Long-Offset Approximations to NMO Function for a Layered Subsurface

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Introduction

Conventional approximations on NMO function assume a modest offset/depth ratio. Conventional velocity analysis uses hyperbolic approximation for the reflection traveltimes:

$$t(x) = \sqrt{T_0^2 + x^2 / V_{RMS}^2} \quad (1)$$

Here T0 is a zero-offset time, x is an offset and VRMS is an RMS velocity. Traveltime function t(x) can be expanded into exact series of x2. It was first done by Bolshih (1956):

$$t(x) = a_0 + a_1 x^2 + a_2 x^4 + a_3 x^6 + ... \quad (A)$$

He gave formulas for the first four coefficients a_k in terms of interval velocities and indicated that all coefficients a_k might be derived one at a time. Taner and Koehler (1969) suggested representing t^2 as a series expansion of x^2:

$$t^2(x) = c_0 + c_1 x^2 + c_2 x^4 + c_3 x^6 + ... \quad (B)$$

where $c_0 = T_0^2$, $c_1 = 1/V_{RMS}^2$. Equations (B) can be obtained by squaring equation (A). Formula (B) is more accurate than (A) because for one layer model, the latter formula gives exact representation for t(x) while equation (A) remains as an infinite series. For the large offsets, hyperbolic approximation (1) is not accurate enough. Because exact formula for traveltimes is an infinite series, many authors proposed some improvements to hyperbola, using more than two parameters. Here we will compare different approximations with new ones.

An NMO formula is an approximate equation that connects offsets and traveltimes. Using more than two terms in the series (B) improves accuracy, but at the same time creates some problems for velocity estimation. Adding extra parameter leads to more expensive velocity analysis and increases coefficient c_1 variance approximately 10 times. This was noticed by Al-Chalabi through random modeling (1974, fig. B3) and can be shown analytically. Because adding fourth term increases c_1 variance about 10 times, from practical point of view, maximum number of estimated coefficients for one gather should be 3. For long offsets (offset/depth ≈ 1.5 – 2.5), we can expect significant improvement of V_{RMS} estimation using three-term velocity analysis, even though it leads to larger standard deviation.

Method

Equations (A) or (B) enable us to find derivatives of time t or t^2 with respect to x^2 at x = 0. There are different forms of approximations, proposed by several researches, but the main idea of all NMO approximations is to keep time t and its derivatives at x = 0 the same as of the exact travel time function t(x). Let's consider any three-term approximation of NMO function

$$t(x) = F(a,b,c,x^2) \tag{2}$$

where F is a function of four variables. To calculate coefficients a, b and c, we have to solve the system of three equations:

$$F(a,b,c,x=0) = T_0\,, \qquad \partial F(a,b,c,x=0)\big/\partial\left(x^2\right) = a_1 \qquad \partial^2 F(a,b,c,x=0)\big/\partial\left(x^2\right)^2 = a_2 \tag{3}$$

From these three equations, we determine a, b and c and find the explicit approximation (2). The main problem here is to choose function F so we can find explicit solution of system (3).

Different NMO Approximations

We will consider three-term approximation a form:

$$t(x) = \sqrt{T^2(0) + \frac{x^2}{V_{RMS}^2} + \frac{S-1}{4t_0^2 V_{RMS}^4}x^4} \tag{4}$$

Here S is a parameter, defined by formula

$$S = \left[\sum_{k=1}^{n} h_k / v_k \sum_{k=1}^{n} h_k v_k^3\right] \Bigg/ \left(\sum_{k=1}^{n} h_k v_k\right)^2 > 1$$

For homogeneous medium, S = 1. The more heterogeneity, the larger is S so we can consider the difference (S-1) as a degree of vertical velocity heterogeneity parameter. Malovichko (1979) derived a representation in a form of shifted hyperbola:

$$t(x) = t_0\left(1 - \frac{1}{S}\right) + \frac{1}{S}\sqrt{t_0^2 + S\frac{x^2}{v_{RMS}^2}} \tag{5}$$

