F-xy Cadzow Noise Suppression

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Summary

Cadzow filtering has previously been applied along constant-frequency slices to remove random noise from 2-D seismic data. Here I extend Cadzow filtering to two or more spatial dimensions. The resulting method is superior to both f-xy prediction (deconvolution) and projection filtering, especially for very noisy data. In particular, it preserves signal better and can be made much harsher.

Introduction

Trickett (2002) described a method for removing random noise from 2-D seismic data based on Cadzow filtering of constant-frequency slices. It works as follows: Suppose we have a series of n equally spaced traces representing, say, a portion of a 2-D stacked section. For a given temporal frequency, extract the complex DFT value $t_1 \; t_2 \; \cdots \; t_n$ for every trace. Now apply Cadzow filtering (Cadzow, 1988) to this series by forming the matrix

$$\mathbf{A} = \begin{bmatrix} t_1 & t_2 & \cdots & t_{n-m+1} \\ t_2 & t_3 & \cdots & t_{n-m+2} \\ \vdots & \vdots & \iddots & \vdots \\ t_m & t_{m+1} & \cdots & t_n \end{bmatrix}$$

This has Hankel structure, meaning the matrix entries are constant along each anti-diagonal. By adjusting m we can change the matrix dimensions. A common strategy is to make the matrix as square as possible by choosing m = n/2. Now replace $\mathbf{A}$ with a rank-reduced approximation by taking the singular-value decomposition (SVD):

$$\mathbf{A} = \mathbf{U}\,\mathbf{S}\,\mathbf{V}^H$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices with column vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ respectively, H is the conjugate-transpose operator, and $\mathbf{S}$ is a real diagonal matrix with diagonal elements $s_i, \; i = 1, \ldots, p$ such that $s_1 \geq s_2 \geq \ldots \geq s_p \geq 0$. For some small integer k form the sum of the first k weighted eigenimages:

$$F_k(\mathbf{A}) = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_k, \quad \mathbf{I}_i = s_i \mathbf{u}_i \mathbf{v}_i^H$$

$F_k(\mathbf{A})$ is the nearest rank-k matrix to $\mathbf{A}$ with respect to both the Frobenius norm and matrix 2-norm. Recover the filtered DFT values $\tilde{t}_1 \; \tilde{t}_2 \cdots \tilde{t}_n$ by averaging the entries of each anti-diagonal of $F_k(\mathbf{A})$. Cadzow recommended iterating between the rank reduction and averaging steps, but I have not found it to give better results for this application. Once all frequencies within the signal bandwidth are noise reduced, take the inverse DFT of each trace.

I referred to the above method as f-x eigenimage filtering since it seemed to be a one-spatial-dimension analogy to f-xy eigenimage filtering (Trickett, 2003). I have since come to regret this label, however, and will henceforth refer to it as f-x Cadzow filtering.

It is easy to prove the following:

> *Theorem 1: If noiseless seismic data has no more than k distinct dips then f-x Cadzow filtering does nothing to it.*

The larger the value of rank k, the greater number of dips that can be faithfully preserved but the weaker the noise suppression. Thus the rank should be taylored to the properties of the data. This theorem does not hold for constant-time slices (that is to say, for t-x filtering), which is why Cadzow filtering is best applied in the frequency domain.

Method

It has proven fruitful to extend noise suppression methods to two or more spatial dimensions, in part because it allows more data to be accessed from a more localized area (Chase, 1992; Ozdemir, et al, 1999; Soubaras, 2000). In general this allows harsher noise reduction and better signal preservation. But how do we do this for f-x Cadzow filtering?

Suppose we have a rectangular p-by-q grid of seismic traces, taken, for example, from a stacked 3-D volume. These traces must be equally spaced along each direction, although the spacing need not be the same in both directions. For a given temporal frequency, suppose the grid values are

$$\begin{matrix} t_{1,1} & t_{1,2} & \cdots & t_{1,q} \\ t_{2,1} & t_{2,2} & \cdots & t_{2,q} \\ \vdots & \vdots & & \vdots \\ t_{p,1} & t_{p,2} & \cdots & t_{p,q} \end{matrix}$$

Following Dologlou, et al, (1996), define $\mathbf{A}$ as a Hankel matrix of Hankel matrices:

