Migration Amplitude Recovery using Curvelets

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In this paper, we recover the amplitude of a seismic image by approximating the normal operator and subsequently inverting it. Normal operator (migration followed by modeling) is an example of pseudo-differential. curvelets are proven to be invariant under the action of pseudo-differential operators under certain conditions. Subsequently, curvelets are forming as eigen-vectors for such an operator. We propose a seismic amplitude recovery method that employs an eigen-value decomposition for normal operator using curvelets as eigen-vectors and to be estimated eigenvalues. A post-stack reverse-time, wave-equation migration is used for evaluation of the proposed method.

Introduction

In mid-90s, Hart Smith[1,2] introduced elements or shapes which are invariant under the action of pseudo-differential. His theory came to practice by design of such an element and its implementation by Candes and Donoho [3]. Since then it has been used in different area of signal and image processing. Candes and Demanent [4] showed how curvelets behave under the action of Fourier integral operators. They stated that under some condition a curvelet maps to another curvelet under the action of Fourier Integral Operator (FIO). Migration is a FIO in weak sense. It means that first order born approximation for the modeling operator obeys the FIO principles. When migration followed by modeling operator they formed normal operator which belong to an operator's class called psudo-differential operators[Stein 5]. Inverting such an operator is in common interest for many researchers in seismic imaging discipline [Mulder, 6] [Claerboat, 7]. By having an almost accurate inverse of such an operator, the amplitudes of the reflectors can be recovered in a seismic image as well as the weak events in the seismic image can be illuminated.

In the following sections, we show the action of normal operator on different curvelets with different locations, frequency supports, angles and scales. We show that under some conditions those curvelets can be considered invariant under the action of such an operator. Following that we propose an eigen-value decompoisition solution for the normal operator using curvelets as eigen-vectors. Finally, we investigate the performance of the method on the amplitude recovery of an imaging problem. In this work, we use a post-stack, two-way, wave-equation migration and modeling.

Curvelets Invariance

A curvelet φ_μ is defined by its index μ which is a triple (j,l,k) with $j=0,1,2,...$ is a scale parameter; $l=0,1,...,2^{[j/2]-1}$ is an orientation parameter ($[x]$ is the integer part of x); and $k=(k_1,k_2)$, $k_1,k_2 \in Z$ is the location parameter.

A distinguished feature of curvelet is that the action of a pseudo-differential operator(ψ) on curvelet elements is in some sense very simple. Roughly speaking, a curvelet φ_μ is mapped into another curvelet at a corresponding index μ', which is close, or identical to μ. To be more specific, a pseudo-differential operator induces a mapping $\mu \to \mu'$ with property that significant curvelet coefficients of $\psi(\varphi_\mu)$ are located very close to μ, itself. Following theorem exists under certain conditions,

Theorem 1. Let ψ be a pseudo-differential operator with a symbol of order zero and ψ is a bounded L_2-operator. Then following error bound exists,

$$\left\| \psi(\varphi_\mu) - a(\mu)\varphi_\mu \right\|_{L_2} \le C_N 2^{[j/2]} \qquad (1)$$

Because of space constraints, we cannot possibly give a proof for theorems 1. However, we can refer to the works have been done in the micro-local analysis of the pseudo-differential operators [6,7,8].

Decomposition

In seismic imaging, Normal operator is defined as action of migration followed by de-migration. This is an important operator since its inverse can correct for the inaccuracy created by migration process. Normal operator defined by $(\psi = K^T K)$, where K is the modeling operator (de-migration) and K^T is the migration operator.

Restating the theorem 1 we propose following decomposition of normal operator

$$\psi = C^T D_\mu C \qquad (2)$$

D_μ is a diagonal matrix which contains the value of $a(\mu)$ for each index μ, C and C^T are curvelet and transpose of curvelet transform.

This decomposition is direct consequence of theorem 1, which states that each curvelet acts as an eigen-vector for the normal operator. Thus we can decompose the normal operator in a form of eigen-value decomposition with curvelets as eigen-vectors and unknown eigen-values.

Estimation of Eigen-Values

To build D_μ in eq.(2), we need to calculate $a(\mu)$ for each index μ which is infeasible. We propose a different method, which utilizes a reference reflectivity vector, r and solve following equation on elements of D_μ,

$$\psi r = C^T D_\mu C r \ \to C^T diag(u)\, d_\mu = \ \psi r \qquad (3)$$

Where $u = Cr$ and d_μ is the diagonal elements of D_μ.

