High-Resolution Deconvolution with Sparsity and Lateral Continuity Constraints

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In this paper we propose a stable high-resolution deconvolution algorithm that combines *a priori* information both within and across seismic traces. Specifically, we assume the reflectivity series is sparse in the time domain but continuous in the space domain. We encode such information in the form of sparse constraints within the trace and spatial smoothing constraints across the trace in the inverse problem. In particular, for the former we use the Cauchy norm to suppress noise, and for the latter we use adaptive FX filtering to enhance the coherence of seismic events across midpoints. The combination of adaptive FX filtering and sparse inversion provides multi-channel solutions that are sparse in the vertical direction and coherent in the lateral direction. The robustness of this technique is validated by tests on both the synthetic wedge model and some real data.

Introduction

Deconvolution can be posed as an inverse problem in which we attempt to remove the signature of the wavelet and therefore increase the resolution. In practice we assume that a seismic trace can be modeled by convolving the reflectivity series with a wavelet and then adding some noise. Deconvolution problems can be classified into deterministic and statistical approaches, depending on whether the wavelet is known or unknown. In this paper, we assume that the wavelet is known and band-limited, having been obtained via wavelet extraction based on well log information or perhaps alternatively via the relaxation method (Canadas, 2002). In the context of land production processing, the proposed technique would typically be applied poststack (i.e., after surface-consistent deconvolution and time-variant spectral whitening have been run in an effort to remove trace-by-trace wavelet fluctuations). Thus, our wavelet can be thought of as the embedded wavelet which exists after conventional wavelet processing. The goal of this paper is to reconstruct the sparse reflectivity series by collapsing this wavelet in the presence of noise.

In recent years, inversion with sparse constraints has been applied to the high-resolution Radon transform (Sacchi and Ulrych, 1995), deconvolution (Oldenburg et al., 1983; Debeye and van Riel, 1990) and least-squares migration (Wang and Sacchi, 2006). One challenge for sparse inversion is that the algorithm can be unstable in the presence of noise and imperfect forward modeling. A remedy to this problem is to impose spatial smoothness constraints in the inversion; however when the geological structure is not flat, simple smoothing will smear the solution. In this paper, we present a new approach which combines adaptive FX filtering and sparseness constraints within the inversion process in order to better preserve and enhance subtle structures.

Theory

A common implementation of trace-by-trace sparse deconvolution (called "Cauchy trace-by-trace deconvolution" hereafter) solves the following cost function:

$$J = \| W\hat{r} - \hat{s} \|^2 + \lambda^2 F(\hat{r}), \tag{1}$$

where W is the convolution matrix containing the known wavelet, $\hat{r}$ is the reflectivity trace, $\hat{s}$ is the seismogram, λ is a trade-off parameter which controls the strength of the constraints, and F is a function to force sparsity. Following Sacchi and Ulrych (1995), we take F to be the Cauchy norm:

$$F(\hat{r}) = \sum_{i=1}^{n} \ln(1 + r_i^2 / \sigma^2), \tag{2}$$

where r_i is the ith element of the reflectivity model, and σ is a scale parameter. The problem can be solved by iterative reweighted least-squares (IRLS) (Scales and Smith, 1994), and the solution at the jth iteration can be expressed in closed form:

$$\hat{r}^j = (W'W + \lambda Q^{j-1})^{-1} W' \hat{s}^j, \tag{3}$$

where Q is a diagonal matrix associated with the reflectivity model, and the superscripts *j* and *j-1* denote iteration numbers. The ith diagonal elements of Q are calculated by

$$Q_{ii} = 1/(1 + r_i^2 / \sigma^2). \tag{4}$$

To make the algorithm more stable, we propose a new multi-channel cost function that embodies the simultaneous constraints of vertical sparseness and lateral continuity:

$$J = \sum_{k=1}^{NTR} (\| W\hat{r}_k - \hat{s}_k \|^2 + \lambda^2 F_k((L_{FX} R)_k)), \tag{5}$$

where $\hat{r}_k$ is the kth reflectivity trace, $\hat{s}_k$ is the kth seismic trace, F_k is the kth regularization function, L_{FX} is the FX filtering operator (Canales, 1984), NTR is the number of traces and R is the multi-channel data ($R = (\hat{r}_1, \hat{r}_2, ..., \hat{r}_{NTR})$). Since it is too expensive to minimize the proposed cost function directly, we break it down into small problems:

$$J_k = \| W\hat{r}_k - \hat{s}_k \|^2 + \lambda^2 F_k(\hat{z}_k), k = 1, NTR, \tag{6}$$

where $\hat{z}_k$ is a FX filtered version of $\hat{r}_k$ (or the kth trace of $L_{FX} R$). Below is a practical processing flow for the algorithm:

