Multi-Step Auto-Regressive Reconstruction of nonuniformly Sampled, Aliased Seismic Records

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Summary

A methodology for the reconstruction of nonuniformly sampled, aliased data is introduced. First, the low frequency (nonaliased) part of the data is reconstructed. Using data inside the reconstructed band, a Multi-Step Auto-Regressive (MSAR) operator extracts the ensemble of prediction filters that are used to reconstruct the high frequency portion of the data. The applicability of the MSAR method to synthetic and real seismic data is discussed.

Introduction

Reconstruction of seismic data using statistical approaches is one of the ongoing research topics in exploration seismology. While they are based on statistical estimation theory, they also utilize information from the physics of wave propagation by taking into account proper apriori information and assumptions. Methods proposed by Spitz (1991), Porsani (1999) and Gulunay (2003) successfully address the problem of removing alias from regularly sampled data. These methods utilize low frequency information to recover high frequency data components. Spitz (1991) computed prediction filters (Auto-Regressive operators) from low frequencies to predict interpolated traces at high frequencies. This methodology is applicable only if the original seismic section is regularly sampled in space. Irregularly sampled data can be reconstructed using Fourier methods. In this case the Fourier coefficients of the irregularly sampled data are retrieved by inverting the inverse Fourier operator with band limiting (Duijndam et al., 1999) and/or sparseness constraints (Liu and Sacchi, 2004).

We introduce a new strategy that combines the strengths of both prediction error methods and Fourier based methods to cope with the problem of reconstructing nonuniformly sampled, aliased data. The proposed algorithm involves the reconstruction of spatial data at low frequencies. The reconstructed low frequency portion of the data is used to extract a suite of prediction error filters (PEFs). Then, the extracted PEFs are used to reconstruct the aliased part of the data in Fourier spectrum.

Theory

Consider a spatio-temporal window of seismic data composed of linear events and missing traces. First by applying the Discrete Fourier Transform (DFT) in the time direction, the data are transformed to the *f-x* domain. Suppose $\mathbf{x}$ is the length-N vector of *f-x* data sampled on a regular grid $x_1, x_2, x_3, ..., x_N$. The vector $\mathbf{y}(f) = [x_{n(1)}(f), x_{n(2)}(f), x_{n(3)}(f), ..., x_{n(M)}(f)]$ indicates the observations, where the set $\mathbf{M} = \{n(1), n(2), n(3), ...n(M)\}$ points out the position of the known samples. First we reconstruct the alias free part of the spectrum using Minimum Weighted Norm Interpolation (MWNI) (Liu, 2004; Liu and Sacchi, 2004). We indicate the minimum and maximum frequencies reconstructed via MWNI as $f_{\min_r}$ and $f_{\max_r}$, respectively. This means the number of frequencies inside the reconstructed band of low frequencies is equal to $\max_r - \min_r + 1$. The prediction filters at αf frequency can be computed by applying Multi-Step forward and backward AR modeling at frequency f:

$$x_k(f) = \sum_{j=1}^{L} P_j(\alpha f) x_{k-\alpha j}(f) \qquad k = \alpha L + 1, \ldots, N, \tag{1}$$

$$x_k^*(f) = \sum_{j=1}^{L} P_j(\alpha f) x_{k+\alpha j}^*(f) \qquad k = 1, \ldots, \alpha N - L, \tag{2}$$

where $f = f_{\min_r}, f_{\min_r+1}, \ldots, f_{\max_r}$, L is the length of the AR operator, k is the trace number, P is the prediction filter components, N is the number of traces and α is the step factor used to indicate that the prediction filter utilized to reconstruct the frequency $f' = \alpha f$ is extracted from reconstructed data at frequency f. It is clear that for a given frequency f' more than one prediction filter can be extracted. In our numerical implementation we have utilized the average prediction filter computed from all possible pairs α, f leading to $f' = \alpha f$.