We choose the reference vector r as migrated image which is our closest possible guess to the ideal choice (i.e., original reflectivity). We need to solve Eq.(3) for d_μ, however this equation is underdetermined since the number of equations (size of model in physical domain) is less than number of unknowns (number of elements $a(\mu)$). The reason is that the curvelet transform is redundant. To find a unique solution for d_μ, we impose additional constraints on it.

We impose a penalty term that avoids abrupt changes in the adjacent elements in the curvelet basis. In other word, we solve following optimization problem for d_μ

$$\min_{d_\mu} \lambda_\theta \| D_\theta d_\mu \|_{L_2} + \lambda_x \| D_x d_\mu \|_{L_2} + \lambda_y \| D_y d_\mu \|_{L_2} \quad subject\ \ to\ \ C^T diag(u) d_\mu = \psi r \quad (4)$$

Where D_θ, D_x and D_y are the differentiation matrices along θ, x and y coordinates in curvelet space. $\lambda_{[\bullet]}$ are the Lagrange multipliers associated with each differentiation.

There is a simple solver for above optimization problem by solving following linear system of equations using linear solvers.

Examples

Figure 1 shows three different curvelets in different locations and angles before and after applying the normal operator. The curvelet with steep angle does not survive after the action of operator since it is not in the aperture of the operator. However, the curvelets that are in the support of the operator remain invariant under the action of normal operator. It can be seen that curvelet with steep angle is more attenuated than the flat curvelet. This can be corrected during the operator decomposition.

Figure 2 shows the performance of our proposed method to approximate the action of normal operator and to recover the amplitudes by inverting it. Figure 2(a) shows the reflectivity model which is SEG-AA' salt model. Figure 2(b) shows the action of normal operator (migrated-demigrated) on the reflectivity shown in (a). The background velocity is SEG-AA' velocity model which is sufficiently smoothed. Figure 2(c) shows the action of approximated normal operator using our proposed method on the reflectivity shown in (a). Relative error in this case is 10%. Figure 2(d) shows the amplitude recovered image by applying the inverse of the approximation ($\psi^{-1} = C^T D_\mu^{-1} C$) on the migrated image.

Conclusion

This work introduces a fast and robust approach for approximation of imaging operator, which can be used in amplitude recovery in seismic image. We formulated the approximation of normal operator as eigenvalue decomposition with curvelets as eigenvector. Applying the inverse of the decomposed operator on migrated image recovers image amplitude. Our proposed method is sufficiently faster than other krylov-based (i.e., CG, LSQR) methods since it evaluates the normal operator only once. In other paper of this proceeding we propose an inversion approach that utilizes this approximation to further enhance the SEG-AA' salt image.

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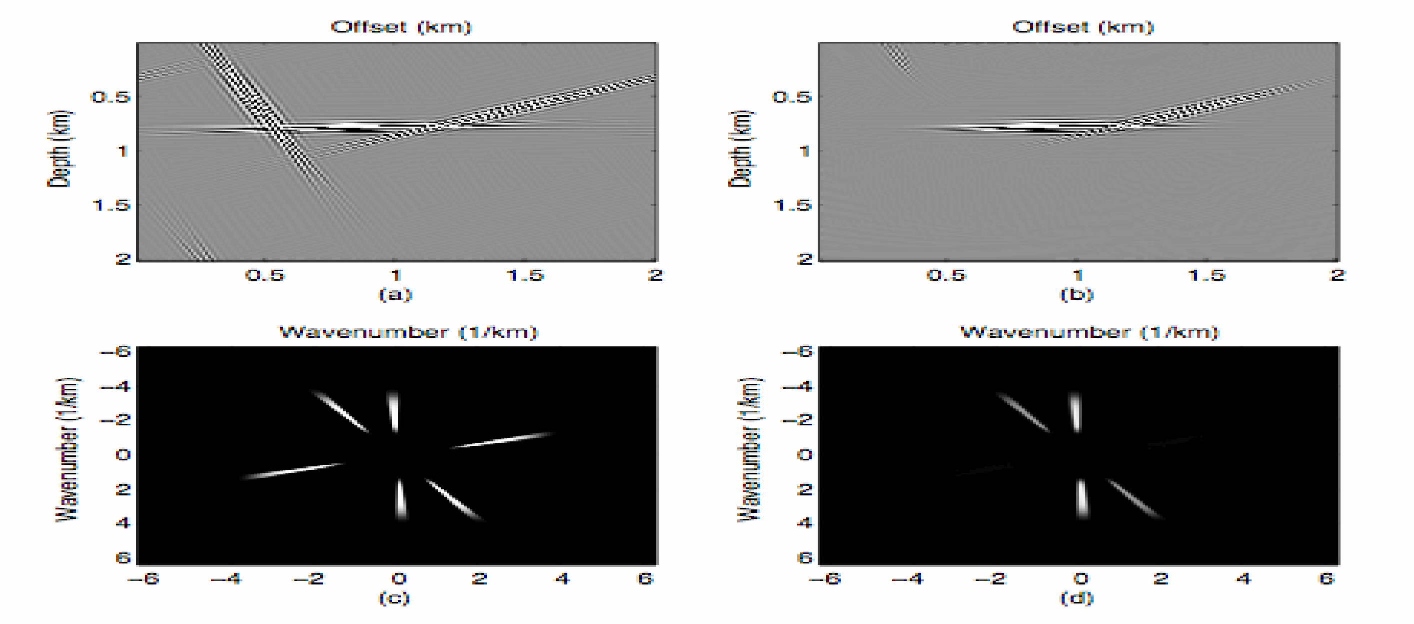