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_{p-r+1} \\ \mathbf{H}_2 & \mathbf{H}_3 & \cdots & \mathbf{H}_{p-r+2} \\ \vdots & \vdots & \iddots & \vdots \\ \mathbf{H}_r & \mathbf{H}_{r+1} & \cdots & \mathbf{H}_p \end{bmatrix} \quad \text{where} \quad \mathbf{H}_i = \begin{bmatrix} t_{i,1} & t_{i,2} & \cdots & t_{i,q-s+1} \\ t_{i,2} & t_{i,3} & \cdots & t_{i,q-s+2} \\ \vdots & \vdots & \iddots & \vdots \\ t_{i,s} & t_{i,s+1} & \cdots & t_{i,q} \end{bmatrix}.$$

We normally set r=p/2 and s=q/2 in order to make matrix $\mathbf{A}$ as square as possible. Now calculate the nearest rank-k matrix $F_k(\mathbf{A})$. Recover each filtered grid value by averaging all of the matrix entries in which its unfiltered grid value was placed.

I refer to the above method as f-xy Cadzow filtering. One can prove the 3-D version of Theorem 1:

> *Theorem 2: If a grid of noiseless 3-D seismic data is made up of plane waves having no more than k distinct dips then f-xy Cadzow filtering does nothing to it.*

It's obvious how to extend Cadzow filtering to more dimensions. In three spatial dimensions, for example, $\mathbf{A}$ is a Hankel matrix of Hankel matrices of Hankel matrices. Theorem 1 extends to any number of spatial dimensions.

Computation time can be a problem. Trickett (2003) describes how to avoid calculating the full SVD using Lanczos bidiagonalization. For f-xy Cadzow filtering, however, an r value of 8 or 10 is required for accuracy, rather than 2 or 3 as is suggested for f-xy eigenimage filtering.

Examples

Let's begin with an artificial example. Figure 1 shows an inline section of a 31-trace by 31-trace stacked volume with 3 events, the events having distinct dips in both spatial directions. To this we add so much random noise that only one of the events is clearly visible. Both f-xy prediction (deconvolution) and projection filtering suffer serious loss of signal no matter what parameters are used. F-xy Cadzow with rank k of 4 preserves far more signal.

Figure 2 shows an inline section from a real stacked 3-D volume. I set the parameters for the three filters so that the results have about the same amount of noise. Cadzow does a better job at preserving low-amplitude conflicting dips than either prediction or (especially) projection. Prediction and projection are also near the limits of their harshness, while Cadzow can be made far stronger (or as mild as you wish) simply by adjusting the rank.

Future Work

This opens the door for further work. With automatic rank determination (Harris and White, 1997), the strength of Cadzow filtering could be made to automatically vary with time, frequency, and space, perhaps resulting in a filter which is safe, powerful, and easy to use. More than two spatial dimensions is possible (e.g., f-xyz Cadzow filtering), and would likely be extraordinarily powerful. The real prize is noise reduction before stack. That shots are not equally spaced on land data is an issue, but this might be overcome by hybridizing Cadzow and f-xy eigenimage (Trickett 2003) filtering, since the latter method does not require equal trace spacing. Cadzow filtering might also be used for interpolation by exploiting algorithms for SVD's with missing data. Computation time can be a problem, particularly for prestack or f-xyz filtering, and so the development of faster low-rank matrix approximation algorithms is worthwhile.

Acknowledgements

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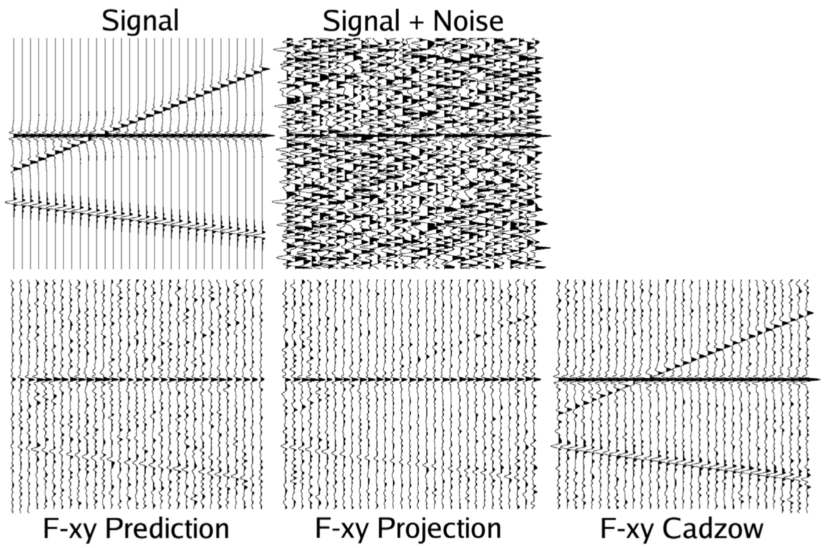

Figure 1: Inline section through a very noisy artificial 3-D volume with three dipping plane waves. Cadzow does a better at recovering the signal than prediction or projection.

