TTI Wave Equation Migration by Phase-Shift Plus Interpolation

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Summary

We describe a phase-shift plus interpolation (PSPI) method for wave-equation migration in TTI media. To apply the PSPI methodology for anisotropy, we generate reference operators based upon phase error criteria with respect to the symmetry axis direction, and exploit correlations between parameters. The method is demonstrated on an elastic synthetic dataset generated over a thrust-belt setting, as found in the Canadian Foothills.

Introduction

Many hydrocarbon reservoirs, such as those in the Rocky Mountain Foothills of western Canada, lie below dipping clastic sequences characterized by tilted transverse isotropy (TTI) (Isaac and Lawton, 2004). Several authors (e.g. Vestrum et al., 1999) have shown the importance of accounting for the tilt of the symmetry axis when imaging such reservoirs using anisotropic depth migration, in order to correctly locate structures laterally. To realize this goal, typically Kirchhoff algorithms have been upgraded to handle TTI.

As for isotropic migration, superior results for significantly greater effort are expected from the use of wave-equation migration methods on TTI data. In contrast to ray-tracing based methods, wave-equation migration is able to handle multi-pathing in a natural way, and is not based upon a high-frequency approximation to the wave equation. Shan and Biondi (2005) have demonstrated both 2-D and 3-D implementations of TTI wave-equation migration, using an implicit operator with explicit correction, applied in the space-frequency (x-y-f) domain. However, efficient x-y-f methods are usually based on circular symmetry. This luxury is not obviously available for TTI, which lacks such symmetry.

An alternative approach to wave-equation migration is based upon applying phase-shift operators in the wavenumber-frequency (k-f) domain. This choice has advantages of operator stability and accurate steep dip behaviour. The main drawback, compared to x-f migration, is that lateral variations in the medium are not naturally accommodated by k-f domain operators. For isotropic migration, a number of methods have been proposed to address this issue, including: phase-shift plus interpolation (PSPI), split-step and Fourier finite difference. Generally, all of these are based upon the idea of migrating with a number of reference velocities and interpolating results, possibly after some correction. We first outline the basic phase-shift operator, and then describe how these PSPI-type methods can be adapted for the TTI algorithm.

Phase-Shift Operator

The acoustic VTI approximation (Alkhalifah, 1998) is given by setting the vertical shear-wave parallel to the symmetry axis equal to zero, to obtain the dispersion relation between the horizontal wavenumbers k_x and k_y, and the vertical wavenumber k_z:

$$k_z^2 = \frac{\omega^2}{V_{P0}^2} \frac{\omega^2 - V_{nmo}^2 (1+2\eta)(k_x^2 + k_y^2)}{\omega^2 - 2V_{nmo}^2 \eta (k_x^2 + k_y^2)}, \tag{1}$$

where V_{P0} is the velocity parallel to the symmetry axis (vertical for VTI), $\eta = (\varepsilon - \delta)/(1+2\delta)$ for Thomsen (1986) parameters ε and δ, and $V_{nmo} = V_{P0}\sqrt{1+2\delta}$.

An acoustic TTI approximation can be obtained by rotating the coordinates of (1). The resulting dispersion is given by a quartic equation (Shan and Biondi, 2005)

$$Ak_z^4 + B(k_x,k_y)k_z^3 + C(k_x,k_y)k_z^2 + D(k_x,k_y)k_z + E(k_x,k_y) = 0\,, \tag{2}$$

where *A, B, C, D* and *E* are polynomials in k_x and k_y of degrees 0, 1, 2, 3 and 4, respectively. There are four solutions to this equation, of which two correspond to the desired P-waves (propagating in opposite directions). The appropriate P-wave solution is selected from these, and is used recursively to drive the extrapolation of the wavefield from depth *z* to *z+Δz*, via

$$P(k_x,k_y,z+\Delta z,\omega) = e^{ik_z\Delta z} P(k_x,k_y,z,\omega). \tag{3}$$

Equation (3) can be used to apply wavefield extrapolation for a medium that varies only with depth, approximating the variation by a series of homogeneous slabs, one for each depth step. In order to address lateral medium variation, equation (3) must be incorporated within a PSPI type of algorithm.