Following Spitz (1991) we can use the prediction filters extracted from low frequencies to reconstruct the high frequencies:

$$\tilde{\mathbf{A}}(\mathbf{P}(f)) \begin{bmatrix} x_{m(1)}(f) \\ x_{m(2)}(f) \\ \\ x_{m(N-M)}(f) \end{bmatrix} = \tilde{\mathbf{B}}(\mathbf{P}(f)) \begin{bmatrix} x_{n(1)}(f) \\ x_{n(2)}(f) \\ \\ x_{n(M)}(f) \end{bmatrix}. \tag{3}$$

The notation $\tilde{\mathbf{A}}(\mathbf{P}(f))$ and $\tilde{\mathbf{B}}(\mathbf{P}(f))$ reflects the fact that these two matrices only depend on the coefficients of the prediction filter. The missing samples are computed using:

$$\mathbf{x}_m = [\tilde{\mathbf{A}}^*(\mathbf{P}(f))\tilde{\mathbf{A}}(\mathbf{P}(f))]^{-1} \tilde{\mathbf{A}}^*(\mathbf{P}(f))\tilde{\mathbf{B}}(\mathbf{P}(f))\mathbf{x}_n. \tag{4}$$

where $\mathbf{x}_m$ and $\mathbf{x}_n$ indicate the unknown and know samples, respectively. The matrix $\tilde{\mathbf{A}}^*$ stands for the transpose and complex conjugate of $\tilde{\mathbf{A}}$.

Examples

In order to examine the performance of the MSAR reconstruction a synthetic section with three linear events, two of them severely aliased, is used after removing 60% of the traces. Figures 1a and 1b show the original section and the section of the missing traces, respectively. The section of the missing traces was reconstructed using both MWNI and MSAR and the results are shown in Figures 1c and 1d, respectively. Figures 2 and 3 show the data in the *f-k* and *f-x* domains. Figures 4a and 4b show the portion of the Fourier spectrum reconstructed with MWNI and used for computing the prediction filters used by the MSAR reconstruction (Figure 1b). Finally, figure 5 shows the number of prediction filters contributing to the reconstruction of each frequency `f'`.

Conclusions

We have proposed an algorithm to interpolate aliased and irregularly sampled data. The method involves the reconstruction of the low frequency portion of the data spectrum using Fourier based reconstruction. The reconstructed low frequencies are used to extract prediciotn error filters that are used to recontruct the high (aliased) frequency portion of the data. An attractive feature of the method is that it does not require the assumption of sparsity in the *f-k* domain. This might be an advantage in certain situations, in particular, when dealing with data windows containing waveforms with complicated *f-k* signatures.

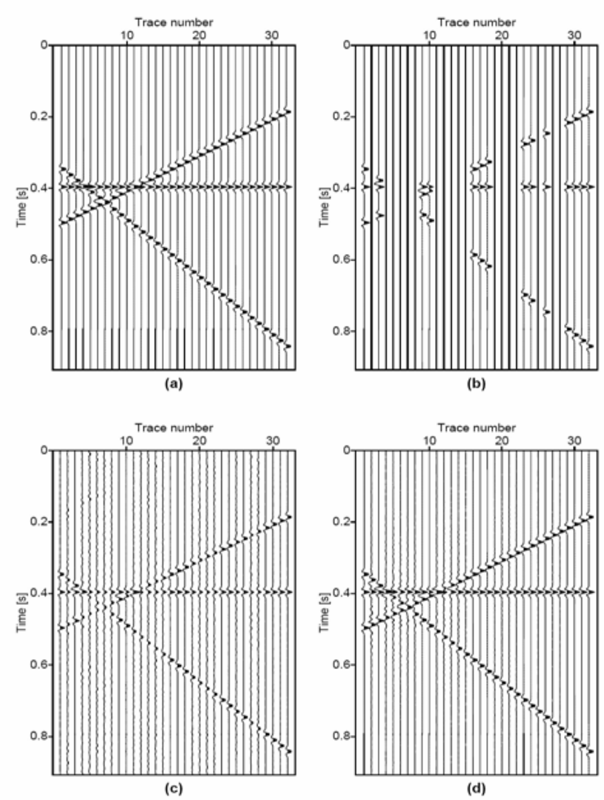

Figure 1. Synthetic section composed of three linear events.
a) Original data.
b) The data after removing 60% of the traces.
c) Reconstructed section using MWNI,
d) Reconstructed section using MSAR.

