CSEG Convention.

Blias, E., 2005b, Bad stacking velocities in the presence of anomaly and their explanation: CSEG Convention.

Castle, R.J., 1994, A theory of normal moveout: Geophysics, **59**, 983-999.

Goldin, S., V., 1986. Traveltime Inversion, SEG.

Malovichko, A.A., 1978, A new representation of the traveltime curve of reflected waves in horizontally layered media: *Applied Geophysics*, **91**, 35-44, (in Russian).

Rutherford S., R. and Williams, R. H., 1989, Amplitude-versus-offset in gas sands: Geophysics, V. 54, p. 680-688

Stork, C. and Clayton, R. W., 1991, Linear aspects of tomographic velocity analysis: Geophysics. **56**. p. 483-495

Stork, C. and Clayton, R. W., 1992, Analysis of the Resolution between Ambiguous Velocity and Reflector Position for Traveltime Tomography, SEG Meeting

Taner, M., T. Koehler, T., 1969, Velocity Spectra: Digital computer derivation and applications of velocity functions: Geophysics**, 34**, p. 859-881.