PSPI for TTI

The obvious and challenging problem in generalizing PSPI for use with anisotropic media is the so-called “curse of dimensionality”. Instead of a single parameter, namely velocity, which can be sampled to define reference operators, we have 3, 4 or 5 parameters depending on whether we are dealing with VTI, TTI in 2-D or TTI in 3-D. We will consider the case of TTI in 2-D, characterized by V_{P0}, the velocity parallel to the symmetry axis, ε and δ, the Thomsen parameters referenced from the symmetry axis, and θ_S, the tilt of the symmetry axis in the plane of propagation.

At first glance, this 4-D parameter space appears potentially intractable. For example, if 5 values are required to adequately represent variation of each parameter within a depth slice, then the total number of reference operators would be 625 within that slice. However, some simple observations imply that this number may be reduced by at least an order of magnitude. First, in many cases there is strong correlation between different parameters, so that the effective dimensionality of the parameter space is reduced. For example, if ε and δ are not assumed to vary independently, it is possible to sample them using a single axis instead of two axes. In practice, limitations on our ability to estimate δ often make this assumption necessary. A second helpful observation is that sampling of θ_s does not need to be equal or uniform for all values of ε and δ. The required sampling of θ_s can be computed, for a given phase error, $\Delta\xi$, by using the relationship

$$\Delta\theta_s = \left(\frac{\partial\xi}{\partial\theta_s}\right)^{-1}\Delta\xi , \qquad \text{where} \qquad \xi = \sqrt{p_x^2 + p_z^2} ,$$

with p_x and p_z representing the horizontal and vertical slowness values respectively, such that $k_x = \omega p_x$ and $k_z = \omega p_z$. A suitable value for $\Delta\xi$ can be obtained by wavelength considerations. Finally, even after computing appropriate combinations of reference parameters using the above, there will still be unused parameter combinations, which may therefore be omitted. Combined, these reductions may allow PSPI extrapolation for TTI at an acceptable cost.

Test on Model Data

An elastic finite difference code was used to generate synthetic shot records, for a model based on thrust belt geology found in the Foothills of the Canadian Rockies. The model contains high velocity Mississippian carbonates (pink colours in figure 1), which have caused displacement and tilt of the overlying Cretaceous clastic sediments (purple to green colours), resulting in a complex TTI overburden. The P- and S-wave velocities and Thomsen parameters ε and δ are constant within each geological unit of the model, with $\delta = 0.05$ and $\varepsilon = 0.1$ for the whole clastic sequence. Both are equal to zero in the carbonates. The symmetry axis varies between –40° and 50°.

The data were migrated using the TTI migration described above and an isotropic WE migration. The results are shown in figure 1, overlaid on the velocity model (in colour), with the velocities being those parallel to the symmetry axis (i.e. V_{P0}). The TTI migration was performed using the full set of model parameters. The isotropic migration used only the velocities along the symmetry axis direction. The results demonstrate the necessity to properly account for tilted anisotropy, and also show the high fidelity result that can be obtained using wave-equation migration. The isotropic migration misplaces events and suffers from poor focusing of the basement reflections. A VTI migration (not shown) produced results which were in general no better than isotropic migration. Furthermore, TTI Kirchhoff migration produced results which were properly imaged, but much noisier.

Conclusions

We have described a PSPI type approach to TTI wave-equation migration which has a high fidelity response. The lateral variation of the medium parameters can be accounted for by a careful choice of reference operators to sample the parameter space optimally, using a phase error criterion with respect to the symmetry axis direction. Results on an elastic synthetic dataset for a complex thrust-belt model show superior imaging from the TTI algorithm.