His derivation was repeated by Castle (1994) using the same complicated approach with Gauss's hypergeometric series. Equation (5) can be derived using system (3). Equation (5) is more accurate than (4) because for large offsets it behaves as x, the same as exact rtraveltime function while function (4) behaves as x^2 for large offsets. Let us consider three-term NMO function similar to Alkhalifah and Tsvankin (1995). Using system (3), we come to NMO formula for isotropic medium:

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{v_{RMS}^2} - \frac{(S-1)x^4}{v_{RMS}^2\left(4t_0^2 v_{RMS}^2 + (3+S)x^2\right)}} \tag{6}$$

Taner et al. (2005) considered NMO representation in the form:

$$t(x)=\sqrt{t_0^2+\frac{x^2}{\left(v_{RMS}+ax^2\right)^2}} \tag{7}$$

where a is named an acceleration factor. System (3) for this approximation leads us to formula: $a=(S-1)/8t_0^2V_{RMS}$. Here we consider another approach to NMO formula. For a homogeneous medium with velocity V, V_{RMS} = V_{Ave} = V, and instead of (1) we can write: $t(x)=\sqrt{T_0^2+x^2/V_{Ave}^2}$, where V_{Ave} is an average velocity. For the layered medium, the last NMO formula should be corrected. For this, let's consider a rational function. To find its exact form, we use system (3), wich gives us a solution:

$$t(x)=\sqrt{\left(t_0^2+x^2/V_{Ave}^2\right)\Big/\left(1+\frac{gx^2}{t_0^2V_{Ave}^2(g+1)}\right)} \tag{8}$$

where g is a vertical heterogeneity factor, suggested by Al-Chalabi (1973, 1974):

$$g=\left(v_{RMS}/v_{Ave}\right)^2-1$$

We can improve formula (6) by using an additional term and the system (3). Then

$$t(x)=\sqrt{\frac{t_0^2+x^2/v_{Ave}^2}{1+gx^2/\left(t_0^2v_{RMS}^2\right)}-\frac{1}{2}\frac{g^2x^4}{v_{RMS}^2\left(t_0^2v_{RMS}^2+(1+g^2)x^2\right)}} \tag{9}$$

We will also consider NMO representation:

$$t(x)=\sqrt{t_0^2+-\frac{x^2}{V_{RMS}^2+\left((S-1)/4t_0^2\right)x^2}} \tag{10}$$

Formulas (8), (9) and (10) are new and derived using system (3).

Model Testing

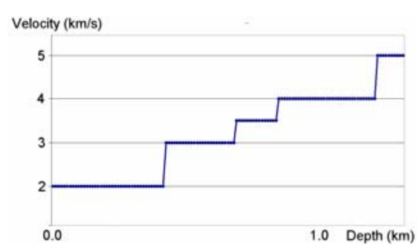

Figure 1. Layered depth velocity model.

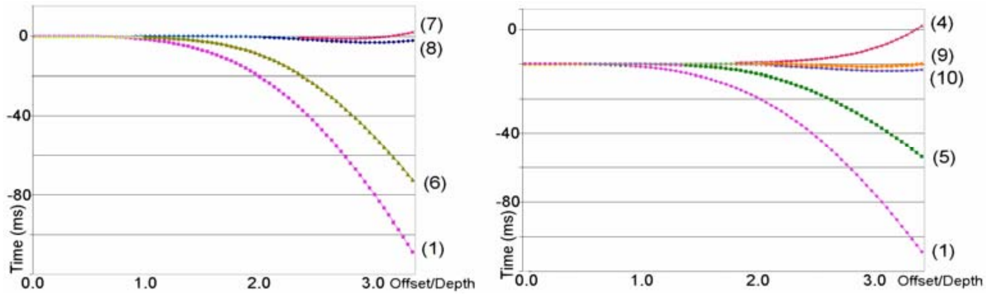

Figure 2. Traveltime residuals for different NMO approximations. Numbers coinside with formula numerations.