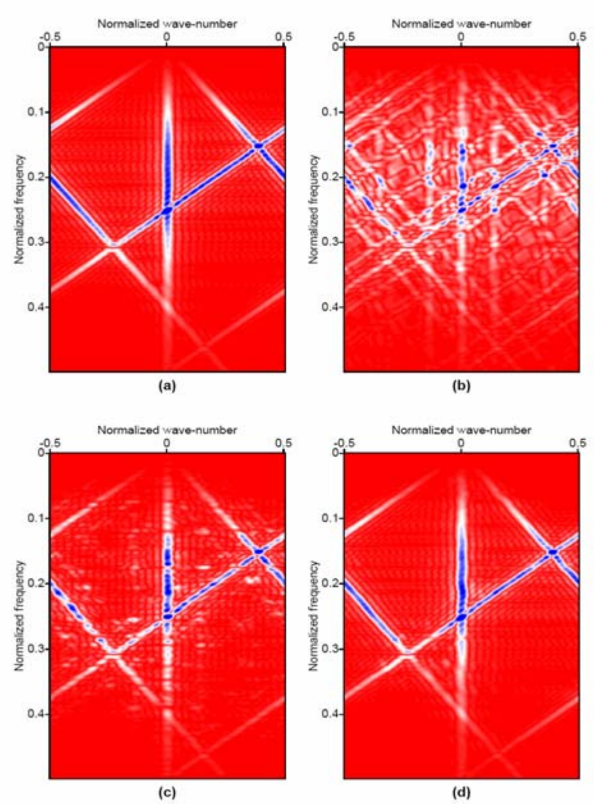

Figure 2. The f-k representation of Figure 1.

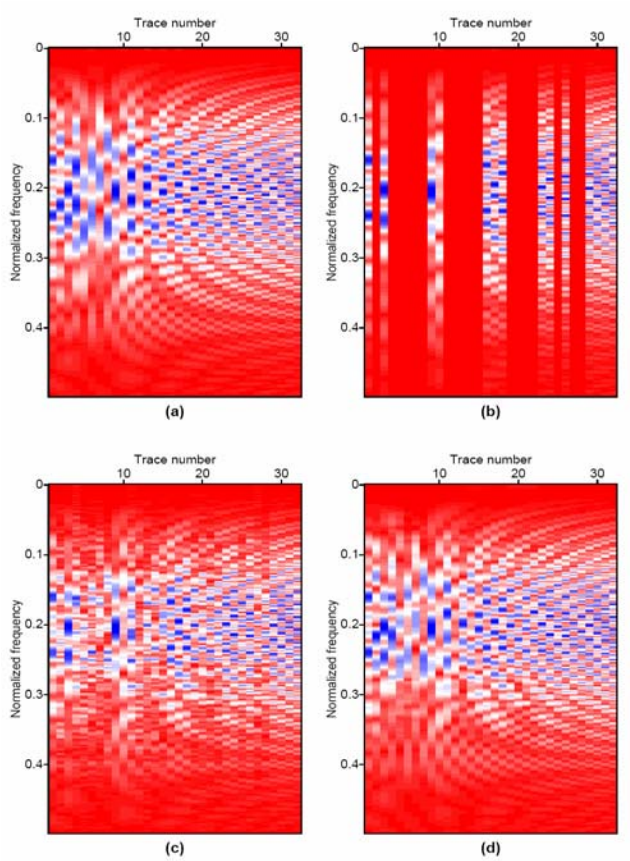

Figure 3. The f-x representation of Figure 1.

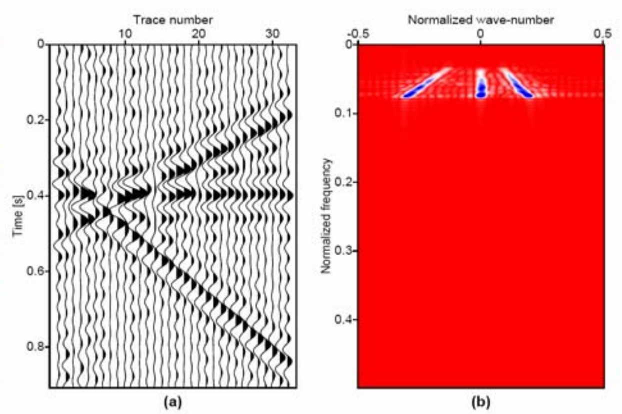

Figure 4. MWNI reconstruction of the low frequency portion of data the data.
a) t-x domain.
b) f-k domain. Prediction error filters utilized to reconstruct the data in Figure 1d where retrieved from these data.

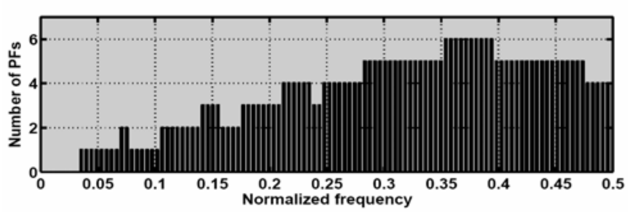

Figure 5. The number of prediction filters contributing to each frequency component in order to reconstruct Figure 1d.

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