- Initialize the diagonal weighting matrices to identity matrices (i.e., ${}^{(k)}Q_{ii}^0 = 1,\ k = 1, NTR$.)
- Solve equation 3 for each seismic trace.
- Apply FX filtering to the deconvolved traces (i.e. to the $\hat{r}_k$ in equation 6).

- Update the diagonal weighting matrices with the FX filtered traces (i.e. replace $\hat{r}_k$ in equation 4 with $\hat{z}_k$ in equation 6).

We repeat the last three steps for 3-4 iterations to acquire a satisfactory solution. It is clear that we do not exactly solve the multi-channel deconvolution problem (equation 5). However by adaptive FX filtering, we gradually adjust the diagonal weighting and balance the energy among neighboring traces. The following tests with synthetic and real data show that the method is robust at suppressing noise and preserving structural information.

Synthetic Example

We evaluated the performance of the algorithm using a wedge model as input. We added significant noise to the data (S/N=2.0) (Figure 1a), then we compared the output of different processing flows. As shown in Figure 1b, the Cauchy trace-by-trace deconvolution efficiently removes the random noise. However, it also removes some coherent information. Another method (Figure 1c), FX filtering followed by trace-by-trace sparse deconvolution, preserves more signal but obviously still shows a degradation in lateral coherency. On the other hand, the proposed method provides a clean and coherent image (Figure 1d)

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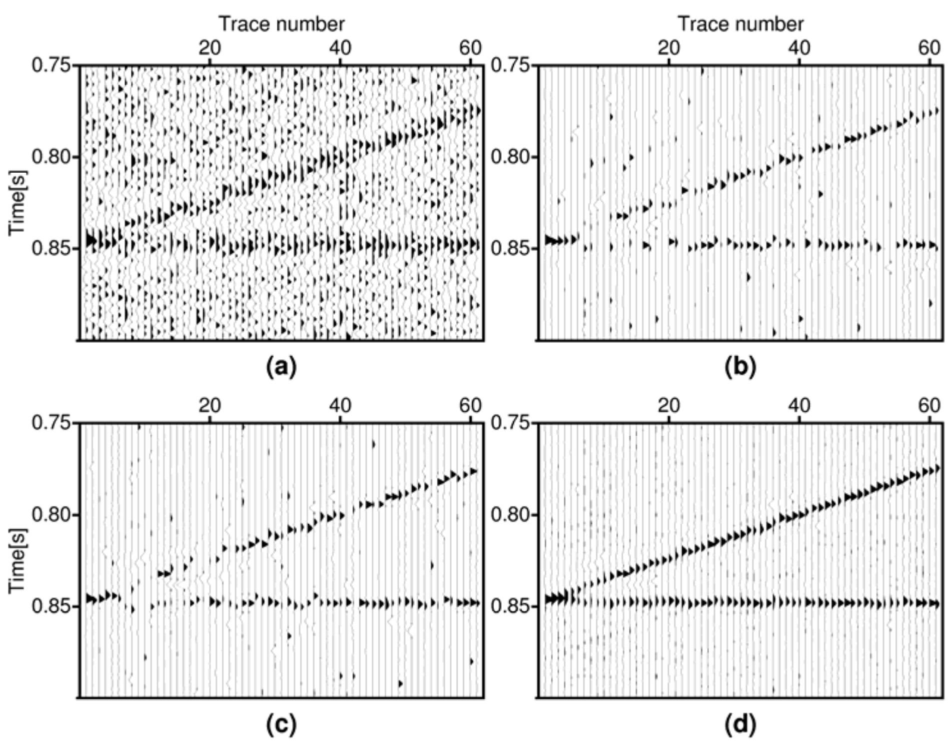

Figure 1. Wedge model data and deconvolution results by different methods. (a) Input data. (b) Cauchy trace-by-trace deconvolution. (c) FX filtering followed by Cauchy trace-by-trace deconvolution. (d) Sparse deconvolution with adaptive FX filtering implementation

Field data Example

We applied the algorithm to some 3D field data. Figure 2 compares the input data (a brute stack of one inline) and the output data. The input data is quite noisy and many events look discontinuous due to some suspicious vertical disturbance. It is clear that with sparsity and lateral continuity regularization, our algorithm has dramatically increased the image quality. All events look coherent in the lateral direction, and the signature of the wavelet is efficiently suppressed.