Let's consider a layered depth model, fig. 1. We calculated NMO functions using different approximations. Fig. 2 and 3 show the approximation errors. For offset/depth less than 1, all the approximations are accurate enough. Our new approximations (8) – (10) fit the exact traveltimes very well at large offsets up to 3 depth.

To investigate the accuracy of RMS velocity estimations through different NMO approximations in the presence of random noise, we simulated it with random time shifts (Al-Chalabi, 1974). Fig. 3 shows RMS velocity estimations obtained with different NMO approximations for offset/depth = 2.5. Formulas (8) and (10) give the most accurate estimation of RMS velocity.

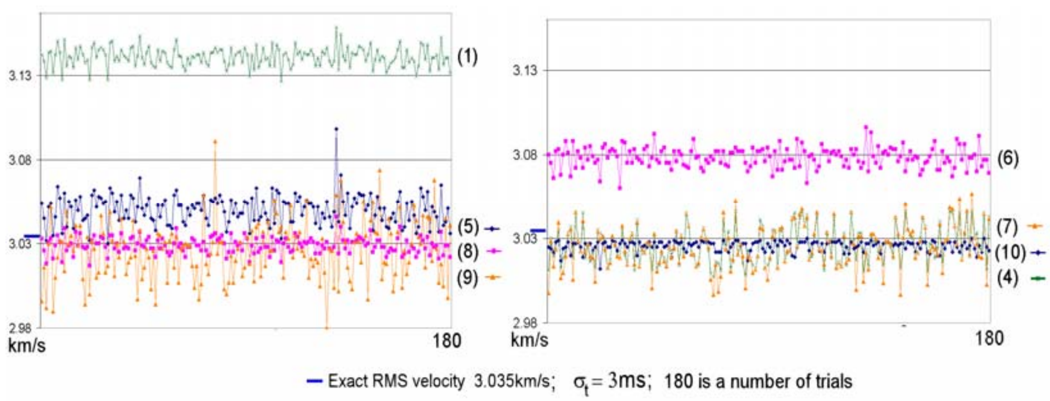

Figure 3. RMS velocity estimations through different NMO approximations in the presence of random noise

Formulas (8) and (9) include average velocity. It implies that we can determine a reflector depth using average velocity instead of RMS. It might lead to a more accurate depth estimation from seismic data. Model data shows that for the offset/depth ≈2, depth estimations, formulas (8) and (9) give the most accurate depth estimation. Fig. 4 shows depth belocity model. Fig. 5 displays depth estimation residuals, determined using different formulas.

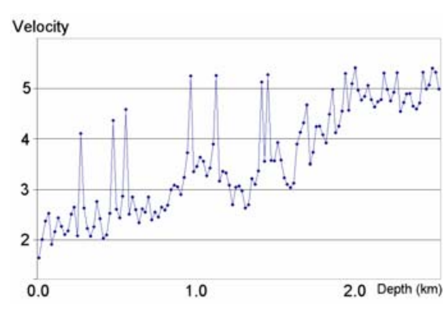

Figure 4. Depth velocity model

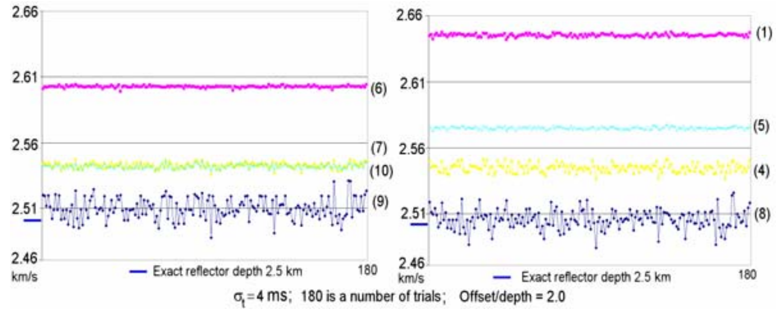

Figure 5. Depth estimation residuals for different NMO approximations

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