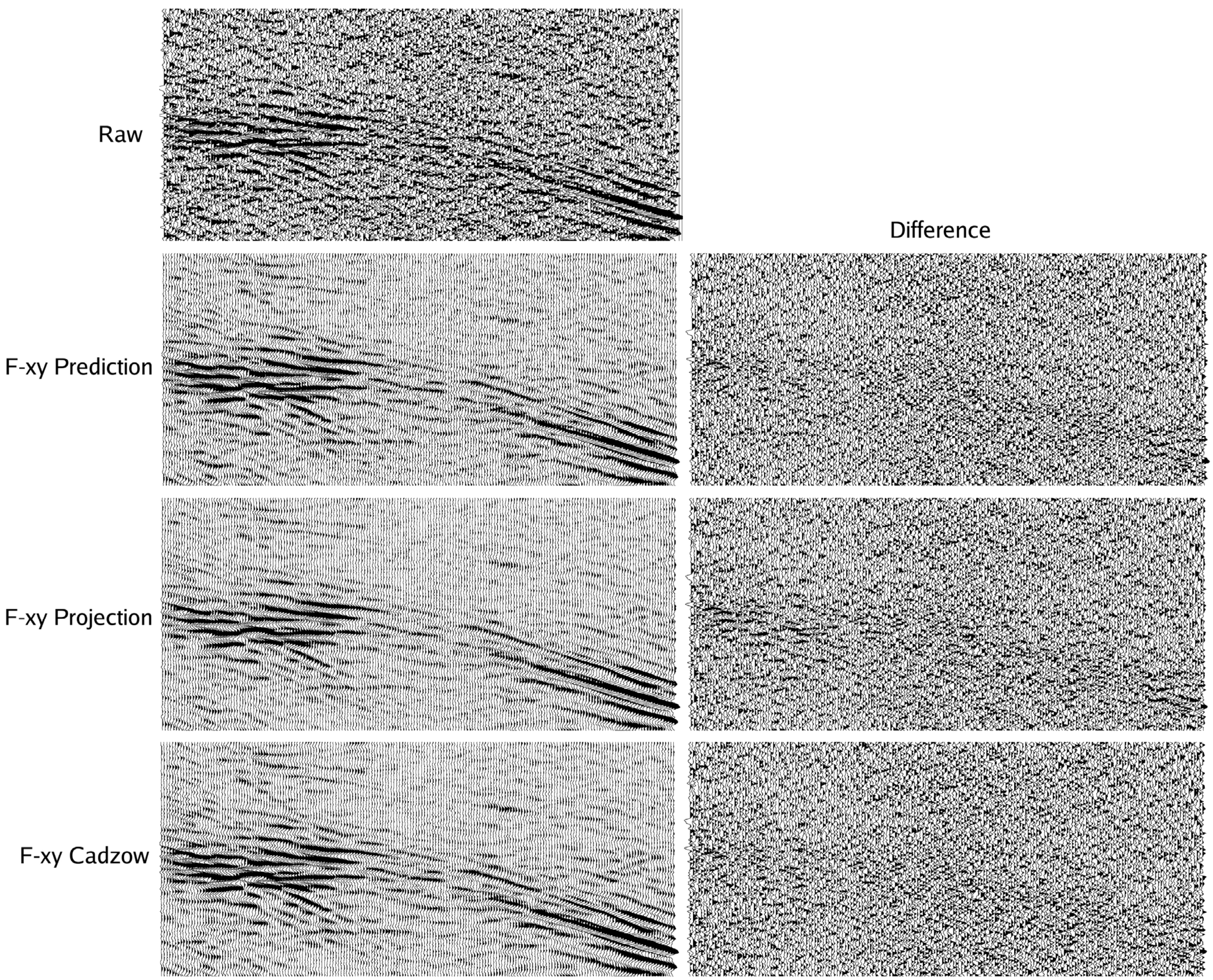

Figure 2: Inline section through a stacked 3-D volume. Both prediction and projection harm low-amplitude conflicting dips. They are also near the limits of their harshess, while Cadzow can be made far stronger.