References

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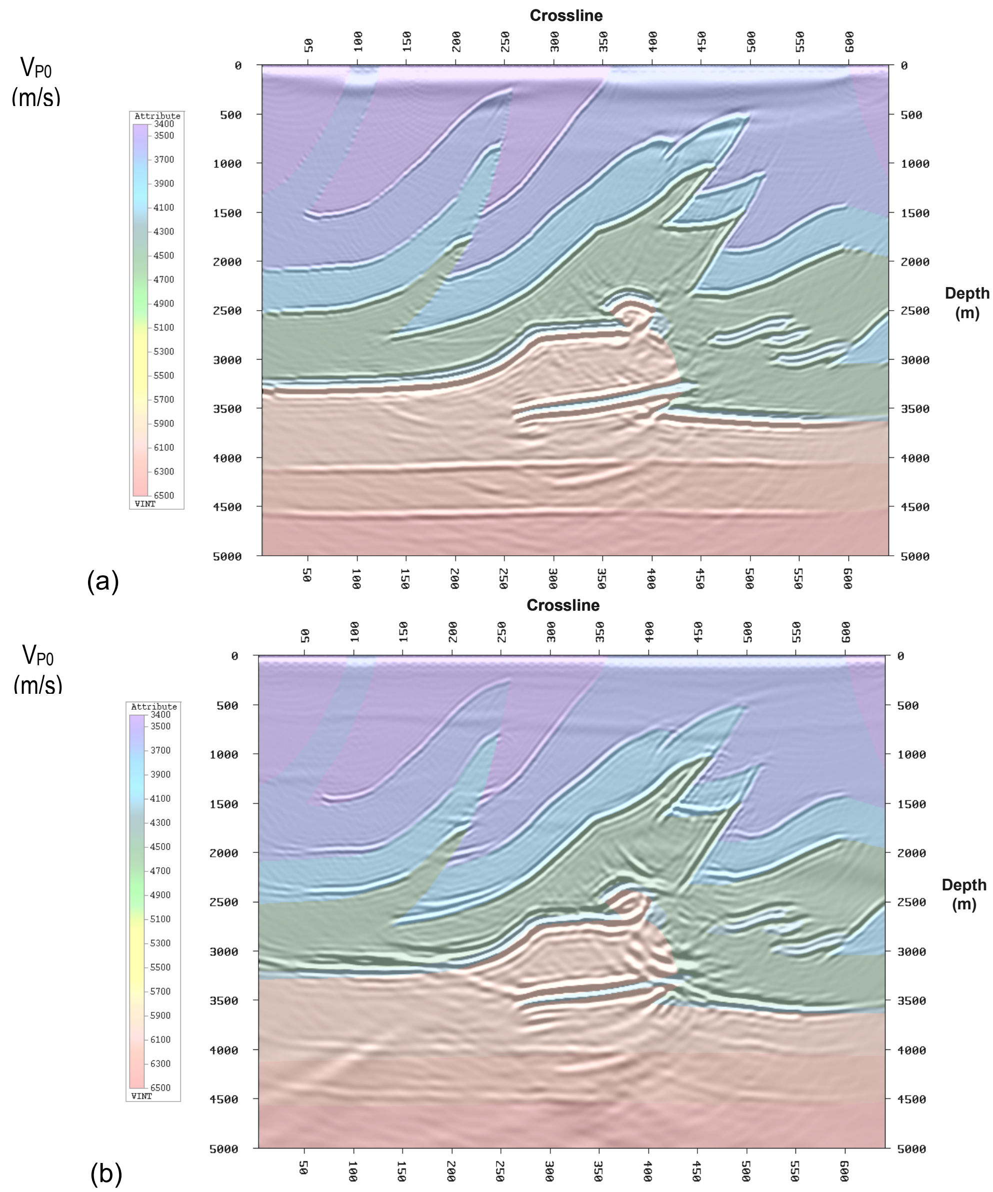

Figure 1. Results of wave-equation migration on elastic modeled data, using: (a) TTI PSPI algorithm described in text, and; (b) isotropic PSPI migration. Both results are overlaid on the velocity model for comparison, ranging from 3400m/s to 6500m/s.