Figure 1. Invariance of curvelets under the action of normal operator, (a) three curvelets (same scale, different angles and positions), (c) Spectrum of the three curvelets shown in (a), (b) Normal operator (migration followed by modeling) applied on three curvelets shown in (a), background velocity is lens model, (d) Spectrum of (b)

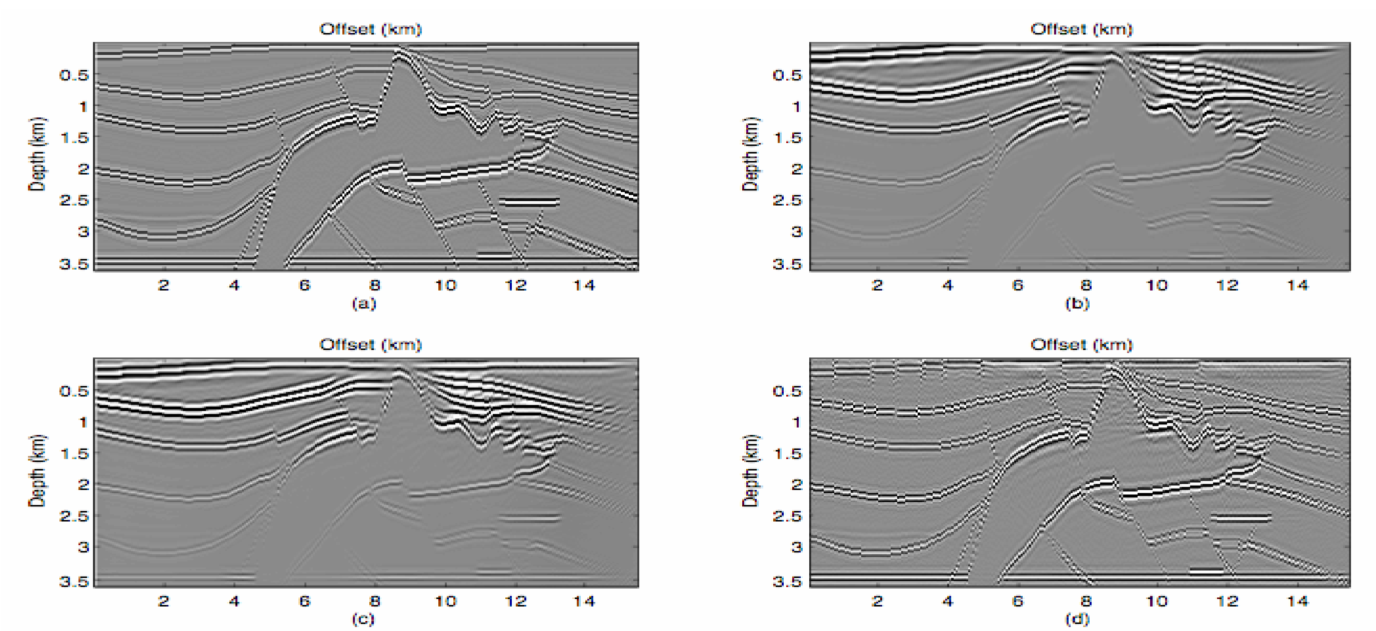

Figure 2. Amplitude recovery using eigenvalue decomposition of the normal operator, (a) SEG-AA' reflectivity model, (b) action of normal operator on (a), (c) action of approximated normal operator on (a), (d) amplitude recovered image applying the inverse of approximated normal operator on (b)