Conclusions and Discussion

We have proposed a robust sparse deconvolution algorithm by combining sparse regularization and FX filtering. Our synthetic and real data tests show that the method can preserve coherent information in various directions.

Although the method is more stable than Cauchy trace-by-trace deconvolution, it is not guaranteed to work well in all circumstances. When the assumption of the sparse reflectivity model is violated, the algorithm will suppress some coherent signal. For instance the algorithm may have difficulty preserving weak events. Interpreters should be aware of this shortcoming in any sparse inversion algorithm.

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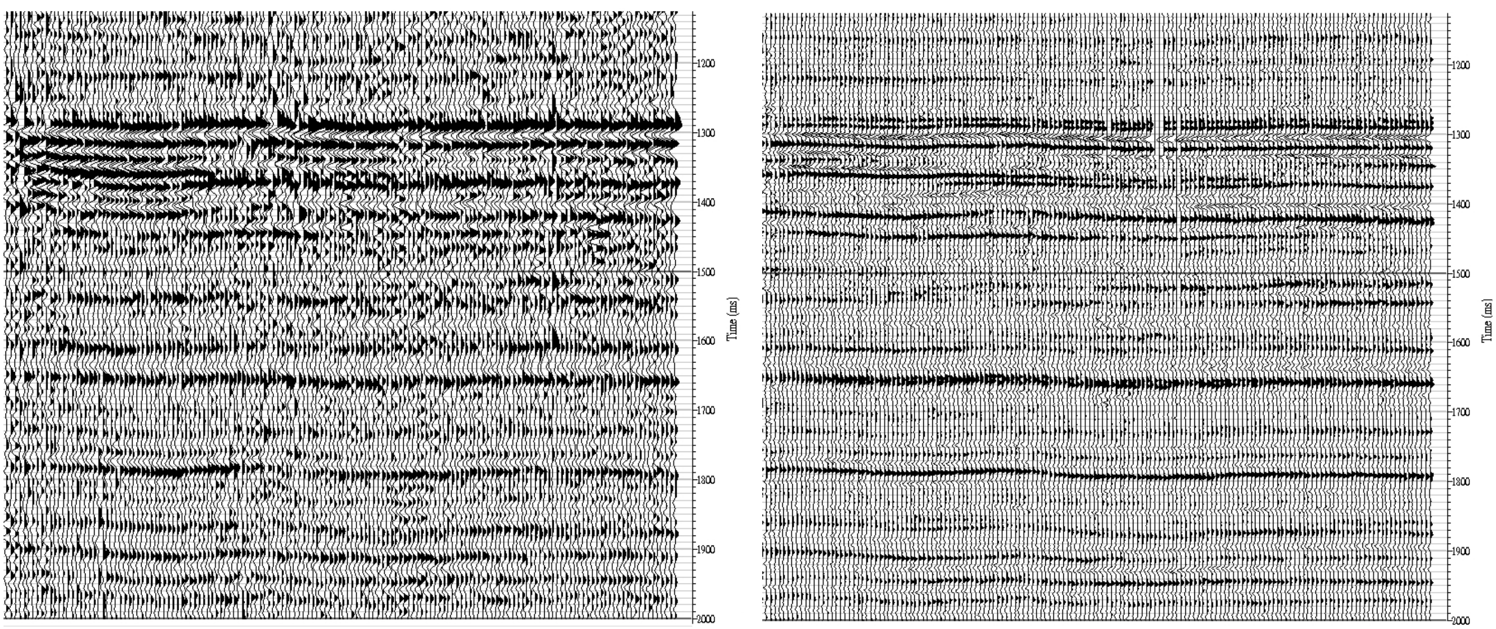

(a) (b)

Figure 2. Field data example. (a) Input data. (b) Result obtained with sparse deconvolution with adaptive FX filtering implementation.